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DEVELOPMENT OF HIGH-FREQUENCY VOLATILITY ESTIMATORS IN PRICING AND TRADING STOCK OPTIONS

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Abstract. Asset return volatility plays a key role in derivative pricing and hedging, risk management and portfolio allocation decisions. This study examined the economic benefit of high-frequency volatility estimators (measures realized) in option pricing and trading. We evaluated the forecasting ability of high-frequency volatility estimators based on the profits that option dealers would derive from trading on the basis of alternative high-frequency volatility forecasts. To this end, we traded European call and put options on Bank of America, Coca-Cola and Microsoft stocks for a period of 24 trading days using high-frequency volatility-based option trading strategies. The study results show that the realized kernel estimators for Bank of America stock options were the only volatility estimators that earned a positive profit from trading (a profit of $20.42 per option over a period of 24 trading days). For Coca-Cola stock options, the best volatility estimator turned out to be the two-time scale covariance estimator. It earned a total profit of $26.88 per option during the same period. For Microsoft stock options, the preferred volatility estimator was the Range-based realized variance estimator. It outperformed all the other competing estimators with a total profit of $54.07 per option which was significantly greater than the profits of the other estimators. It was concluded that high-frequency volatility forecasts by the realized kernel, two-time scale realized variance and realized range-based variance estimators yield accurate volatility forecasts and are very useful in pricing and trading Bank of America, Coca-Cola and Microsoft stock options, respectively.

Keywords: Volatility, realized volatility, realized measures, high-frequency volatility forecast, HAR model, Black-Scholes-Merton model, option trading strategies

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РАЗВИТИЕ МЕТОДОВ ОЦЕНКИ ВОЛАТИЛЬНОСТИ ДОХОДНОСТИ ПРИ ЦЕНООБРАЗОВАНИИ И ТОРГОВЛЕ ФОНДОВЫМИ ОПЦИОНАМИ

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Аннотация. Волатильность доходности активов играет ключевую роль в ценообразовании и хеджировании производными финансовыми инструментами, управлении рисками, а также принятии решений о распределении портфеля как трейдера, так и инвестора. В этом исследовании изучалась экономическая выгода высокочастотных оценок волатильности (реализованных показателей) при ценообразовании и торговле опционами. Мы оценили качество прогнозов высокочастотных оценщиков волатильности на основе прибыли, которую дилеры опционов могли бы получить от торговли на основе альтернативных прогнозов высокочастотной волатильности. С этой целью мы анализировали торговлю европейскими опционами "колл" и "пут" на акции Bank of America, Coca-Cola и Microsoft в течение двадцати четырех торговых дней, используя стратегии торговли опционами на основе анализа высокочастотной волатильности. Результаты исследования показывают, что для опционов на акции Bank of America реализованные оценки ядра были единственными оценщиками волатильности, которые получили положительную прибыль от торговли (прибыль в размере 20,42 доллара США за опцион в течение двадцати четырех торговых дней). Для опционов на акции Coca-Cola лучшей оценкой волатильности оказалась оценка ковариации в двухкратном масштабе. За тот же период он получил общую прибыль в размере 26,88 доллара за опцион. Для опционов на акции Microsoft предпочтительной оценкой волатильности была оценка реализованной дисперсии на основе диапазона. Он превзошел всех других конкурирующих оценщиков с общей прибылью в размере 54,07 доллара США за опцион, что было значительно больше, чем прибыль других оценщиков. Был сделан вывод о том, что высокочастотные прогнозы волатильности с помощью реализованного ядра, оценки реализованной дисперсии в двухкратном масштабе и оценки реализованной дисперсии на основе диапазона дают точный прогноз волатильности и очень полезны при ценообразовании и торговле опционами на акции Bank of America, Coca-Cola и Microsoft, соответственно.

Ключевые слова: Волатильность, реализованная волатильность, реализованные показатели, прогноз высокочастотной волатильности, модель HAR, модель Блэка-Шоулза-Мертона, стратегии торговли опционами

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Introduction

Over the past two decades, research on volatility measurement has focused on constructing non-parametric estimators of asset return volatility using high-frequency price data. It has been well-established that high-frequency financial data can serve to better understand and forecast financial volatility. The main advantage of using high-frequency financial data to estimate volatility is in the increased quality of volatility forecast. For instance, a recent study measuring the asset return volatility has built on the above advantage to propose new realized volatility estimators (realized measures) "that are more efficient, are robust to market microstructure effects, and can estimate the variation due to the continuous part of the price process separately from the variation due to the "jump" part of the price process." [1]

As a matter of fact, the approach adopting high-frequency data for measuring and forecasting asset return volatility has provided a record number of realized volatility estimators. These estimators are based on a variety of different assumptions about the price process and take many different functional forms. In addition, they are based on different sampling schemes and price series. One particular technique is based upon sampling in calendar time with trade price, while another utilizes tick time sampling with quote price. Some of the realized volatility estimators further require choices about tuning parameters such as a kernel bandwidth or 'block size' for their practical application. For example, implementing quantile-based realized variance requires choices to be made with respect to the number of blocks (K), block length (m_K), quantiles, (λ) and quantile weights (α). These as well as other tractability issues often make asymptotic comparison of the estimators impossible. Selecting a particular estimator for empirical analysis is also rather complicated because there are no clear procedures for choosing the optimal estimators.

