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Pile group effect at vertical vibrations

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Abstract. Analysis of the applicability of wave model solution to evaluate the variation of the dynamic stiffness in the pile foundation versus the distance between piles at the vertical vibrations; both experimental data from references and our own measurements in field are involved. To justify the reliability of the solutions of the wave models used to determine the dynamic stiffness values of the pile foundations at the vertical vibrations, we use the data obtained experimentally for the determination of natural frequencies in the cap-bound groups. The data obtained for the natural frequencies of 3×3 floating piles with different distances between them are considered. In addition, the data found at the forced vertical vibrations of the cap-bound groups of 2×2 piles under different loads and at different distances between the piles are involved. Processing of available amplitude-frequency curves involves the solution of the inverse problem with the theory of nonlinear vibrations to determine the parameters of the "pile group – soil" system, namely the effective mass, stiffness, and damping. The correlation between the measured and predicted data is evaluated by using the data obtained at the description of the pile groups in soil behavior. It has been found that the relations obtained at the solution of wave models and used to calculate the dynamic stiffness at the vertical vibrations of pile foundations consider the mutual effect of the piles in the group and permit satisfactory accuracy of the results. The maximum discrepancy between the results and experimental data is 15 %.

1. Introduction

High accuracy of the evaluated characteristics of the vibrations in the pile foundations installed under the machines with dynamic loadings is always a topical task [1–25]. The dynamic interaction between the pile and pile foundation and soil is among the least understood issues, and the combined effect of grouped piles makes it even more complex. In the cases when the distance between piles is large, the group stiffness can be evaluated via simple summing up the stiffnesses of individual piles. However, the piles located close to each other demonstrate the essential mutual effect and finally their efficiency may change considerably. To evaluate the dynamic interaction of the piles, both between each other and with the soil, the theories describing the processes and experiments, and experimental researches are necessary to verify their applicability.

Note that the activities to determine the amplitude-frequency characteristics of the pile foundations have been performed for a long time but still are far from over. The accurate theoretical solution of the problem of the dynamic interaction between the pile and soil is complicated by the non-linear character of the process, thus approximate approaches are applied. Particularly, the approaches proposed in [9–12] are among them. Many researches are devoted to the interaction of one pile with the soil under the dynamic loading, but at the same time, the behavior of pile groups is studied, too. Numerical simulation methods involving finite or boundary elements are commonly used to evaluate the dynamic condition of pile structures in complex application environment [16–19].

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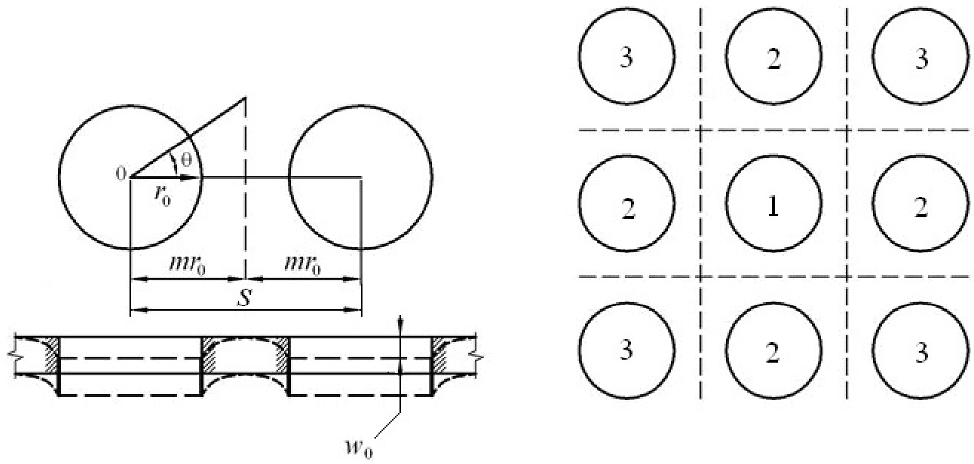
For most engineering tasks, the interaction between the pile and soil is usually successfully explained by the elasticity theory and, as is shown by many theoretical and experimental works, wave models simulate quite accurately the vibration process of the pile foundations in soil [1–5, 14, 25]. The Lamb analysis of the reactions of the semi-infinite elastic medium excited by a periodical vertical force acting along the vertical axis is the first investigation in this field (1904) [26]. Today, the solutions found for the tasks of the vibrating infinite plate with a round cut are used successfully to determine the amplitude-frequency characteristics of the pile foundations under the dynamic loadings [14]. However, generalization of this result for the cases when there is more than one cut is of practical interest. To find the link between motions and reactions on the side surface of embedded solid bodies, either in-lined or grouped, the authors of [16, 27, 28] proposed the solution for the task with the vertical vibrations of the plate with several round cuts; they also derived the formulas to find the stiffness and damping characteristics of the system (see schematic in Fig. 1). But the issue of reliability and accuracy of these results still remains open.

