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CONTROL OF THE SPECTRUM OF LYAPUNOV CHARACTERISTIC EXPONENTS IN NONLINEAR LARGE-SCALE SYSTEMS

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Abstract. The article deals with the control problem for a large-scale nonlinear system with chaotic dynamics based on a centralized and decentralized controller structure. The control is based on the feedback principle, which makes it possible to implement in a closed system a given spectrum of Lyapunov characteristic exponents to suppress chaotic dynamics and transfer the system to stable periodic movements or to a state of equilibrium. To change the spectrum, a modal control procedure is proposed, generalized for nonlinear large-scale systems. An example of the application of this technique to suppress chaotic oscillations in a system consisting of three synchronous generators is considered. Computational experiments confirm the workability of centralized and decentralized management. The article considers the use of the proposed method for the synthesis of decentralized control through the example of a system consisting of three synchronous generators. The results of the study confirmed the suppression of chaotic oscillations and the provision of a regular mode in a closed system. The advantage of the proposed decentralized control is the reduction of computational costs for the synthesis and implementation of control systems for large-scale systems.

Keywords: nonlinear large-scale systems, deterministic chaos, control of the spectrum of Lyapunov characteristic exponents, modal control, Sylvester's matrix algebraic equation

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УПРАВЛЕНИЕ СПЕКТРОМ ХАРАКТЕРИСТИЧЕСКИХ ПОКАЗАТЕЛЕЙ ЛЯПУНОВА В НЕЛИНЕЙНЫХ КРУПНОМАСШТАБНЫХ СИСТЕМАХ

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Аннотация. Рассмотрена задача управления нелинейной крупномасштабной системой с хаотической динамикой на основе централизованной и децентрализованной структуры регулятора. Управление строится по принципу обратной связи, позволяющей реализовать в замкнутой системе заданный спектр характеристических показателей Ляпунова для подавления хаотической динамики и перевод системы к устойчивым периодическим движениям или в состояние равновесия. Для изменения спектра предложена процедура модального управления, обобщенная для нелинейных крупномасштабных систем. Описано использование предлагаемой методики синтеза децентрализованного управления на примере системы, состоящей из трёх синхронных генераторов. Результаты исследования подтвердили подавление хаотических колебаний и обеспечение в замкнутой системе регулярного режима. Преимущество предлагаемого децентрализованного управления состоит в уменьшении вычислительных затрат на синтез и реализацию систем управления крупномасштабными системами. Синтезированная обратная связь обеспечивает подавление хаотических колебаний не в малой области фазового пространства, а в области существования решения уравнений динамики нелинейной системы.

Ключевые слова: нелинейные крупномасштабные системы, детерминированный хаос, управление спектром характеристических показателей Ляпунова, модальное управление, матричное алгебраическое уравнение Сильвестра

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Introduction

One of the most important problems in the modern theory of nonlinear systems is the development of methods for the analysis and synthesis of controls for chaotic dynamics. Systems of this class are of interest not only because of the abundance of new mathematical problems but also in connection with the broad applications of the theory of control of chaotic systems in solving practical problems. In some systems, the modes of deterministic chaos are useful, for example, in cryptography [1, 2], for others – harmful (vibrations of various structures [3, 4], chaotic oscillations in power systems [5, 6]). Therefore, one of the most important tasks of the theory of nonlinear dynamic systems is the development of methods for controlling chaos [7–9].

At present, approaches based on the development of methods of the theory of automatic control are used to solve control problems in nonlinear systems with deterministic chaos. Papers [10, 11] consider the application of the method of analytical design of aggregated controllers to the synthesis of nonlinear systems with chaotic dynamics. The synthesis of adaptive control as applied to systems of this class is presented in [12].

The study of chaotic regimes in electric power systems is considered in works [13, 14]. The synthesis of stabilizing control in small energy systems is considered in [15, 16]. The work [17] is devoted to the elimination of voltage and frequency deviations and the suppression of chaotic oscillations in electrical systems. Robust stabilization as applied to power systems was proposed in [18].

The method of decentralized control of large-scale linear systems is considered in [19, 20]. Methods for suppressing and amplifying chaos based on modal control generalized for nonlinear systems are presented in [21]. This paper is devoted to the suppression of chaotic oscillations using decentralized control in large-scale nonlinear systems. Large-scale systems are understood as systems that: are described by differential or difference equations of high dimension; consist of subsystems that interact with each other.