Unfortunately, previous studies on selecting realized volatility estimators of asset return variation have centered on "recommending a sampling frequency based on the underlying theory using plug-in type estimators of nuisance parameters. For some estimators, a formula for the optimal sampling frequency under a set of assumptions is derived and can be computed using estimates of higher order moments. However, these formulas are usually heavily dependent on assumptions about the microstructure noise and efficient price process, such as independence of the noise from the price and a lack of serial correlation in the noise" [2].

Furthermore, the empirical performance of the realized volatility estimators is not known. Papers that introduce new realized volatility estimators only provide evidence about the new estimator's advantages over previous estimators "...in the form of theoretical properties of the estimators such as consistency, asymptotic efficiency, rate of convergence, or results from Monte Carlo simulations using common stochastic volatility models. These comparisons inevitably require making specific assumptions on important properties of the price process" [2].

A number of papers have implemented and analyzed the properties of the realized volatility approach (see for example, Andersen, Bollerslev, Diebold and Labys [3], Andersen, Bollerslev, Diebold and Ebens [4], Ebens [5], Areal and Taylor [6], etc.). Although the findings of these studies support the realized volatility approach, they are mainly statistical in nature. A separate question is whether using realized volatility measures for estimating and forecasting asset return volatility can considerably affect the decisions that depend on conditional volatility estimates. Unfortunately, "it is not clear, whether using realized volatility leads to more accurate option prices or better investment management decisions" [7].

Objectives

The goal of this study is to evaluate high-frequency volatility estimators based on the profits that option dealers would derive from trading based on alternative high-frequency volatility estimates (forecasts). With this in mind, we examined the economic benefit of realized volatility estimators in option pricing and trading. The specific objectives of the study are:

1) to estimate the volatility of Bank of America, Coca-Cola and Microsoft stocks using high-frequency volatility estimators;

2) to determine the price of European call and put options on Bank of America, Coca-Cola and Microsoft stocks using high-frequency volatility estimate/forecast;

3) to ascertain the high-frequency volatility-based option trading strategy that yields the optimal profit or loss from trading.

The object of the research is the US stock market while the subject is methods for estimating the volatility of the financial assets returns based on high-frequency data.

Empirical Literature Review

Few existing studies have evaluated the performance of realized volatility estimators based on economic criteria. Most of the recent studies in this area have employed mainly traditional statistical criteria in their evaluations.

Ping and Li [8] studied the performance of the truncated two-scale realized volatility estimator (TTS-RV) in forecasting the realized variance (RV) of the SSE Composite Index (SSEC index). The results of the study show that the TTSRV can describe the continuous and jump processes of RV with higher accuracy: in fact, the TTSRV outperformed the other competing models in both statistical and economic evaluations.

Chin and Lee [9] examined the predictive ability of high-frequency long-memory volatility models with multipower variation volatility estimators. The findings of the study indicate that higher-power variation volatility proxies (bipower, tripower and quadpower) perform better than the realized volatility and fractionally integrated ARCH models for both in-sample and out-of-sample evaluations.

Zhou [10] analyzed the accuracy of realized volatility measures in the measurement of the daily volatility of real estate investment trust (REIT) returns in major global REIT markets (Australia, Japan, UK and US.). The study revealed that no single estimator performs best in all markets and that the performance of the estimators is market-dependent. The study identified the following as the best realized volatility estimators: RV_1m for Australia, RVac1_1m for Japan, RVac1_5m for UK, and RK_1m for US.

Studies which employ economic criteria (utility and profit-based evaluation approaches) to assess the quality of high-frequency volatility forecasts include Bandi and Russell [11], Bandi and Russell [12], Bandi, Russell and Yang [13], De Pooter, Martens and Van Dijk [14] (in a multivariate context), Fleming, Kirby and Ostdiek [15] and Fleming, Kirby and Ostdiek [7] (in the no noise case). Bandi and Russell [11] assessed the forecasting performance of the optimally-sampled and fixed-interval realized variance estimators in a portfolio choice problem and in option pricing. Based on the long-run utility that a mean-variance representative investor derives from alternative variance forecasts, the study found that “a risk-averse investor is willing to pay between 25 and 300 basis points per year to employ variance forecasts based on optimal intervals” [11].