In view of this, the present work analizes the wave model solution applied to the evaluation of the dynamic stiffnesses varying of the pile foundations at the vertical vibrations, with due regard to the distance between piles; we use the theoretical evaluations from [27, 28] and compare them with the experimental data from references [7–9, 29, 30]. The results of series of field experiments performed by the authors are considered; the experiments aim to determine the natural frequencies of the pile foundations with floating cap-bound pile groups 3×3 ($s/d = m =$ 2, 3, 5, d is the pile diameter, s is the distance between central axes of neighboring piles) [29, 30]. The results obtained experimentally are compared with theoretical solutions. Along with it, the measurement results obtained in field for the forced vertical vibration of the cap-bound pile groups 2×2 at $s/d = m =$2, 3, 4 and under different loadings are considered [7–9]. The effective mass, stiffness, and damping of the pile system in soil are determined by the measured non-linear amplitude-frequency characteristics at the inverse task solution. Then, the theory of non-linear vibrations is used to calculate the amplitude-frequency curves which are then compared with experimental results.

2. Methods

Agreement between the measured and predicted data was evaluated for the description of the non-linear behavior of the pile and soil system regarding the distances between piles.



a **b**

Figure 1. Arrangement of cuts in a wavering thin plate: a) two neighboring ones, b) according to the 3×3 scheme.

In [14], considering warping axisymmetric vibrations of the infinitely thin layer with one round cut (radius r_0) described by the equation of motion of the elastic medium at zero volume forces in the cylindrical system of coordinates (r,t) as

$$\frac{\mu}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r} = \rho\frac{\partial^2 w}{\partial t^2},$$

with the boundary condition on the contour

$$w(r_0,\theta,t) = w_0 e^{i\omega t},$$

the authors determine that the reaction of the single-thickness soil layer applied to the pile side surface is described as

$$S_{w0}(kr_0)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w1,0} + iS_{w2,0}),$$

where the real $S_{w1,0}$ and imaginary $S_{w2,0}$ dimensionless parts of S_{w0} can be presented as

$$S_{w1,0}(kr_0) = 2\pi kr_0 \frac{J_0(kr_0)J_1(kr_0) + Y_0(kr_0)Y_1(kr_0)}{J_0^2(kr_0) + Y_0^2(kr_0)},$$
$$S_{w2,0}(kr_0) = \frac{4}{J_0^2(kr_0) + Y_0^2(kr_0)}. \quad (1)$$

Here, J_n, Y_n are the Bessel 1st and 2nd kind functions, $w = w(r,t)$ is the motion along the axis z, ρ is the density; $\mu = V_s^2 \rho$ is the Lame coefficient, $k = \omega / \sqrt{\mu/\rho}$.

The warping vibrations of the layer with several round cuts in line are described in [16]; the cuts radii are r_0, the centers are located within the distance of $s = 2r_0 m$ or m diameters from each other, $m > 1$ (see schematic in Figure 1a), as well as for the inner cut in line as is shown in Figure 1b, for which the reaction of the single-thickness soil layer attached to the pile side surface is, according to [27],

$$S_{w1}(kr_0, kr_g)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w1,1} + iS_{w2,1}),$$

where, $r_g = mr_0$, and $S_{w1,1}, S_{w2,1}$ are the real and imaginary dimensionless components, so S_{w1} can be presented as

$$S_{w1,1}(kr_0, kr_g) = S_{w1,0} - \frac{3}{2} S_{w1cor}(kr_0, kr_g),$$

$$S_{w2,1}(kr_0, kr_g) = S_{w2,0} - \frac{3}{2} S_{w2cor}(kr_0, kr_g),$$

$$S_{w1cor}(kr_0, kr_g) = \pi kr_0 \frac{J_0(kr_0)J_1(kr_g) + Y_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0) + Y_0^2(kr_0)} C,$$

$$S_{w2cor}(kr_0, kr_g) = \pi kr_0 \frac{Y_0(kr_0)J_1(kr_g) + J_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0) + Y_0^2(kr_0)} C,$$

$$C = \frac{\sum_{n=1}^{2} J_{2n-1}(kr_0)\left[Y_{2n-2}(kr_0) - Y_{2n}(kr_0)\right] - Y_{2n-1}(kr_0)\left[Y_{2n-2}(kr_0) - Y_{2n}(kr_0)\right]}{\sum_{n=1}^{2} J_{2n-1}(kr_0)\left[Y_{2n-2}(kr_g) - Y_{2n}(kr_g)\right] - Y_{2n-1}(kr_0)\left[Y_{2n-2}(kr_g) - Y_{2n}(kr_g)\right]},$$

here, opposite to (1), there are additive terms regarding the effect of neighboring cuts.

The expressions describing the reaction of the boundary (not corner) cut (schematic in Fig. 1b) were found in [28]

$$S_{w2}(kr_0, kr_g)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w1,2} + iS_{w2,2}),$$

$$S_{w1,2}(kr_0,kr_g)=S_{w1,0}(kr_0)-\frac{5}{4}S_{w1cor}(kr_0,kr_g),$$

$$S_{w2,2}(kr_0,kr_g)=S_{w2,0}(kr_0)-\frac{5}{4}S_{w2cor}(kr_0,kr_g),$$

and for the corner cut

$$S_{w3}(kr_0,kr_g)w_0e^{i\omega t}=\mu w_0e^{i\omega t}(S_{w1,3}+iS_{w2,3}),$$

$$S_{w1,3}(kr_0,kr_g)=S_{w1,0}(kr_0)-\frac{7}{8}S_{w1cor}(kr_0,kr_g),$$

$$S_{w2,3}(kr_0,kr_g)=S_{w2,0}(kr_0)-\frac{7}{8}S_{w2cor}(kr_0,kr_g).$$

Thus, the dynamic stiffnesses S_{wj} are described by the complex functions which depend on the vibration frequency ω, cut size r_0. The reactions go ahead the respective motions by time intervals Δ_j which are found in accordance with $\Delta_j=\arctan(S_{w2j}/S_{w1j})$. The parameters characterizing the motion amplitude can be estimated from the relation $A_j=(S_{w1j}^2+S_{w2j}^2)^{0.5}$.

