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THE MOTION OF AN UNCHARGED RELATIVISTIC PARTICLE: AN ANALYSIS OF ITS INTEGRABLE MOTION INTEGRALS DYNAMICS

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Abstract. In this study, the dynamics of a relativistic particle that does not have an electric charge and is under the action of an external force has been analyzed on the basis of the special theory of relativity. The accelerated motion of such a particle was also investigated in the absence of an external electromagnetic field specified by the scalar and vector potentials. An analytical method for efficient writing of classical equations of relativistic dynamics was developed, and an estimate of integrals of motion was carried out. The motion integral was established to be valid both for charged particles and uncharged ones. The use of motion integrals made it possible to describe the relationship between dynamic parameters. The dependence of the ξ space-time coordinate on the integral of motion was also obtained.

Keywords: motion integral, relativistic particle energy, radiative friction, radiation intensity

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АНАЛИЗ ДИНАМИКИ ИНТЕГРИРУЕМЫХ ИНТЕГРАЛОВ ДВИЖЕНИЯ НЕЗАРЯЖЕННОЙ РЕЛЯТИВИСТСКОЙ ЧАСТИЦЫ

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Аннотация. На основе специальной теории относительности в работе проанализирована динамика релятивистской частицы, не имеющей электрического заряда, под действием внешней силы. Исследовано ускоренное движение такой частицы и в отсутствие внешнего электромагнитного поля, заданного скалярным и векторным потенциалами. Разработан аналитический метод эффективной записи классических уравнений релятивистской динамики и проведена оценка интегралов движения. Установлено, что интеграл движения справедлив и для заряженных частиц, и для незаряженных. Использование интегралов движения позволило описать связь динамических параметров. Получена также зависимость пространственно-временной координаты ξ от интеграла движения.

Ключевые слова: интеграл движения, энергия релятивистской частицы, радиационное трение, интенсивность излучения незаряженной частицы

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Introduction

The motion equation of a particle of mass m and a charge q in the electromagnetic field has been studied extensively in classical relativistic electrodynamics, and has the following form [1]:

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}[\mathbf{V} \times \mathbf{H}], \tag{1}$$

where the momentum $\mathbf{p}$ of the particle and its velocity $\mathbf{V}$ are related as follows:

$$\mathbf{p} = \frac{m\mathbf{V}}{\sqrt{1 - \frac{V^2}{c^2}}}. \tag{2}$$

The change $d\varepsilon$ in the energy of the particle was widely established to be determined by the equation

$$\frac{d\varepsilon}{dt} = q\mathbf{E} \cdot \mathbf{V}, \tag{3}$$

where

$$\varepsilon = \frac{mc^2}{\sqrt{1 - \frac{V^2}{c^2}}} = \sqrt{m^2c^4 + c^2p^2}. \tag{4}$$

From Eqs. (1) and (4) the relationship between the energy ε and the longitudinal component $\mathbf{n}{\cdot}\mathbf{p}$ of the momentum $\mathbf{p}$ of the particle can be represented as an integral of motion in the following form:

$$\varepsilon - \mathbf{n}{\cdot}\mathbf{p}c = \gamma_0 c,\ \gamma_0 = \frac{mc\left(1 - \mathbf{n}{\cdot}\boldsymbol{\beta}\right)}{\sqrt{1 - \beta^2}},\ \boldsymbol{\beta} = \frac{\mathbf{V}}{c}, \tag{5}$$

where $\mathbf{n}$ is the normal vector directed along the particle's trajectory.

It was demonstrated in Ref. [2] that the integral (5) was the same for a plane monochromatic electromagnetic wave and a constant uniform magnetic field.

The acceleration and radiation of relativistic particles were explored in several works [3, 4]. However, generally, the dynamics of particle acceleration was considered under the action of external ponderomotive forces of electromagnetic nature [5 – 9].

In Refs. [2, 10 – 14], the motion integrals of a charged particle were obtained, and the particle momentum was demonstrated to be written as an explicit function of the zero coordinate ξ ($\xi = t - z/c$ where t, z are the laboratory time and coordinate respectively, and c is the velocity of light in vacuo).

In the absence of external forces acting on the particle, the particle's motion is free and obeys the basic dynamics equation. In this study, we focused on the non-electromagnetic 2D force action upon a particle. We intend to show that the motion integral (5) is valid for any particle motion in free space.