Bandi and Russell [12] consider volatility forecasting for the purpose of option pricing. Their findings show that explicit optimization of the finite sample mean square error properties of the realized variance results in accurate forecasts and considerable economic gains. However, a related study by Bandi, Russell and Yang [13] also established that although “estimators with superior finite sample mean squared-error properties generate higher average profits and higher Sharpe ratios, the optimal forecasts are in general not the forecasts derived from mean square error-optimal estimates” [13].

Other studies which provide a comprehensive analysis of the forecasting performance of the realized volatility estimators include Andersen, Bollerslev and, Meddahi [16] and Ghysels and Sinko [17]. However, the forecasting metrics in these studies are statistical in nature. Andersen et al. [16] use Mincer–Zarnowitz-style regression models to predict the variance while Ghysels and Sinko [17] employ MIDAS regressions.

Data and Methods

Data

This study used option and stock price data on Microsoft, Bank of America, and Coca-Cola for the analysis. Because historical option data are not available, we downloaded the option quotes (option chains) for Microsoft, Bank of America, and Coca-Cola stocks at the close of each trading from Yahoo Finance. The sample period for the option chains covered 2021.08.02–2021.09.03. The download of the option quotes started on 2021.08.02 and the options had an expiration date of 2021.09.03. As a result, we acquired option quotes for only 24 trading days to be used for our analysis. We also obtained historical close prices (5-minute close price data) for the stocks underlying the option chains from Finam for the period from

2020.02.06 to 2021.09.02. Finally, we obtained the continuously compounded risk-free rate for the US from the US Department of the Treasury.

Methods

Methods of High-frequency Volatility Estimation

To estimate the volatility of the stocks underlying the option contracts, this study used the recently proposed high-frequency volatility estimators (realized measures) available in the literature on volatility. Let us briefly describe the estimators used.

1) Realized Variance/Realized Volatility (rRVar) calculates the daily Realized Variance. The realized variance or realized volatility (RV) is the sum of the squared intraday returns.

2) Realized covariances via subsample averaging (rAVGCov) calculates realized variances by averaging RVs across partially overlapping grids. It was first introduced by Zhang et al. [18]

3) Modulated realized covariance (rMRCov) calculates the univariate or multivariate pre-averaged estimator of Hautsch and Podolskij [19].

4) Two-time scale covariance estimator (rTSCov) calculates the two-time scale covariance matrix proposed in Zhang et al. [18] and Zhang [20].

5) Robust two-time scale covariance estimator (rRTSCov) calculates the robust two-time scale covariance matrix proposed in Boudt and Zhang [21].

6) Realized kernel estimator (rKernelCov) calculates realized covariance using a kernel estimator. The types of available kernels are Rectangular, Bartlett, Second-order, Epanechnikov, Cubic, Fifth, Sixth, Seventh, Eighth, Parzen, Tukey–Hanning and modified Tukey–Hanning. For this study, we used the Epanechnikov, Parzen and modified Tukey–Hanning kernel estimators.

7) Realized bipower covariance (rBPCov) calculates the Realized BiPower Covariance (rBPCov), defined by Barndorff-Nielsen and Shephard [22].

8) Min Realized Variance (rMinRVar) calculates the rMinRVar, defined by Andersen et al. [23].

9) Median Realized Variance (rMedRVar) calculates the rMedRVar, defined by Andersen et al. [23].

10) Threshold Covariance (rThresholdCov) calculates the threshold covariance matrix proposed by Gobbi and Mancini [24].

11) Hayashi-Yoshida covariance (rHYCov) calculates the Hayashi-Yoshida Covariance estimator [25].

12) Realized outlyingness weighted covariance (rOWCov) calculates the Realized Outlyingness Weighted Covariance (rOWCov) defined in Boudt et al. [26].

13) Realized semi-variance of high-frequency return series (rSVar) calculates the realized semi-variances, defined by Barndorff-Nielsen et al. [27]. In this this study, we estimated both realized semi-variance_downside and semi-variance_upside.

14) Range-based Realized Variance (RRV) calculates the realized range-based estimator suggested by Christensen and Podolskij [28]. For this estimator, we chose $M_k = 10$ and $\lambda = 0.7$ where M_k is the block length, i.e., the number of high-frequency returns in each non-overlapping block (K) and λ is the variance factor. Notice that $M_k = 10$ and $\lambda = 0.7$ is for moderately liquid assets.

15) Quantile-based Realized Variance (QRV) calculates the quantile-based realized variance developed by Christensen, Oomen and Podolskij [29]. For this estimator, we chose the following hyperparameters based on the guidelines provided by Christensen et. al: i) $M_k = 40$, $\lambda = (0.9,0.93,0.95)$ and $\alpha = (0.3,0.3,0.4)$, ii) $M_k = 20$, $\lambda = (0.8,0.85,0.9,0.95)$ and $\alpha = (0.1,0.2,0.3,0.3)$. Here M_k is the block length, λ, is the quantiles of the returns and α is the quantile weight. This gave us two estimators under this approach, Quantile Realized Variance (QRV) and modified Quantile Realized Variance (mQRV).