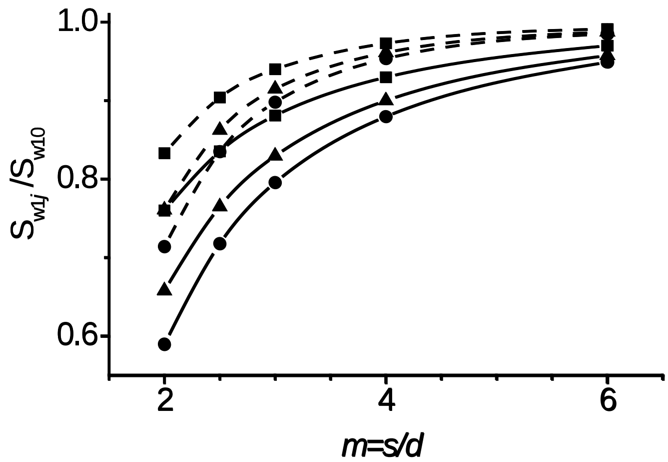


Figure 2. Variation of the relative stiffness at $kr_0=$ 0.05 (dashed curves) and $kr_0=$0.35 (solid curves) from the distance between the piles and their position in a group

• (*j*=1), ▲ (*j*=2), ■ (*j* = 3)

Fig. 2 illustrates the variation of the relative dynamic stiffness S_{w1j}/S_{w10}, $j=1$, 2, 3 at $kr_0=$0.05, 0.35 versus the distance between the piles and their position in the group in accordance with the schematic in Fig. 1b. The presented results lead to the conclusion that the reducing distance between the piles may cause the stiffness reduction up to 40 %.

To verify the presented theoretical evaluations of the dynamic interaction for the “pile-soil” system, the results of a series of field measurements are used; the experiments were carried out with the cap-bound groups of hanging piles 3×3 (schematics in Fig. 3 and Fig. 1b) and were purposed to determine the natural frequencies of the pile foundations. All three test pile foundations were made as a monolithic reinforced cap with the sizes 1.0×1.0×0.2 m supported by 9 rigidly fastened piles with the diameter $d=$ 76 mm $(r_0=d/2)$, the working length $h=$1.4 m; the piles are made from metal tubes, with the wall thickness of 3.5 mm. The distance between pile axes was 2*d*, 3*d* and 5*d*. Mass of the whole structure was $M=$ 690 kg. The cap has no contact with the soil. The impulse loading was carried out by a steel ballast – a 6 kg parallelepiped freely falling on the surface of each tested pile foundation from the height of 0.5 m.

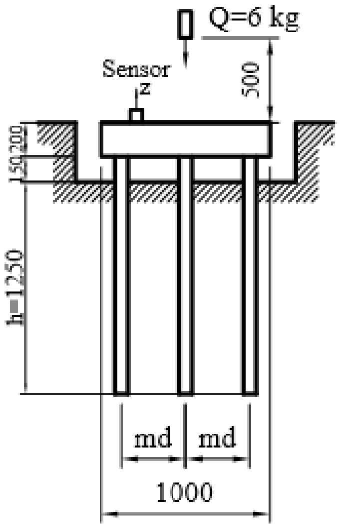


Figure 3. Schematics of experimental researches of tested foundations.

The soil is the test field contained down to 9.3 m from slightly wet loessial sand clay, the density $\rho = 17.0$ kN/m³ and deformation modulus $E =$ 14 MPa, medium-hard loam is the sub-soil. No ground water in the field. The value of the cross waves rate for the test field soil was found experimentally as $V_s =$ 146 m/s.

Investigation results obtained in the experimental field [29, 30] to determine the natural frequencies of foundations are presented in Fig. 4 and in Table 1.

Table 1. Measured natural frequencies.

Distance between piles	Measured frequency f_z, Hz (the average value) [29, 30]
2*d*	82.90
3*d*	91.36
5*d*	101.05

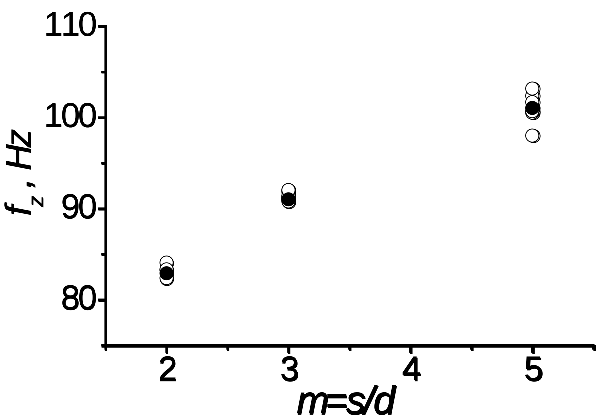


Figure 4. Measured natural frequencies of the pile foundations 3×3 at different s/d
o – experiment, • – average value [29, 30].