Formulation of the problem of distributed control of nonlinear large-scale systems

Mathematical model of a nonlinear system. Let the disturbed motions of a nonlinear dynamic object be described by a vector differential equation:

$$\dot{x}(t) = dx(t)/dt = \varphi(x(t), u(t)), \;\; x(0) = x_0, \tag{1}$$

where $x(t) \in \mathrm{R}^n$ is a state vector, $u(t) \in \mathrm{R}^m$ is a control vector, $m \le n$, $\varphi(x(t), u(t)) = (\varphi_i(x(t), u(t)))_{i=1}^{n}$ is a vector function, $\varphi_i(x(t), u(t))$ are real functions that are defined and continuous in a domain $\Omega = \{(x,u) \| \|x\| + \|u\| < \wp, \; \wp = \text{const} > 0\} \subset \mathrm{R}^n \otimes \mathrm{R}^m$ and have continuous partial derivatives in it, which are bounded in a closed domain $\Omega_1 = \{(x,u) \| \|x\| + \|u\| \le \wp_1 < \wp\} \subset \mathrm{R}^n \otimes \mathrm{R}^m$.

The set of admissible controlled processes Ξ is defined as the set of triples $\xi = (x(t), u(t), t)$ that satisfy the conditions:

1) the functions $x(t), u(t)$ are defined on an interval $[0, \infty)$, $x(t)$ is continuous and piecewise differentiable, $u(t)$ is piecewise continuous;
2) the functions $x(t), u(t)$ satisfy differential connection (1);
3) for all $t \in [0, \infty)$ the pair $(x(t), u(t)) \in \Omega \subset \mathrm{R}^n \otimes \mathrm{R}^m$;
4) the values $x_0 = x(0) \in \Omega_0 \subset \mathrm{R}^n$.

The state of the i-th isolated (non-interacting) subsystem is determined by the expression:

$$\dot{x}_i = g_i(t, x_i), \;\; x_i(0) = x_{i0}, \;\; g_i(t, 0) \equiv 0, \;\; i = \overline{1, N}. \tag{2}$$

Here $x_i \in \mathrm{R}^{n_i}$ is the state vector of the i-th subsystem, $\sum_{i=1}^{N} n_i = n$; $g_i(t, x_i) : \mathrm{R} \times \mathrm{R}^{n_i} \to \mathrm{R}^{n_i}$ – vector functions that determine the state of isolated subsystems; N – the number of subsystems in the system.

The functions $h_i(t, x) : \mathrm{R} \times \mathrm{R}^n \to \mathrm{R}^{n_i}$ equal to

$$h_i(t, x) = f_i(t, x) - g_i(t, x_i), \;\; i = \overline{1, N}, \tag{3}$$

describe the relationship of the i-th subsystem with other subsystems.

The behavior of the i-th interacting subsystem can be represented by the equation:

$$\dot{x}_i = g_i(t, x_i) + h_i(t, x), \;\; i = \overline{1, N}. \tag{4}$$

Equation (3) describes the relationships between isolated subsystems (2), and equation (4) – the behavior of large-scale system (1), represented in the form of interacting subsystems. Large-scale systems include systems with a large dimension of the state vector, represented as subsystems interacting with each other.

Linearization of nonlinear system. Let equation (1) describe the deviations of the phase coordinates of a nonlinear object from a certain trajectory x^S, on which it is held by the control action u^S. Using the Taylor formula under the assumption that the components of the function $\varphi\left(x(t),u(t)\right)=\left(\varphi_i\left(x(t),u(t)\right)\right)_{i=1}^{n}$ are differentiable in a neighborhood $\xi^S=\left(x^S,u^S\right)$, equation (1) can be transformed to the quasilinear form:

$$\dot{x}(t)=A\left(\xi^S\right)x(t)+B\left(\xi^S\right)u(t)+f\left(\xi^S\right),\ x(0)=x_0. \tag{5}$$

In system (5), the coefficients $A\left(\xi^S\right)$ and $B\left(\xi^S\right)$ are calculated at a point ξ^S by the following formulas:

$$A\left(\xi^s\right)=\left.\begin{bmatrix} \partial\varphi_1/\partial x_1 & \dots & \partial\varphi_1/\partial x_n \\ \dots & \dots & \dots \\ \partial\varphi_n/\partial x_1 & \dots & \partial\varphi_n/\partial x_n \end{bmatrix}\right|_{\substack{x(t)=x^S \\ u(t)=u^S}}, \tag{6a}$$

$$B\left(\xi^s\right)=\left.\begin{bmatrix} \partial\varphi_1/\partial u_1 & \dots & \partial\varphi_1/\partial u_m \\ \dots & \dots & \dots \\ \partial\varphi_n/\partial u_1 & \dots & \partial\varphi_n/\partial u_m \end{bmatrix}\right|_{\substack{x(t)=x^S \\ u(t)=u^S}}. \tag{6b}$$