For ease of tabular presentations, we abbreviated the above high-frequency volatility estimators as follows: RV, AV, MRC, TS, RTS, Epa, Par, mTH, BP, MiRV, MeRV, Thr, HY, OW, SV.do, SV.up, RRV, QRV, mQRV.

Models for Forecasting the Realized Volatility

To forecast the realized volatility, the study adopted the Heterogeneous Autoregressive model of Realized Variance (HAR-RV model) developed by Corsi [30]. The HAR-RV model is a predictive model for the daily integrated volatility. It predicts future volatility using a daily, a weekly and a monthly component. The HAR-RV model assumes that volatility can be depicted as the sum of volatilities created by specific groups of market players with each of them having different time boundaries. The dynamics of the model are given by:

$$RV_{t+1d}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \varepsilon_{t+1d}^{(d)},$$

where $RV_t^{(d)}$ is the realized variances for day t, $RV_t^{(w)}$ is the average realized variance over the last week (the last 5 trading days) calculated as follows:

$$RV_t^{(w)} = \frac{1}{5}\left(RV_t^{(d)} + RV_{t-1}^{(d)} + RV_{t-2}^{(d)} + RV_{t-3}^{(d)} + RV_{t-4}^{(d)} \right)$$

and $RV_t^{(m)}$ is the average realized variance over the last month (the last 22 trading days) calculated as follows:

$$RV_t^{(m)} = \frac{1}{22}\left(RV_t^{(d)} + RV_{t-1}^{(d)} + \ldots + RV_{t-20}^{(d)} + RV_{t-21}^{(d)} \right).$$

The HAR model was estimated by ordinary least squares under the assumption that at time t, the conditional mean of $\varepsilon_{t+1d}^{(d)}$ is equal to zero.

For the purposes of this study, we used a HAR-RV model with the following details:

- Type: HAR;
- Lags: 1 5 22;
- Window Type: “rolling”;
- Maximum lags 22.

We used a 5-minute return series (30,808 5-returns) for the period from 2020.02.06 to 2021.09.02 to forecast the volatility of the stocks underlying the option chains. We assumed that essentially all the information from the high-frequency data is contained in the 5-minute data, hence the decision to estimate and forecast the volatility of the stocks using a sampling frequency of 5 min. Other works considered in choosing a model for the study included Rodionov et al. [31], Rudskaya et al. [32], Zaitsev [33], Zaitsev et al. [34]

Option Pricing Model

The Black–Scholes–Merton (BSM) model is still widely used today and is regarded as one of the best ways of determining the fair price of options. In view of this, we used the BSM formulas for the prices of European call and put options in pricing the option contracts. These formulas are

$$c = S_0 N(d_1) - K e^{-rT} N(d_2),$$
$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

In the formulas above, c and p represent the European call and put option prices respectively, T is the maturity date of the options contract, S_0 is the current stock price, K is the strike price, σ is the volatility of the underlying stock, r is the continuously compounded risk free rate per annum and the function $N(x)$ is the cumulative probability distribution function for a variable with a standard normal distribution.

Method of Volatility Forecast Evaluation

Well-posed economic loss functions are important measures for the performance of alternative volatility estimators. Several studies (see for example Degiannakis and Filis [35], Leitch and Tanner [36], Satchell and Timmermann [37]; etc.) argue that volatility forecasts should be evaluated based on their economic use, rather than statistical loss functions. Using the profit measure, Leitch and Tanner [36] and Satchell and Timmermann [37] show that a forecast can be of low value according to forecast error statistic but at the same time be very profitable (the authors found a weak relationship between the statistical forecast accuracy and the forecast's profit). In view of the above, in this study we used economic criteria to evaluate the forecasting ability of the high-frequency volatility estimators.

According to Engle, Kane and Noh [38], a reasonable criterion for choosing the best method for forecasting the volatility of asset return from a set of competing methods would be the expected incremental profit from replacing the lesser forecast with the better one. The central idea of this profit-based evaluation approach is to construct a trading rule and examine which forecasting method or model produces the highest return on average either on an unadjusted or on a risk-adjusted basis. This evaluation approach has a number of advantages. In the first place, it is an economic value model. Secondly, other approaches which assess volatility forecasting methods in isolation only identify the most accurate volatility forecast, that is the forecast which best fits the actual data. In the profit-based evaluation approach, the best forecasting method could be any of the competing methods. Here, the test is not how well the estimators fits the data, but how well the estimator identifies the direction of movement in the data irrespective of size. The question that the authors pose is therefore whether a method/model can indicate if volatility is under or over priced in a manner that allows to make an economic gain.