The natural frequencies λ_z at the vertical vibrations of the pile foundations and stiffness K_z, in the presence of damping, are related as

$$\lambda_z = 2\pi f_z \approx \sqrt{K_z/M}, \quad K_z \approx \lambda_z^2 M,$$

where, M is the mass of the whole structure.

The stiffnesses of the pile groups were found for various s involving the measurement results from Table 1 and formula

$$K_z^g(m) = [2\pi f_z(m)]^2 M \quad (2)$$

Theoretical evaluations of the stiffness for the pile groups were found with the relations

$$K_z^g(m) = GhS_{w1}^g(kr_0, mkr_0) \quad (3)$$

hence it follows that the value of the grouped pile stiffness factor S_{w1}^g is related to the dimensionless frequency of vibrations kr_0 and pile position in the cap. For the considered pile foundations from 3×3 piles, S_{w1}^g is determined in accordance with the schematic in Fig. 1b by the formula

$$S_{w1}^g(kr_0, mkr_0) = S_{w1,1}(kr_0, mkr_0) + 4S_{w1,2}(kr_0, mkr_0) + 4S_{w1,3}(kr_0, mkr_0),$$

where $m = s/d$, $k = \omega / V_s$, ω is the angular frequency of vibrations, V_s is the rate of transversal waves in soil, $G = V_s^2\rho = \mu$ is the shear modulus. In the cases under consideration, the dimensionless vibration frequency $kr_0 = 0.15$ is the average value within the range from 0.13 to 0.17.

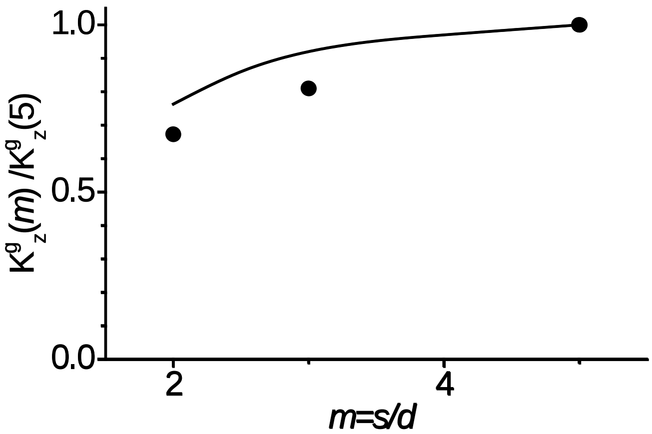


Figure 5. Changes in system stiffness versus the distance between piles (solid line – result of calculation, • –experiment [29, 30]).

The results obtained by the engineering calculations involving formulas (2), (3) are shown in Fig. 5. The calculated curve and dots illustrate the varying stiffnesses in respect to the value at $s/d = 5$. The values of the natural frequencies of the foundations with different distances between piles found by the formula (3) show the maximal difference with the test data, which is below 14 %. This result proves that the calculations in the framework of the used approximations permit obtaining the satisfactory agreement with experimental findings [29, 30].

In order to determine the dynamic stiffness and damping at the vertical vibrations of the pile-soil system, the inverse task [31] was solved involving the theory of non-linear vibrations at the processing of the amplitude-frequency curves available in [7, 8]. First, the effective mass, stiffness and damping are determined by the measured values of the vibration amplitudes and frequencies. Then, these parameters are used to verify the obtained formulas by means of comparison of the calculation and dynamic test results in the field for the frequencies and amplitudes of the vibrations of the cap-bound 2×2 floating piles [7–9].

Let us consider the action of the harmonically changing force with the amplitude proportional to the frequency square ω on the pile foundation

$$P_z = r_e m_e \omega^2,$$

then the motion equation can be written for the system under consideration as follows

$$M\ddot{z} + \Phi K_z \dot{z} + K_z z = m_e r_e \omega^2 \sin \omega t.$$

Here, M is the effective mass which includes M_0 and the mass of the attached soil vibrating together with the pile foundation. The solution of this equation is written as

$$z = \frac{m_e r_e \omega^2}{\sqrt{(K_z - M\omega^2)^2 + (K_z \Phi \omega)^2}} \sin \omega t$$

whereas the variation of the maximal amplitude A is described by the formula

$$A = \frac{m_e r_e}{M} \frac{\omega^2}{\sqrt{(K_z / M - \omega^2)^2 + (\Phi \omega K_z / M)^2}}. \tag{4}$$

It follows from (4) that the amplitude depends on the impact force frequency in a complex manner. In the onset regime, the vibration amplitude is $A_\infty = m_e r_e / M$ Before the system comes into the onset regime, the resonance is possible

$$A_{res} = \frac{A_\infty}{\Phi \sqrt{(K_z / M)(1 - \Phi^2 K_z / 4M)}}, \tag{5}$$

at the frequency

$$\omega_{res} = \sqrt{\frac{K_z / M}{1 - \Phi^2 K_z / 2M}}, \quad \Phi^2 K_z / M < 2. \tag{6}$$