Suppose for all

$$\xi^S\in S\left(x^S,u^S,\rho\right)=$$
$$=\left\{\left(x^S,u^S\right):\left\|x-x^S\right\|+\left\|u-u^S\right\|\le\rho,\rho>0\right\}\subset \mathrm{R}^n\otimes\mathrm{R}^m,$$

the following estimates are true

$$\left\|f\left(\xi^S\right)\right\|\le q\left\|\xi\right\|. \tag{7}$$

If the Jacobian matrix is calculated by formula (6*a*) and condition (7) is satisfied, then equation (5) takes the form of a linearized system (or equations in variations):

$$\dot{y}(t)=Ay(t)+Bu(t). \tag{8}$$

System (8) can be used to design a control that stabilizes system (1) in the vicinity of a particular solution. The real parts of the eigenvalues of the Jacobian matrix determine the geometric picture of the behavior of the trajectories of the original nonlinear system.

Statement of the control problem. The type of trajectories of system (1) is determined by the Lyapunov characteristic exponents. A nonlinear system in the presence of chaotic dynamics is Lyapunov unstable in the small and Poisson stable in the large (in asymptotic). The Lyapunov characteristic exponents are a quantitative measure of instability. Among the entire set of Lyapunov characteristic exponents, the largest (senior) exponent $\chi_1 = \chi_{\max}$ is the most important. The characteristic exponents, in descending order $\chi_1 \ge \chi_2 \ge \ldots \ge \chi_n$, define the Lyapunov spectrum of a nonlinear dynamic system.

In nonlinear systems, in addition to stable singular points and limit cycles, strange attractors can be attractors as well. In n-dimensional nonlinear systems, the signature of the Lyapunov spectrum can take the following form:

$$\underbrace{(-, -, -, \ldots, -, -, -)}_{n} \text{ – equilibrium status;} \tag{9a}$$

$$(0, \underbrace{-, -, \ldots, -, -, -)}_{n-1} \text{ – limit cycle;} \tag{9b}$$

$$\underbrace{(+, \ldots, +,}_{s} 0, -, \ldots, -) \text{ – strange attractor, } s \ge 1. \tag{9c}$$

The problem of chaos stabilization (suppression) consists in transforming the chaotic mode of system (1), which is characterized by Lyapunov spectrum (9c), into a regular mode with a spectrum of characteristic exponents (9a) or (9b), that is, to provide an attractor in the form of a singular point or limit cycle.

To solve this problem, let us look for control in the form of feedback over the phase vector of the nonlinear system (1)

$$u(t) = -Lx(t),\, L \in \mathrm{R}^{m \times n}, \tag{10}$$

which will provide in a closed system

$$\dot{x}(t) = \varphi\big(x(t), Lx(t)\big),\;\; x(0) = x_0, \tag{11}$$

a spectrum of Lyapunov characteristic exponents

$$\sigma(\varphi) = \left\{\chi_i(\varphi),\;\; i = \overline{1,n}\right\},$$

that is equal to the desired (required) spectrum

$$\sigma(G) = \left\{\chi_i(G),\;\; i = \overline{1,n}\right\}. \tag{12}$$

The desired spectrum (12) is determined by the required character of the regular motion of system (1).

To reduce the computational costs of synthesis, the control of nonlinear system (1) must be implemented in the form of controller (10) with a decentralized structure

$$\begin{gathered} u_i(x_i) = -L_{ii}x_i,\;\; i = \overline{1,N} \Leftrightarrow u(x) = -L_D x, \\ L_D = \operatorname{blockdiag}\left\{L_{ii}\right\}_{i=1}^{N}. \end{gathered} \tag{13}$$

A decentralized regulator is a set of local regulators (13) that implement feedback on the phase vector of subsystems (2).

Synthesis of control of chaotic dynamics of a nonlinear system

Synthesis of control over the spectrum of Lyapunov characteristic exponents. Synthesis of control of a nonlinear system by introducing feedback consists in changing the spectrum of Lyapunov characteristic exponents to achieve the desired result – the transition to regular motion.

To solve the problem of changing the spectrum of Lyapunov characteristic exponents, the fact that they are determined by the eigenvalues of the Jacobian matrix of the linearized system is used. A change in the eigenvalues of the Jacobian matrix, the real parts of which determine the characteristic exponents of the linearized system, entails a change in the Lyapunov characteristic exponents of the nonlinear system. The desired eigenvalues can be assigned to the Jacobian matrix using the modal control synthesis technique based on solving the matrix algebraic Sylvester equation.

The validity of this approach is substantiated by the theorems on structural stability (roughness) of nonlinear dynamical systems, formulated in [22], and the topological equivalence of a nonlinear system and a hyperbolic linearized model [23, 24]. The theorems imply that if a linearized system is hyperbolic (has no purely imaginary eigenvalues), then the nonlinear system has stable or unstable manifolds, which are smooth analogs of stable or unstable spaces of the linearized system. Otherwise, the nonlinear system and the linearized system have the same number of singular points and limit cycles.