This study adopted the profit-based evaluation approach to compare the performance (forecasting ability) of the high-frequency volatility estimators. We analyzed the forecasting ability of the high-frequency volatility estimators in the actual options market as opposed to a hypothetical/simulated option market based on the profit that option dealers would derive from trading on the basis of alternative high-frequency volatility forecasts. To this end, we traded (bought and sold) European call and put options on Coca-Cola, Microsoft, and Bank of America stocks for a period of 24 trading days (2021.08.03–2021.09.03). Using our high-frequency volatility forecasts, we created and followed the option trading strategies below for a period of 24 trading days:

1) Covered call/Reverse covered call strategy;
2) Protective put/Reverse protective put strategy;
3) Bull call spread/Bear call spread strategy;
4) Bear Put spread/Bull Put spread strategy;
5) Long straddle/Short straddle strategy;
6) Long strangle/Short strangle strategy.

We then computed the total (aggregated) profit of the volatility estimators (Methods) and made comparisons. The estimator with the highest dollar profit was regarded as the best volatility estimator. Outlined below are the specific steps involved in our evaluation methodology:

1) Agents are provided with information about a European call and put option satisfying specific selection criteria. The call and put options are not required to be at the money options; the nearest to the "**at the money**" options were selected. This was done to ensure that it was appropriate to use the Black-Sholes-Merton model and to minimize the effect of volatility smile since the Black-Sholes-Merton model assumes that volatility is constant.

2) Agents price a call and put option using their high-frequency volatility forecasts and the Black–Scholes–Merton model.

3) If an agent's price for the call option is ***higher*** than the market price, then the agent will buy the call option at the market price and vice versa. Also, if an agent's price for the put option is ***lower*** than the market price, then the agent will buy the put option at the market price and vice versa. (NB: agents traded on only one option.)

4) Agents create the option trading strategies identified above. Agents hold the options for one trading day and the payoff/profit from the strategies are computed. Positive profits mean that the method for forecasting volatility accurately forecasts under/over pricing in the market as indicated by implied volatility measures which are too low or high.

5) Agents ascertain the total payoff/profit of each volatility estimator by summing the total payoffs/profits of the option trading strategies estimator-by-estimator. The estimator with the highest profit is regarded the best volatility estimator.

Below is a brief description of the option trading strategies we created and followed during our investigations:

1) Covered call strategy: Short European call + Long position in a stock. It is created when the estimated option price is lower than the market price (Sell Strategy). We are taking the position that the stock price will not increase, and the option will expire worthless, so we sell a call option and RECEIVE the premium.

2) Reverse of the covered call strategy: Long European call + short position in a stock. It is created when the estimated option price is higher than the market price (Buy Strategy). We think that the stock price will rise so we buy a call option.

3) Protective put strategy: Long European put + Long position in a stock. It is created when the estimated option price is lower than the market price. We think the stock price will fall so we BUY a put option.

4) Reverse of the protective put strategy: Short European put + short position in a stock. It is created when the estimated option price is higher than the market price. We don't think the stock price will fall so we SELL a put option to earn the premium.

5) Bull call spread strategy: Long European call with strike K_1 + short European call with strike K_2 on the same stock, $K_1 < K_2$. The options have the same expiration date. It is created when the estimated option price is higher than the market price.

6) Bear call spread strategy: Long European call with strike K_1 + short European call with strike K_2 on the same stock, $K_1 > K_2$. The options have the same expiration date. It is created when the estimated option price is lower than the market price.

7) Bear put spread strategy: Long European put with strike K_1 + short European put with strike K_2 on the same stock, $K_1 > K_2$. The options have the same expiration date. It is created when the estimated option price is lower than the market price.

8) Bull put spread strategy: Long European put with strike K_1 + short European put with strike K_2 on the same stock, $K_1 < K_2$. The options have the same expiration date. It is created when the estimated option price is higher than the market price.

9) Long straddle: Long call + Long put (Buy Strategy). The call and put options have the same expiration date and strike price. They also have the same underlying security. It is created when i) the estimated price of the call option is higher than the market price and ii) the estimated price of the put option is lower than the market price.

10) Short straddle: Short call + Short put (Sell Strategy). The call and the put options have the same expiration date and strike price. They also have the same underlying security. It is created when i) the estimated price of the call option is lower than the market price and ii) the estimated price of the put option is higher than the market price.

11) Long strangle: Long call + Long put. The strike prices are different but, the same expiration date and the same underlying security.

12) Short strangle: Short call + Short put. The strike prices are different but the same expiration date and the same underlying security.

Results and Discussion

Below we present the results obtained during the study on the problem posed. First, we consider the findings obtained for Bank of America stock. This is then followed by a summary of the findings obtained for Coca-Cola and Microsoft stocks respectively.

Bank of America Stock (BAC)

Table 1 below summarizes the results of the profit-based evaluation of the high-frequency volatility estimators for Bank of America stock options. The table shows the total (aggregated) profit of the volatility estimators from six option trading strategies (Covered call/Reverse covered call strategy, Protective put/Reverse protective put strategy, Bull call spread/Bear call spread strategy, Bear Put spread/Bull Put spread strategy, Long straddle/Short straddle strategy, and Long strangle/Short strangle strategy) over a period 24 trading days (2021.08.03–2021.09.03 inclusive).