The relations (5), (6) permit evaluating Φ and K_z / M , if the values A_{res} and f_{res} ($\omega_{res} = 2\pi f_{res}$) are found during the measurements on site

$$K_z / M = \omega_{res}^2 \sqrt{1 - \left(A_\infty / A_{res}\right)^2}, \quad \Phi = \omega_{res}^{-1} \sqrt{\frac{2 - 2\sqrt{1 - \left(A_\infty / A_{res}\right)^2}}{\sqrt{1 - \left(A_\infty / A_{res}\right)^2}}}.$$

The performed experimental investigations focused on determination of the vibration frequencies and amplitudes of the groups of cap-bound floating 2×2 piles at different values of the vertical harmonic action [7–9]. All experimental foundations were made as a monolithic reinforced cap with overall sizes 0.57×0.57×0.25 m supported by 4 rigidly fixed concrete piles with the diameter $d =$ 100 mm, the working length $h =$ 1.5 m. The distance between the pile axes s was 2d, 3d and 4d. The caps do not touch the soil. The tests were carried out at different eccentric momenta $m_e r_e =$ 0.0187; 0.0278; 0.0366 and 0.0450 where m_e is the mass of the eccentric rotary element in the vibrator, and r_e are the mass eccentricities. The methodology of the vibration tests is described in [8]. The mass of each foundation is $M =$ 1200 kg including the rotary element mass. The shear modulus value G of the experimental field soil is from $14 \cdot 10^6$ N/m^2 to $26 \cdot 10^6$ N/m^2, the transverse wave V_s velocities lie in the range of 95–150 m/s, they depend on the depth [9]. Table 2 presents the determined resonance frequencies f_{res} and amplitudes A_{res}.

Table 2. Resonant frequencies f_{res} and amplitudes A_{res}.

$m_e r_e$, kg·m	s/d = 2		s/d = 3		s/d = 4	
	A_{res}, (μm)	f_{res}, (Hz)	A_{res}, (μm)	f_{res}, (Hz)	A_{res}, (μm)	f_{res}, (Hz)
0.0187	0.0358	29.61	0.0317	35.45	0.0262	38.21
0.0278	0.0510	29.22	0.0422	34.41	0.0381	36.71
0.0366	0.0633	28.95	0.0589	33.35	0.0501	35.18
0.0450	0.0832	28.46	0.0707	32.73	0.0619	33.50

The data from Table 2 with the resonance frequencies f_{res} and amplitudes A_{res} $(s = 4d)$ are used to determine the values of K_z^g / M and Φ for the pile group at different s. The results of calculation are given in Table 3.

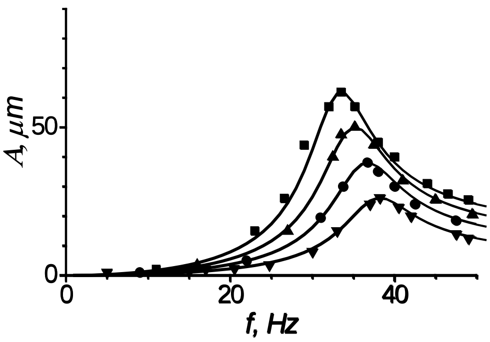
Table 3. Calculated K_z^g / M and Φ.

$m_e r_e$, (kg·m)	$s/d = 2$		$s/d = 3$		$s/d = 4$	
	K_z^g / M, (1/s²)	Φ, (s)	K_z^g / M, (1/s²)	Φ, (s)	K_z^g / M, (1/s²)	Φ, (s)
0.0187	3.41·10⁴	0.89·10⁻³	4.88·10⁴	0.84·10⁻³	5.62·10⁴	0.95·10⁻³
0.0278	3.32·10⁴	0.94·10⁻³	4.57·10⁴	0.97·10⁻³	5.18·10⁴	1.00·10⁻³
0.0366	3.25·10⁴	1.01·10⁻³	4.31·10⁴	0.94·10⁻³	4.71·10⁴	1.05·10⁻³
0.0450	3.15·10⁴	0.96·10⁻³	4.14·10⁴	0.98·10⁻³	5.31·10⁴	1.10·10⁻³

As we know $m_e r_e$, according to [28], the effective mass M can be evaluated involving the experimental values for A_∞ – $M = m_e r_e / A_\infty$, and then it should be specified for further calculations. The resulting M is 3200 kg that is much bigger than the mass of the structure $M_0 =$ 1200 kg. This result is in good agreement with the estimations of the effective mass presented in [7, 8] which were obtained by another method [32].

3. Results and Discussion

Figure 6a presents the non-linear amplitude-frequency curves calculated with the found K_z^g / M and Φ at different $m_e r_e$ [7, 8] for the pile group at $s = 4d$. Extra calculations were made for $s = 2d$ at $M = 3200$ kg and respective values of K_z^g / M and Φ as an extra verification (Fig. 6b).