The feedback synthesis algorithm for a nonlinear large-scale system (11) includes the following steps [25].

1. The phase space is divided into small cells E_i and the invariant measure p_i is calculated (the probability of a trajectory visiting a nonlinear system of a cell E_i):

$$p_i = \frac{N_i}{N}, \tag{14}$$

here, N_i is the number of trajectory points in the cell E_i; N is the total number of points on the trajectory of a nonlinear system, which is considered for a sufficiently long time interval after it hits a strange attractor.

The size of the cells is selected as follows:

$$h_j = \frac{1}{S(T)-S(T_0)} \sum_{k=S(T_0)}^{S(T-1)} \left| x_j(k+1) - x_j(k) \right|, \tag{15}$$

where T_0 is the time of the beginning of the calculation of the invariant measure, T is the time of the end of the calculation; $S(t)$ is the step number corresponding to the time t. Thus, for each phase coordinate x_j, the cell size h_j is chosen so that its side is equal to the difference between the coordinate values x_j for each next and previous point of the trajectory, averaged over time.

2. Nonlinear system (11) after linearization in the center of each cell with side (15) has the form:

$$\dot{y}_i(t) = J(x_i)\, y_i(t) + B(x_i)\, L_i y_i(t). \tag{16}$$

3. The required eigenvalues of the Jacobian matrix corresponding to the center of each cell are calculated by the formula:

$$\bar{v}\left(\tilde{J}(x_i)\right) = v\left(J(x_i)\right) + \alpha \cdot \mathrm{Re}\left(v\left(J(x_i)\right)\right), \tag{17}$$

where $v\left(J(x_i)\right)$ are the eigenvalues of the Jacobian matrix of the original system, calculated in the center x_i of the cell E_i; α is a coefficient that affects the shift of the eigenvalues of the matrix along the real axis of the complex plane and depends on the problem of chaos control being solved. When chaotic dynamics are suppressed, the coefficient α is selected to be less than or equal to zero; when chaos is amplified, the coefficient α is greater than zero to increase the entropy of a nonlinear system.

4. Based on the required eigenvalues of the Jacobi matrix of each cell, the feedback coefficients are calculated $L_i,\ i=\overline{1,N},$ which provide a given location of the eigenvalues of the Jacobian matrix of the closed-loop system (16). The calculations are carried out according to formula (23) given in the next paragraph of this section.

5. The feedback coefficient (10) of a nonlinear system is defined as the average value over all cells E_i. The average value is found taking into account the invariant measures (14):

$$L=\sum_{i=1}^{N} L_i p_i. \tag{18}$$

6. Let us check the spectrum of Lyapunov characteristic exponents of the nonlinear system (11) for compliance with one of the spectra (9*a*) or (9*b*), depending on the control problem being solved.

Synthesis of control of a linearized system. The problem of positioning the poles of the system is considered, in which the determination of the controller parameters is reduced to solving the matrix Sylvester equation.

Centralized administration. For system (8), it is necessary to find a stabilizing controller in the form of feedback on the state vector

$$u\left(y(t)\right)=-Ly(t) \tag{19}$$

such that the spectrum of the closed system

$$\dot{y}(t)=\left(A-BL\right)y(t)=A_y y(t) \tag{20}$$

coincides with or is a subset of the prescribed spectrum given by the sequence $\mu=\{\mu_1,\ldots,\mu_n\}$

$$\rho\left(A_y\right)=\rho\left(-F\right), \tag{21}$$

here, $F=\mathrm{diag}\left(\mu_i\right)_{i=1}^{n}\in \mathrm{R}^{n\times n}$ is the matrix, on the main diagonal of which the numbers μ_i are located, which are chosen on the basis that the spectra of the matrices A_y and $(-F)$ coincide; $\rho\left(A_y\right)=\left\{\lambda_1\left(A_y\right),\right.$ $\left.\ldots,\lambda_n\left(A_y\right)\right\}$ and $\rho\left(-F\right)=\left\{\lambda_1\left(-F\right)=-\mu_1,\ldots,\lambda_n\left(-F\right)=-\mu_n\right\}$ are the spectra of matrices A_y and $(-F)$.