Table 1. Profit (US$) of the volatility estimators from all the option trading strategies for 24 days-BAC

Volatility Estimator	Total Profit or Loss	Rank
RV	–10.73	4
AV	–68.24	17
MRC	–61.18	9
TS	–65.56	16
RTS	–61.08	8
Epa	20.42	1
Par	20.42	1
mTH	20.42	1
BP	–69.73	18
MiRV	–73.87	19
MeRV	–64.20	14
Thr	–61.30	12
HY	–46.99	6
OW	–61.18	10
SV.do	–61.18	11
SV.up	–62.80	13
RRV	–46.03	5
QRV	–64.22	15
mQRV	–56.43	7

Note. The cell highlighted in green indicates the best volatility estimator based on profit from trading.

As evident from Table 1, the best volatility estimators based on profit from trading for Bank of America stock are the realized kernel estimators (Epa, Par, mTH). These estimators made a total profit of $20.42

each during the 24 trading days. The positive profits earned by these volatility estimators imply that they correctly forecast under/over pricing in the market as indicated by implied volatility measures which are too low/high. Table 1 also shows that all the other volatility estimators recorded losses. Table 2 below shows a breakdown of the total profit of the volatility estimators according to the option trading strategies.

Table 2. Profit (US$) of the option trading strategies for 24 trading days-BAC

Volatility_ Estimators	total_Cov_ RCov.Call	total_Prot_ RProt.Put	total_BullCall_ BearCall	total_BearPut_ BullPut	total_ LongStrad_ ShortStrad	total_ LongStrang_ ShortStrang
RV	–13.74	4.38	1.47	6.77	2.62	–12.23
AV	5.65	–13.82	1.43	–5.50	–29.72	–26.29
MRC	15.54	–15.28	–1.05	–4.38	–29.72	–26.29
TS	11.35	–15.28	–1.25	–4.38	–29.72	–26.29
RTS	15.54	–15.28	–0.95	–4.38	–29.72	–26.29
Epa	–15.54	6.45	0.85	5.58	14.36	8.71
Par	–15.54	6.45	0.85	5.58	14.36	8.71
mTH	–15.54	6.45	0.85	5.58	14.36	8.71
BP	0.65	–12.71	0.60	–3.72	–28.80	–25.75
MiRV	1.65	–13.82	–1.12	–5.50	–28.80	–26.29
MeRV	11.75	–15.28	–0.29	–4.38	–29.72	–26.29
Thr	15.62	–15.28	–1.25	–4.38	–29.72	–26.29
HY	13.75	–15.28	–1.25	–4.38	–13.55	–26.29
OW	15.54	–15.28	–1.05	–4.38	–29.72	–26.29
SV.do	15.54	–15.28	–1.05	–4.38	–29.72	–26.29
SV.up	13.82	–15.28	–0.95	–4.38	–29.72	–26.29
RRV	–8.96	–7.70	–1.28	–0.10	–13.88	–14.11
QRV	12.39	–15.28	–0.95	–4.38	–29.72	–26.29
mQRV	–0.45	–12.71	–2.56	–4.38	–17.86	–18.47

Notes. i. The highlighted cells indicate the maximum profit under each option trading strategy. ii. The best option trading strategy is the covered call-reverse covered call strategy. The preferred volatility estimator under this strategy is the threshold estimator.

Table 2 shows that the best option trading strategy for Bank of America stock is the covered call/reverse covered call strategy (**Cov-RCov.Call**) with a total maximum profit of $15.62. The preferred volatility estimator under this strategy is the Threshold Covariance estimator (**Thr**). Following the Threshold Covariance estimator are the kernel estimators (**Epa**, **Par**, **mTH**) with a profit of $14.36 under the long straddle/short straddle strategy.

Coca-Cola Stock (KO)

Table 3 summarizes the results of the profit-based evaluation of the high-frequency volatility estimators for Coca-Cola stocks options. The table shows the total (aggregated) profit earned by the volatility estimators from six option trading strategies (Covered call/Reverse covered call strategy, Protective put/Reverse protective put strategy, Bull call spread/Bear call spread strategy, Bear Put spread/Bull Put spread strategy, Long straddle/Short straddle strategy, and Long strangle/Short strangle strategy) over a period of 24 trading days (2021.08.03–2021.09.03 inclusive).

The results in Table 3 above confirm that the best volatility estimator based on profit from trading for Coca-Cola stock is the two-time scale covariance estimator (TS). It had a total profit of $26.88 during the 24 trading days. Closely following the TS estimator are the Realized Semi-variance_downside (**SV.do**), Realized Semi-variance_upside (**SV.up**) and Threshold Covariance (Thr) estimators with the total profits of \$25.30, \$25.24 and \$25.22 respectively. The positive profits imply that the volatility estimator correctly forecasts under/over pricing in the market as indicated by implied volatility measures which are too low/high. Table 4 provides further analysis for the performance of the volatility estimators under each option-trading strategy.