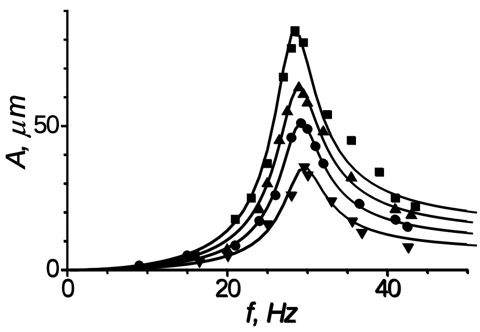


Figure 6. Experimental and calculated frequency curves of the pile group 2×2 at *s* = 4*d* (a), at *s* = 2*d* (b), for different values $m_e r_e =$ 0.0187 (▼), 0.0278 (•), 0.0366 (▲), 0.0450 (■) (kg·m) [7, 8] (curves – results of calculations)

The data presented in Fig. 6 show that the theoretical results satisfactory agree with the measurement data from the references [7, 8].

The values of K_z^g / M for the pile group permit evaluating the natural frequencies λ_z at the vertical vibrations pile foundations and damping – $\lambda_z \approx \sqrt{K_z / M}$ The results given in Table 4 lead to the conclusion that the natural frequencies drop as the excitation intensity rises, which agrees with the conclusions of [31] about the non-linear behavior of the considered “piles-soil” system. The decay modulus varies weakly, its average value is evaluated as $\Phi \approx$ 0.97·10⁻³ s. Comparison with the experiment proves that the analytical methods permit describing the major peculiarities of the amplitude-frequency behavior of the pile groups at low vertical vibrational actions. In the considered cases, the effective mass and damping preserve their values as the excitation intensity rises. Fig. 7 presents the variation of the dynamic stiffness determined by the experimental results in respect to the maximal value at $s/d = 4$.

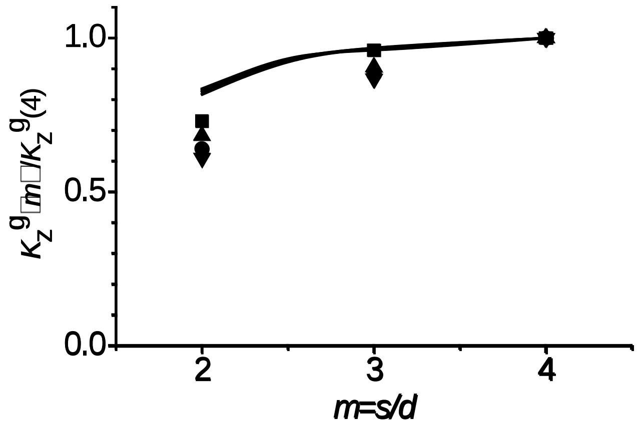


Figure 7. Variation of the relative stiffness of the pile group 2×2 versus the distance between the piles in the group: experiment at different values $m_e r_e = 0.0187$ (▼), 0.0278 (•), 0.0366 (▲), 0.0450 (■) (kg·m) [8], (curves – computation results)

Theoretical evaluation of the stiffness for the pile group is found with the relations

$$K_z^g(m) = 4GhS_{1,3}(kr_0, mkr_0),$$

at the dimensionless vibration frequency $kr_0 =$ 0.08, the average value within the range 0.06–0.1. The closely spaced curves in Fig. 7 illustrate the variation of the computed stiffnesses in respect to the maximal value at $s/d = m = 4$ for different $m_e r_e$.

It follows from the comparison of the theoretical evaluation and experimental findings that the considered analytical method permits predicting the peculiarities of the dynamic interaction between the piles in the group. The maximal difference between the experimental results and theoretical evaluations which involves the interaction effect between the piles under the action of the vertical vibration loadings is about 15 % at $s/d = 2$, whereas at $s/d = 3$, the difference in results is maximum 5 %. Note at the same time that we ignore the interaction with soil under the pile end, which may be essential. However, the final conclusion of the satisfactory agreement of the calculation and experiment [7, 8] is valid; it is evident though that the development of theoretical evaluations of the interaction of pile groups between each other and soil needs further improvement. Experimental data obtained after the tests with full-scale pile foundations in the field conditions should be involved.

4. Conclusions

1. The results obtained from engineering calculations involving analytical expressions to determine the natural frequencies of the foundations with different distances between piles have the maximal disagreement with the experimental findings of 14 %, which proves the adequacy of the used approximations.

2. The analytical methods under consideration permit predicting the major peculiarities of the interaction between piles in a group under the action of vertical vibration loadings on the foundation. The maximal difference between the experimental data and theoretical evaluations which consider the effect of pile interaction is about 15 % at $s/d = 2$ and is maximum 5 % at $s/d = 3$.

3. At the solution of the inverse task, use of the measured values of the foundation vibration amplitudes and frequencies permit determining the parameters of the system “cap-bound pile group – soil”, i.e. the effective mass, stiffness and damping with the satisfactory agreement of the calculation results and experimental data. It was found that the stiffness reduces as the intensity of vertical actions rises. The calculated amplitude-frequency curves involving the calculated mass of the soil-pile system, stiffness and damping agree with the values measured for the vertical vibrations.

4. The performed investigations lead to a conclusion that the relations obtained in the framework of the wave models and used to calculate the dynamic stiffnesses at the vertical vibrations of pile foundations, allow considering the mutual effect of piles in the group and satisfactory accuracy of the results, which is confirmed by the comparison with the experimental findings.