For systems with several inputs $m>1$, the solution to the pole placement problem is not unique, and the question arises of describing the set of stabilizing controllers. The problem of finding the matrix L that determines the “depth” of the feedback from the full state vector is reduced to solving the Sylvester matrix equation:

$$AP+PF=BG \tag{22}$$

with respect to a matrix $P\in \mathrm{R}^{n\times n}$ with an arbitrary matrix $G\in \mathrm{R}^{m\times n}$ and solving the matrix equation

$$LP=G,\ \ L=GP^{-1}. \tag{23}$$

For dynamical system (8), the conditions for the existence of a solution to the pole placement problem and the method for synthesizing a stabilizing control are contained in the theorem given in [19].

The parameters of the controller (19), ensuring the fulfillment of condition (21) in closed-loop system (20), are determined from relation (23), where the matrix P is the solution to Sylvester's equation (22). Matrix $A \in \mathrm{R}^{n\times n}$ – Jacobian matrix.

Decentralized control. Implementation of control in a centralized structure requires complete information about the system, which is a serious limitation due to the increase in memory and computer time costs, the complexity of organizing the transmission of information about the state of subsystems in the event of their geographic dissociation. In addition, centralized control is not resistant to structural disturbances (changes in connections between subsystems).

Let us represent the matrix $A \in \mathrm{R}^{n\times n}$, the matrix of parameters of system (8), as a sum $A = A_D + A_O$, where $A_D = \mathrm{blockdiag}\{A_{ii}\}_1^N$ is the block diagonal matrix, the elements of which characterize the parameters of isolated subsystems; $A_O = \mathrm{block}\{A_{ij}\}_{i,j=1}^N$, $A_{ij} \neq 0,\ i \neq j$ is the block nondiagonal matrix, each block A_{ij} of which determines the intensity of the effects of the j-th subsystem on the i-th subsystem; $B = \mathrm{blockdiag}\{B_{ii}\}_1^N \in \mathrm{R}^{n\times m}$ is the block diagonal input matrix.

Based on the structural decomposition, system (8) is represented as a set of interacting subsystems:

$$\dot{x}_i = A_{ii}x_i + B_{ii}u_i + h_i,\ x_i(0) = x_{i0},\ h_i = \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}x_j, \tag{24}$$

here, $x_i \in \mathrm{R}^{n_i}$ is the state vector of the i-th subsystem; $\sum_{i=1}^{N} n_i = n$; $u_i \in \mathrm{R}^{m_i}$ is the vector of control actions of the i-th subsystem; $h_i : \mathrm{R}^n \to \mathrm{R}^{n_i}$ is a vector function characterizing the influence on the i-th subsystem of all other subsystems; $B_{ii} \in \mathrm{R}^{n_i\times m_i}$ is the matrix of controls of the i-th subsystem.

Let us choose matrices G and F with a structure similar to the matrix A: $G = G_D + G_O$ and $F = F_D + F_O$. Here, $G_D = \mathrm{blockdiag}\{G_{ii}\}_{i=1}^N$, $G_O = \mathrm{block}\{G_{ij}\}_{i,j=1}^N$, $F_D = \mathrm{blockdiag}\{F_{ii}\}_{i=1}^N$, $F_O = \mathrm{block}\{F_{ij}\}_{i,j=1}^N$. Then Sylvester's equation (22) takes the form:

$$(A_D + A_O)P + P(F_D + F_O) = B(G_D + G_O).$$

This equation is equivalent to two equations: the equation for diagonal blocks

$$A_D P + PF_D = BG_D \tag{25}$$

and the equation for nondiagonal blocks

$$A_O P + PF_O = BG_O.$$

With a diagonal structure of block matrices A_D, F_D, B and G_D included in equation (25), it is equivalent to the N equations:

$$A_{ii}P_{ii} + P_{ii}F_{ii} = B_{ii}G_{ii},\ i = \overline{1,N}, \tag{26}$$

which correspond to isolated subsystems.

Under these conditions, equation (23) takes the diagonal form:

$$LP = G_D \Leftrightarrow \left(L_{ii}P_{ii} = G_{ii}, i = \overline{1,N}\right),$$

and regulator (19) is the desired decentralized structure.

Decentralized control ensures the equality of the closed-loop system spectrum to the spectrum of the reference matrix: $\rho\left(A_y\right) = \rho\left(-F\right)$. Reducing computational costs is achieved by decomposing the Sylvester equation of dimension n into N equations of dimension $n_i\left(n_i << n\right)$, corresponding to the subsystems, and implementing local controllers in the form of feedback on the phase vector of the subsystems.

Research of processes in the system of synchronous generators

The proposed method for the synthesis of control of a nonlinear large-scale system is considered through the example of control of chaotic oscillations arising in the operation of an electric power system, presented in the form of a system of three interconnected synchronous generators.

Model of a three-machine system. To analyze the chaotic behavior of the electric power system, the classical model of a synchronous generator is used, which allows for a qualitative and quantitative analysis of the processes, indicating the irregular nature of the deviation of the rotor angle and frequency.