Table 3. Profit (US\$) of the volatility estimators from all the option trading strategies for 24 days-KO

Volatility Estimator	Total Profit or Loss	Rank
RV	–18.20	15
AV	7.28	11
MRC	23.94	6
TS	26.88	1
RTS	24.16	5
Epa	–22.08	17
Par	–22.08	17
mTH	–22.08	17
BP	–3.64	13
MiRV	5.62	12
MeRV	7.80	10
Thr	25.22	4
HY	13.23	8
OW	23.54	7
SV.do	25.30	2
SV.up	25.24	3
RRV	–19.94	16
QRV	13.10	9
mQRV	–8.68	14

Note. The cell highlighted in green indicates the best volatility estimator based on profit from trading.

Table 4. Profit (US\$) of the option trading strategies for 24 trading days-KO

Volatility_ Estimators	total_Cov_ RCov.Call	total_Prot_ RProt.Put	total_BullCall_ BearCall	total_BearPut_ BullPut	total_ LongStrad_ ShortStrad	total_ LongStrang_ ShortStrang
RV	–9.20	8.63	1.86	–1.41	–10.03	–8.05
AV	2.12	1.09	2.44	–2.81	–4.37	8.81
MRC	9.90	–9.53	2.44	–2.59	10.31	13.41
TS	9.52	–5.89	2.44	–2.07	9.47	13.41
RTS	9.90	–9.53	2.44	–2.37	10.31	13.41
Epa	–9.92	8.27	1.88	–1.27	–11.79	–9.25
Par	–9.92	8.27	1.88	–1.27	–11.79	–9.25
mTH	–9.92	8.27	1.88	–1.27	–11.79	–9.25
BP	–1.98	4.13	2.44	–1.07	–7.95	0.79
MiRV	0.20	2.79	2.44	–1.07	–5.11	6.37
MeRV	2.20	0.41	2.44	–2.77	–3.29	8.81

Thr	9.90	–8.47	2.44	–2.37	10.31	13.41
HY	3.90	0.41	2.44	–1.73	–5.20	13.41
OW	9.92	–9.95	2.44	–2.59	10.31	13.41
SV.do	9.92	–8.41	2.44	–2.37	10.31	13.41
SV.up	9.92	–8.47	2.44	–2.37	10.31	13.41
RRV	–9.00	6.93	2.06	–1.01	–10.63	–8.29
QRV	0.86	0.99	2.44	–0.73	0.41	9.13
mQRV	–3.90	5.35	2.66	–1.01	–8.29	–3.49

Notes. i. The highlighted cells indicate the maximum profit under each option trading strategy. ii. The best option trading strategy is Long strangle-Short strangle strategy. The preferred volatility estimators under this strategy are MRC, TS, RTS, Thr, HY, OW, SV.do and SV.up.

The results in Table 4 suggest that the best option trading strategy for Coca-Cola stock options is the long strangle/short strangle strategy (**LongStrang-ShortStrang**). It had a total profit of $13.41 (max). The preferred volatility estimators under this strategy are the Modulated Realized Covariance (**MRC**), Two-time Scale Covariance (**TS**), Robust Two-Time Scale Covariance (**RTS**), Threshold Covariance (**Thr**), Realized Outlyingness Weighted covariance (**OW**), Realized Semi-Variance_downside (**SV.do**) and Realized Semi-Variance_upside (**SV.up**) estimators. All these estimators made a profit of $13.41 each during the period in question. However, we can observe that their performance declines for other option trading strategies.

Microsoft Stock (MSFT)

Table 5 gives the results for the profit-based evaluation of the volatility estimators for Microsoft stock options. The table summarizes the total profit of the volatility estimators from six option trading strate-

Table 5. Profit (US$) of the volatility estimators from all the option trading strategies for 24 days-MSFT

Volatility Estimator	Total Profit or Loss	Rank
RV	16.87	3
AV	–70.57	14
MRC	–76.01	16
TS	–23.03	8
RTS	–74.51	15
Epa	–6.27	4
Par	–6.27	4
mTH	–6.27	4
BP	–15.21	7
MiRV	–57.71	12
MeRV	–62.47	13
Thr	–106.83	19
HY	–24.45	9
OW	–76.01	17
SV.do	–81.29	18
SV.up	–34.85	10
RRV	54.07	1
QRV	–56.39	11
mQRV	19.33	2

Note. The cell highlighted in green indicates the best volatility estimator based on profit from trading.

gies (Covered call/Reverse covered call strategy, Protective put/Reverse protective put strategy, Bull call spread/Bear call spread strategy, Bear Put spread/Bull Put spread strategy, Long straddle/Short straddle strategy, and Long strangle/Short strangle strategy) over a period 24 trading days (2021.03–2021.09.03 inclusive).