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References

1. Prakash, S., Puri, V.K. Foundations for machines: analysis and design [Online]. New York: J. Wiley & Sons, 1988. 656 p.
2. Prakash, S., Sharma, H.D. Pile foundations in engineering practice [Online]. New York: John Wiley & Sons, 1990. 768 p
3. Pyatetzkij, V.M., Aleksandrov, B.K., Savinov, O.A. Sovremennye fundamenty mashin i ih avtomatizirovannoe proektirovanie [Modern machine foundations and their automated design]. Moscow: Strojizdat, 1993. 415 p. (rus)
4. Savinov, O.A. Sovremennye konstrukcii fundamentov pod mashiny i ih raschet [Modern construction foundations of machines and their calculation] [Online]. Leningrad: Strojizdat. 1979. 200 p. (rus)
5. Guo, W.D. Theory and practice of pile foundations [Online]. CRC Press, 2013. 576 p.
6. Novak, M., El Sharnouby, B. Evaluation of dynamic experiments on pile group [Online]. Journal of Geotechnical Engineer-ing. 1984. 110(6). Pp. 738–756.
7. Rashidifar, M.A., Rashidifar, A.A., Abertavi, A. Nonlinear characteristics of the pile soil system under vertical vibration. Universal Journal of Engineering Science. 2016. 4(4). Pp. 59–65. DOI: 10.13189/ujes.2016.040401
8. Manna, B., Baidya, D.K. Dynamic nonlinear response of pile foundations under vertical vibration – Theory versus experiment. Soil Dynamics and Earthquake Engineering. 2010. 30(6). Pp. 456–469. DOI: 10.1016/j.soildyn.2010.01.002
9. Das, S.K., Manna, B., Baidya, D.K. Prediction of pile-separation length under vertical vibration using ANN [Online]. 14th Asian Regional Conference on Soil Mechanics and Geotechnical Engineering 2011. Hong Kong, China. Pp. 887–892.
10. El Naggar, M., Novak, M. Nonlinear axial interaction in pile dynamics. Journal of Geotechnical Engineering. 1994. 120(4). Pp. 678–696. DOI: 10.1061/(ASCE)0733-9410(1994)120:4(678)
11. Matlock, H., Foo, S.H.C., Bryant, L.M. Simulation of lateral pile behavior under earthquake motion [Online]. Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, XI. 1978. Pp. 600–619.
12. Novak, M., Sheta, M. Approximate approach to contact problems of piles. Proceedings: Dynamic Response of Pile Foundation: Analytical Aspects. NY: ASCE. 1980. Pp. 53–79.
13. Veletsos, A.S., Dotson, K.W. Vertical and torsional vibration of foundations in inhomogeneous media. Journal of Geotechnical Engineering. 1988. 114(9). Pp. 1002–1021. DOI: 10.1061/(ASCE)0733-9410(1988)114:9(1002)
14. Baranov, V.A. O raschete vynuzhdennyh kolebanij zaglublennogo fundamenta [About the computation of the forced vibrations of the embedded pile foundation]. In Issues of dynamics and strength questions. Proceedings of the Riga Polytechnical Institute. Riga 1967. No. 14. Pp. 195-209.
15. Nuzhdin, L.V., Zabylin, M.I. Zhestkost' i dempfirovanie vertikal'nyh kolebanij svajnogo fundamenta v sloistyh osnovaniyah [Stiffness and damping of vertical vibrations of the pile foundation in the layered lower stratum]. Izvestiya vuzov. Building and Architecture. 1990. No. 12. Pp. 36–41. (rus)
16. Kolesnikov, A., Popov, V. Estimation of dynamic responses on contours of several round cuts at thin plate vibrations. Proceedings of ISEC – 9th International Structural Engineering and Construction Conference: Resilient Structures and Sustainable Construction. Valencia, Spain. 2017. DOI: 10.14455/ISEC.res.2017.69
17. Wu, W., Wang, K., Zhang, Z., Leo, C. Soil-pile interaction in the pile vertical vibration considering true three-dimensional wave effect of soil. International Journal for Numerical and Analytical Methods in Geomechanics. 2013. 37(17). Pp. 2860–2876. DOI: 10.1002 /nag.2164
18. Medina, C., Aznárez, J., Padrón, L., Maeso, O. Effects of soil-structure interaction on the dynamic properties and seismic response of piled structures. Soil Dynamics and Earthquake Engineering. 2013. 53. Pp. 160–175. DOI: 10.1016/j.soildyn.20-13.07.004
19. Allani, M., Holeyman, A. Numerical evaluation of effects of nonlinear lateral pile vibrations on nonlinear axial response of pile shaft. Soils and Foundations. 2013. 53(3). Pp. 395–407. DOI: 10.1016/j.sandf.2013.04.002
20. Khalil, M.M., Hassan, A.M., Elmamlouk, H.H. Dynamic behavior of pile foundations under vertical and lateral vibrations. HBRC Journal. 2019. 15(1). P. 55–71. DOI: 10.1080/16874048.2019.1676022
21. Amiri, A.M., Ghanbari, A.A., Derakhshandi, M. An analytical model for estimating the vibration frequency of structures located on the pile group in the case of floating piles and end-bearing pile march. Civil Engineering Journal. 2018. 4(2). Pp. 450–468. DOI: 10.28991/cej-0309105
22. Biswas, S., Manna, B. Experimental and theoretical studies on the nonlinear characteristics of soil-pile systems under coupled vibrations. Journal of Geotechnical and Geoenvironmental Engineering. 2018. 144(3). P. 04018007. DOI: 10.1061/(ASCE)-GT.1943-5606.0001850
23. Biswas, S., Manna, B., Baidya, D.K. Experimental and theoretical study on the nonlinear response of full-scale single pile under coupled vibrations. Soil Dynamics and Earthquake Engineering. 2017. 94. Pp. 109–115. DOI: 10.1016/j.soildyn.2017.01.012
24. Gazetas, G., Markis, N. Dynamic pile-soil-pile interaction. Part I: Analysis of axial vibration. Earthquake Engineering and Structural Dynamics. 1991. 20(2). Pp. 115–132. DOI: 10.1002/eqe.4290200203
25. Novak, M. Dynamic Stiffness and Damping of Piles [Online]. Canadian Geotechnical Journal. 1974. 11(4). Pp. 574–598.
26. Lamb, H. On the propagation of tremors over the surface of an elastic solid [Online]. Philosophical Transactions. London, Ser. A. 1904. 203. Pp. 1–42.