The equations of the mathematical model of the three-machine electric power system, which has unequal inertia of the rotors of the generators included in it, has the form [26]:

$$\begin{gathered}\frac{d\delta_1}{dt} = \omega_1,\\ \frac{d\omega_1}{dt} = -B_1 \cdot \sin\left(\left(1+\frac{1}{\sqrt{2}}\right)\cdot\delta_1 + \frac{1}{\sqrt{2}}\cdot\delta_3\right) - C_{13}\cdot\sin\left(\delta_1-\delta_3\right) + P_1,\end{gathered} \tag{27a}$$

$$\begin{gathered}\frac{d\delta_2}{dt} = \omega_2,\\ \frac{d\omega_2}{dt} = -B_2 \cdot \sin\left(\left(1+\frac{1}{\sqrt{2}}\right)\cdot\delta_2 + \frac{1}{\sqrt{2}}\cdot\delta_3\right) - C_{21}\cdot\sin\left(\delta_2-\delta_1\right) + P_2,\end{gathered} \tag{27b}$$

$$\begin{gathered}\frac{d\delta_3}{dt} = \omega_3,\\ \frac{d\omega_3}{dt} = -B_3 \cdot \sin\left(\left(1+\frac{1}{\sqrt{2}}\right)\cdot\delta_1 + \frac{1}{\sqrt{2}}\cdot\delta_3\right) - C_{31}\cdot\sin\left(\delta_3-\delta_1\right) + P_3,\end{gathered} \tag{27c}$$

where δ_1, δ_2, δ_3 – deviations of the angle of rotation of the rotor of the generator relative to the synchronously rotating axis; ω_1, ω_2, ω_3 – deviation of the angular frequency; P_{c13}, P_{c21}, P_{c31} – synchronizing power between generators; P_1, P_2, P_3 – change in the power supplied to the network by generators; ε_{01}, ε_{02}, ε_{03} – the initial values of the power supplied to the network by the generators in the event of a network disturbance;

The studies were carried out at the following values of the model parameters:

$$B_1 = \frac{P_1}{T_{j1}} = 1,\ C_{13} = \frac{P_{c13}}{T_{j1}} = 0.1,\ P_1 = \frac{\varepsilon_{01}}{T_{j1}} = 0.4,$$

$$B_2 = \frac{P_2}{T_{j2}} = 1,\ C_{21} = \frac{P_{c21}}{T_{j2}} = 0.1,\ P_2 = \frac{\varepsilon_{02}}{T_{j2}} = 0.4,$$

$$B_3 = \frac{P_3}{T_{j3}} = 1,\ C_{31} = \frac{P_{c31}}{T_{j3}} = 0.1,\ P_3 = \frac{\varepsilon_{03}}{T_{j3}} = 0.3.$$

Introducing the phase vector of system (27)

$$x(t) = \left(x_1(t) = \delta_1,\ x_2(t) = \omega_1,\ x_3(t) = \delta_2,\ x_4(t) = \omega_2,\ x_5(t) = \delta_3,\ x_6(t) = \omega_3\right)^T \in \mathrm{R}^6$$

it can be written as

$$\dot{x}(t) = F(x(t)).$$

Chaotic properties of a system without control. The study of system (27) for the presence of chaotic oscillations was carried out under the initial conditions:

$$\delta_1(0) = 0.6;\ \omega_1 = 0.3;\ \delta_2(0) = 0.6;\ \omega_2 = 0.3;\ \delta_3(0) = 0.6;\ \omega_3 = 0.3.$$

The singular point of system (27) has coordinates:

$$x_0 = \begin{pmatrix} -10.1818; & 0; & -6.5625; \\ 0; & 1.8609; & 0 \end{pmatrix}^T.$$

For the indicated values of the parameters and initial conditions, the Lyapunov characteristic exponents of system (27) are:

$$\begin{aligned} \lambda_1 &= 0.0036; & \lambda_4 &= -0.0054; \\ \lambda_2 &= 0.0027; & \lambda_5 &= -1.0456; \\ \lambda_3 &= 0.0012; & \lambda_6 &= -3.1895. \end{aligned}$$

Fig. 1 shows the projection of the phase portrait of system (27) onto the plane $x_3 = \delta_2$ and $x_4 = \omega_2$.

Since the spectrum contains positive Lyapunov characteristic exponents, there is therefore a chaotic regime in system (27). Fig. 1 shows that the projection of the trajectory of the system in the phase space is a strange attractor, which is also inherent in the irregular regime.