As seen from Table 5, the best volatility estimator for Microsoft stock is the Range-based Realized Variance estimator (RRV) with a total profit of $54.07 during the 24 trading days. This is followed by the modified Quantile Realized Variance (mQRV) and Realized Variance (RV) estimators with the total profits of $19.33 and $16.87, respectively. The positive profits earned by the RRV, mQRV, and RV estimators imply that these volatility estimators correctly forecast under/over pricing in the market as indicated by implied volatility measures which are too low/high. Table 5 also shows that all the other competing volatility estimators recorded losses. Table 6 presents analysis of the total profits under each of the option trading strategies.

Table 6. Profit (US$) of the option trading strategies for 24 trading days-MSFT

Volatility_ Estimators	total_Cov_ RCov.Call	total_Prot_ RProt.Put	total_BullCall_ BearCall	total_BearPut_ BullPut	total_ LongStrad_ ShortStrad	total_ LongStrang_ ShortStrang
RV	–51.48	37.05	15.27	4.66	32.62	–21.25
AV	–14.78	–22.47	–0.81	–7.24	9.30	–34.57
MRC	69.64	–71.57	6.37	–7.10	–37.58	–35.77
TS	–22.44	–14.31	–1.07	–7.24	35.94	–13.91
RTS	69.64	–70.07	6.37	–7.10	–37.58	–35.77
Epa	–47.12	27.85	15.37	2.42	18.72	–23.51
Par	–47.12	27.85	15.37	2.42	18.72	–23.51
mTH	–47.12	27.85	15.37	2.42	18.72	–23.51
BP	–32.00	–22.47	3.47	–10.68	36.58	9.89
MiRV	–32.00	–17.75	–4.43	–10.68	9.36	–2.21
MeRV	–18.96	–9.93	–1.07	–7.24	9.30	–34.57
Thr	35.00	–63.13	4.23	–9.58	–37.58	–35.77
HY	–22.44	–14.31	–0.83	–7.24	34.28	–13.91
OW	69.64	–71.57	6.37	–7.10	–37.58	–35.77
SV.do	62.86	–70.07	6.37	–7.10	–37.58	–35.77
SV.up	67.40	–63.13	6.37	–14.16	–15.32	–16.01
RRV	–40.48	28.27	11.93	–8.74	47.48	15.61
QRV	14.58	–20.45	–1.41	–7.24	–31.18	–10.69
mQRV	–44.80	–12.71	–2.03	–8.90	46.24	41.53

Note. i. The highlighted cells indicate the maximum profit under each option trading strategy. ii. The best option trading strategy is the Covered call/Reverse covered call strategy. The preferred volatility estimator under this strategy are MRC, RTS and OW.

The results in Table 6 confirm that the best option trading strategy for Microsoft stock options is the Covered Call/Reverse Covered Call strategy (**Cov-RCov.Call**) with a total profit of $69.64 (max). The preferred volatility estimators under this strategy are Modulated Realized Covariance (**MRC**), Robust Two-Time Scale Covariance estimator (**RTS**) and Realized Outlyingness Weighted Covariance (**OW**). The next best volatility estimator is the Realized Semi-Variance_upside (**SV.up**) with a profit of $67.40.

Conclusion

This study assessed the forecasting ability of high-frequency volatility estimators based on profit that option dealers derive from trading based on alternative high-frequency volatility forecasts. The findings of the study revealed that:

1) for Bank of America stock, the best volatility estimator based on profit from trading is the realized kernel estimators (**Epa**, **Par**, **mTH**). These estimators made a total profit of $20.42 each during the 24 trading days;

2) for Coca-Cola stock, the best volatility estimator based on profit from trading is the Two-time scale covariance estimator (**TS**). It had a total profit of $26.88 during the 24 trading days;

3) for Microsoft stock, the preferred volatility estimator in terms of profit from trading is the Range-based Realized Variance estimator (**RRV**) with a total profit of $54.07 over a period of 24 trading days;

4) the optimal high-frequency volatility-based option trading strategy for Bank of America and Microsoft stock options based on profit/ loss from trading is the Covered call/reverse covered call strategy (**Cov-RCov.Call**) while for Coca-Cola stock options the Long strangle/short strangle strategy (**Long-Strang-ShortStrang**) is preferred.

Based on the above findings, it is concluded that high-frequency volatility forecasts by the Realized Kernel estimators, Two-Time Scale Covariance estimator and Range-based Realized Variance estimator are useful in pricing and trading Bank of America, Coca-Cola and Microsoft stock options, respectively, and could result in significant economic benefits (profit from trading).

Directions for further research

This study analyzed the performance of high-frequency volatility estimators by focusing on stock returns on the US market. Since different asset classes and markets exhibit different volatility behavior and patterns, it is recommended that future studies investigate the performance of realized volatility estimators for other asset classes and in different markets.

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