27. Kolesnikov, A.O., Popov, V.N. Ocenka dinamicheskih reakcij na konturah neskol'kih kruglyh vyrezov pri kolebaniyah plastiny [Evaluation of the dynamic reactions on the contours of several round cuts at the plate vibrations] [Online]. Structural Mechanics of Engineering Constructions and Buildings. 2014. No 3. Pp. 37–43.
28. Kolesnikov, A.O., Popov, V.N. Dinamicheskie reakcii na konturah krugovyh vyrezov s uchyotom ih vzaimnogo raspolozheniya pri kolebaniyah plastiny [Dynamic reactions on round cut contours regarding their mutual location at the plate vibrations] [Online]. Izvestiya VNIIG B.E. Vedeneeva. 2017. 283. Pp. 3–10. (rus)
29. Kolesnikov, A.O., Popov, V.N., Kostyuk, T.N. Issledovanie vliyaniya zaglubleniya i raspolozheniya svaj na sobstvennye chastoty pri kolebaniyah fundamentov [Analysis of the effect of embedment and position of piles on the natural frequencies at pile foundation vibrations]. Izvestiya vuzov. Building. 2019. No. 5. Pp. 39–52. DOI 10.32683/0536-1052-2019-725-5-39-52 (rus)
30. Kolesnikov, A.O., Kostiuk, T.N., Popov, V.N. Calculation of foundation vertical stiffness with the piles mutual influence effect. Structural Mechanics of Engineering Constructions and Buildings. 2019. 15(3). Pp. 229–236. DOI: 10.22363/1815-5235-2019-15-3-229-236 (rus)
31. Manna, B., Baidya, D.K. Nonlinear vertical dynamic response of pile groups. Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering. IOS Press, 2009. Pp. 1128–1131. DOI: 10.3233/978-1-60750-031-5-1128
32. Novak, M. Data reduction from nonlinear response curves. Journal of Engineering Mechanics. 1971. 97(EM4). Pp. 1187–1204.
33. Lorenz, H. Elasticity and damping effects of oscillating bodies on soil. Symposium on Dynamic Testing on Soils. ASTM STP, 1953. No. 156. Pp. 113–122. DOI: 10.1520/STP49606S
34. Gan, S., Zheng, C., Kouretzis, G., Ding, X. Vertical vibration of piles in viscoelastic non-uniform soil overlying a rigid base. Acta Geotechnica. 2020. 15. Pp. 1321–1330. DOI: 10.1007/s11440-019-00833-7
35. Kolesnikov, A.O., Kostyuk, T.N., Popov, V.N. Attenuation of the soil vibration amplitude at pile driving. Magazine of civil engineering. 2020. 3(95). Pp. 49–56. DOI: 10.18720/MCE.95.5
36. Amiri, A. M., Ghanbari, A., Derakhshandi, M. Analytical model for natural frequency of SDOF system considering soil–pile–structure interaction. International Journal of Civil Engineering. 2018. 16(4). Pp. 1–13. DOI: 10.1007/s40999-018-0284-1
37. Li, Q., Shu, W., Cao, L., Duan, W., Zhou, B. Vertical vibration of a single pile embedded in a frozen saturated soil layer. Soil Dynamics and Earthquake Engineering. 2019. No. 122. Pp. 185–195. DOI: 10.1016/j.soildyn.2019.03.032
38. Luan, L., Ding, X., Cao, G., Deng, X. Development of a coupled pile-to-pile interaction model for the dynamic analysis of pile groups subjected to vertical loads. Acta Geotechnica. 2020. 15. Pp. 3261–3269. DOI: 10.1007/s11440-020-00972-2
39. Cui, C., Meng, K., Wu, Y., Chapman, D. Dynamic response of pipe pile embedded in layered visco-elastic media with radial inhomogeneity under vertical excitation. Geomechanics and Engineering. 2018. 16(6). Pp. 609–618. DOI: 10.12989/gae.2018.16.6.609

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