Research of processes under centralized control. Let us introduce into the system the control of the frequency of each generator; then the control vector has a dimension of 6×3 and the matrix B is equal to

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

and the equations of system (27) with centralized control take the form:

$$\dot{x}(t) = F(x(t)) - BLx(t).$$

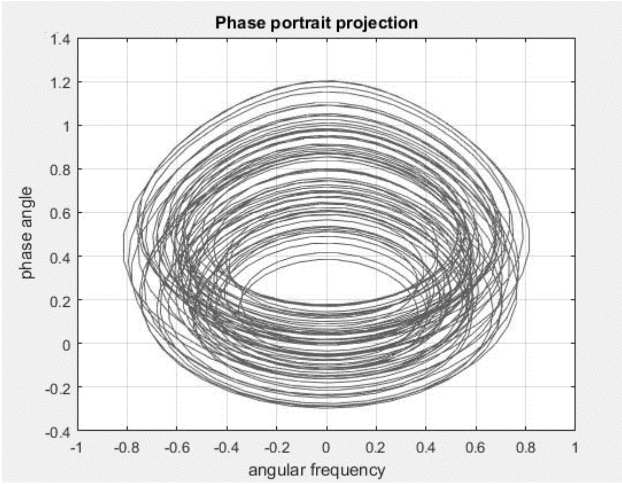

Fig. 1. Projection of the phase portrait of the system onto the plane $x_3 = \delta_2$ and $x_4 = \omega_2$

The Jacobian matrix of system (27) has the form:

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \partial f_2/\partial x_1 & 0 & 0 & 0 & \partial f_2/\partial x_5 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \partial f_4/\partial x_1 & 0 & \partial f_4/\partial x_3 & 0 & \partial f_4/\partial x_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \partial f_6/\partial x_1 & 0 & 0 & 0 & \partial f_6/\partial x_5 & 0 \end{bmatrix},$$

where

$$\frac{\partial f_2}{\partial x_1} = -\frac{\cos(\delta_1-\delta_3)}{10} - \left(\frac{1}{\sqrt{2}}+1\right)\cos\left(\delta_1\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right),$$

$$\frac{\partial f_2}{\partial x_5} = \frac{\cos(\delta_1-\delta_3)}{10} - \frac{\sqrt{2}}{2}\cos\left(\delta_1\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right),$$

$$\frac{\partial f_4}{\partial x_1} = \frac{\cos(\delta_1-\delta_2)}{10},$$

$$\frac{\partial f_4}{\partial x_3} = -\frac{\cos(\delta_1-\delta_2)}{10} - \left(\frac{1}{\sqrt{2}}+1\right)\cos\left(\delta_2\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right),$$

$$\frac{\partial f_4}{\partial x_5} = -\frac{\sqrt{2}}{2}\cos\left(\delta_2\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right),$$

$$\frac{\partial f_6}{\partial x_1} = \frac{\cos(\delta_1-\delta_3)}{10} - \left(\frac{1}{\sqrt{2}}+1\right)\cos\left(\delta_1\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right),$$

$$\frac{\partial f_6}{\partial x_5} = -\frac{\cos(\delta_1-\delta_3)}{10} - \frac{\sqrt{2}}{2}\cos\left(\delta_1\left(\frac{1}{\sqrt{2}}+1\right)+\frac{\delta_3}{\sqrt{2}}\right).$$

The feedback coefficient calculated by the method of synthesis of the centralized controller taking into account (16) and (17) is equal to

$$L = (-5.8045; \quad -9.0067; \quad -7.1735)^T .$$

The spectrum of Lyapunov characteristic exponents has the form:

$$\lambda_1 = 0,\ \lambda_2 = -4.5682,\ \lambda_3 = -5.2761,\ \lambda_4 = -7.5076,\ \lambda_5 = -10.2082,\ \lambda_6 = -15.8423.$$

The senior characteristic exponent is zero, the remaining characteristic exponents are less than zero; this indicates that the system is brought to regular movement – the limit cycle.

Fig. 2 shows the projection of the phase portrait of the system with centralized control on the coordinate plane $x_3 = \delta_2$ and $x_4 = \omega_2$.

Research of processes in decentralized management. Let us decompose system (27) into subsystems that correspond to the equations of one generator with phase coordinates – deviation of the rotor angle of rotation and deviation of the generator frequency. The mathematical model of subsystem (24), in this case, is, for example, equation (27a). That is, there are three subsystems of dimension two.

Jacobian matrices for each of the subsystems:

$$J_{11} = A_{11} = \begin{bmatrix} 0 & 1 \\ \dfrac{\partial f_2}{\partial x_1} & 0 \end{bmatrix}; \quad J_{22} = A_{22} = \begin{bmatrix} 0 & 1 \\ \dfrac{\partial f_4}{\partial x_3} & 0 \end{bmatrix}; \quad J_{33} = A_{33} = \begin{bmatrix} 0 & 1 \\ \dfrac{\partial f_6}{\partial x_5} & 0 \end{bmatrix}.$$

Formulas for calculating partial derivatives $\partial f_j / \partial x_k$, $j = 2, 4, 6,\ k = 1, 3, 5$ are given in the previous paragraph. The Jacobian matrices for each of the subsystems are the diagonal blocks of the Jacobian matrix for the system as a whole.

For each of the subsystems, the feedback coefficient is calculated using the decentralized control synthesis technique when solving equation (26)

$$L_{11} = -1.8620,\quad L_{22} = -0.7354,\quad L_{33} = -2.7388.$$

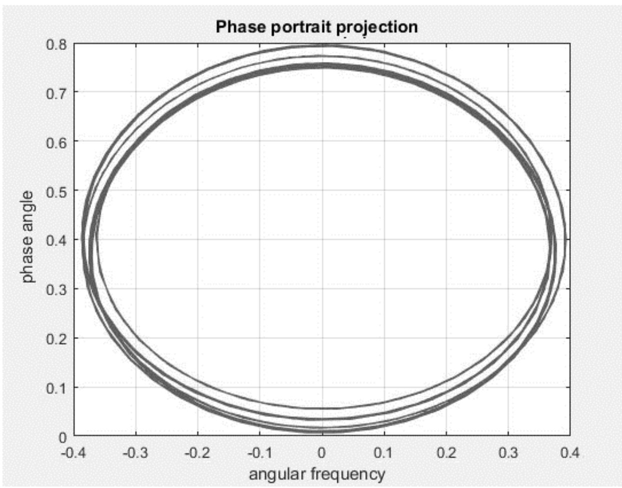

Fig. 2. Projection of the phase portrait of a system with centralized control onto a plane $x_3 = \delta_2$ and $x_4 = \omega_2$

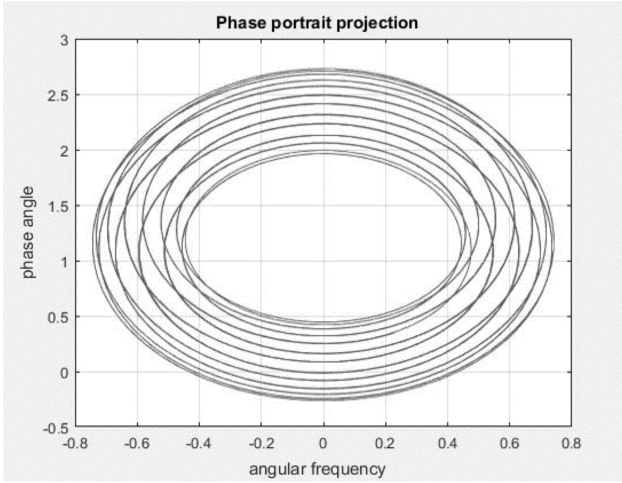

Fig. 3. Projection of the phase portrait of a closed-loop decentralized control system onto the plane $x_3 = \delta_2$ and $x_4 = \omega_2$

Lyapunov characteristic exponents in a system closed by a decentralized controller are equal to

$$\lambda_1 = 0,\ \lambda_2 = 0,\ \lambda_3 = -0.0896,\ \lambda_4 = -1.0628,\ \lambda_5 = -3.9880,\ \lambda_6 = -6.8304.$$

Fig. 3 shows the projection of the phase portrait of a nonlinear system with decentralized control on a plane $x_3 = \delta_2$ and $x_4 = \omega_2$.

The spectrum of Lyapunov characteristic exponents and the projection of the phase portrait of a system closed by decentralized control are calculated using a mathematical model (27) that takes into account the mutual influence of generators. The spectrum of Lyapunov characteristic exponents and the projection of the phase portrait of a system closed by decentralized control indicate the presence of a regular regime.

Conclusion

A technique for the synthesis of control for suppressing chaotic oscillations in a nonlinear large-scale system using phase vector feedback is presented. The feedback coefficient providing a given spectrum of Lyapunov characteristic exponents is calculated by the modal control method based on the solution of the matrix algebraic Sylvester equation extended to nonlinear large-scale systems with chaotic dynamics.

The article considers the use of the proposed method for the synthesis of decentralized control through the example of a system consisting of three synchronous generators. The results of the study confirmed the suppression of chaotic oscillations and the provision of a regular mode in a closed system due to the formation of a spectrum with negative Lyapunov characteristic exponents.

The advantage of the proposed decentralized control is the reduction of computational costs for the synthesis and implementation of control systems for large-scale systems. The synthesized feedback provides suppression of chaotic oscillations not in a small region of the phase space, but in the region of existence of solutions to the equations of the dynamics of a nonlinear system.

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