The objective of this study is to analyze the dynamics of an uncharged relativistic particle in the absence of external electromagnetic fields, i. e., $\mathbf{E} = \mathbf{H} = 0$, scalar $\varphi = 0$, and vector field potentials $\mathbf{F}$ for the following force acting $\mathbf{A} = 0$, on the particle is of the form

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \neq 0, \tag{6}$$

where $\mathbf{p}$ is the relativistic momentum of the particle (see Eq. (2)).

Eventually, we show that the integral of motion is also applicable in the absence of fields ($\mathbf{E} = \mathbf{H} = 0$, $\varphi = 0$, $\mathbf{A} = 0$). We consistently derive formulas for the space-time coordinate ξ, coordinate $\mathbf{r}$, velocity $\boldsymbol{\beta}$, momentum $\mathbf{p}$, energy ε and radiation intensity $I = -d\varepsilon/dt$ of the particle, depending on the motion integrals Q_t and Q_ξ.

Finally, we demonstrate the dependence of the dynamic parameters (ξ, $\mathbf{r}$, $\boldsymbol{\beta}$, $\mathbf{p}$, ε and I) on the motion integral γ.

The main goal of this work is to obtain invariant forms of the integrals of motion

$$\gamma = \gamma(\mathbf{n}\cdot\mathbf{r}, t, Q_t) \text{ and } Q_t = Q_t(\mathbf{n}\cdot\mathbf{r}, t, \gamma)$$

in 1 + 1 dimensions, which are mutually expressed in terms of the coordinate $\mathbf{n}\cdot\mathbf{r} = \mathbf{n}\cdot\mathbf{r}(t, Q_t, \gamma)$ and proper time $t = t(\mathbf{n}\cdot\mathbf{r}, Q_t, \gamma)$ of the particle, based on the law of conservation of energy – momentum for a relativistic particle (4).

Moreover, the aim is to search for an invariant form as the motion integrals for energy $E = E(r, t, \gamma, Q_t)$ and momentum $\mathbf{n}\cdot\mathbf{P} = \mathbf{n}\cdot\mathbf{P}(\mathbf{r}, t, \gamma, Q_t)$ of the particle.

An analysis of the space – time coordinate ξ

We introduce the space – time coordinate ξ [1, 2, 10 – 15] such as

$$\xi = t - f\frac{\mathbf{n}\cdot\mathbf{r}}{c}, \tag{7}$$

where t is the laboratory time, $\mathbf{n}$ is the normal vector, $\mathbf{r}$ is the laboratory coordinate, c is the speed of light, and f characterizes the direction of motion of a particle or wave and takes a value of +1 or –1 when the particle moves to the right or left, respectively, relative to the observer located at the initial space – time point ($\mathbf{r}_0$, t_0).

Differentiating (7) with respect to time t, we obtain

$$\frac{d\xi}{dt} = \frac{dt}{dt} - f\frac{\mathbf{n}}{c}\frac{d\mathbf{r}}{dt} = 1 - f\mathbf{n}\frac{\mathbf{V}}{c} = 1 - f\mathbf{n}\cdot\boldsymbol{\beta}, \tag{8}$$

where $\boldsymbol{\beta} = \mathbf{V}/c$, $\mathbf{V} = d\mathbf{r}/dt$.

From Eq. (7), we can express t as follows:

$$t = \xi + f\frac{\mathbf{n}\cdot\mathbf{r}}{c}, \tag{9}$$

and by differentiating (9) with respect to ξ, we obtain

$$\frac{dt}{d\xi}=\frac{d\xi}{d\xi}+f\frac{\mathbf{n}}{c}\frac{d\mathbf{r}}{d\xi}=1+f\frac{\mathbf{n}}{c}\frac{d\mathbf{r}}{d\xi}. \tag{10}$$

Multiplying Eqs. (8) and (10), and considering that $\frac{d\xi}{dt}\frac{dt}{d\xi}=1,$ we obtain:

$$\frac{d\mathbf{r}}{dt}-\frac{d\mathbf{r}}{d\xi}+\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{dt}\frac{d\mathbf{r}}{d\xi}=0. \tag{11}$$

By substituting $\frac{d\mathbf{r}}{dt}=\frac{d\mathbf{r}}{d\xi}\frac{d\xi}{dt}$ into Eq. (11), we can find out that the condition given by Eq. (8) is satisfied, and the following equalities are true:

$$\frac{d\mathbf{r}}{dt}=f\mathbf{n}c\left(1-\frac{d\xi}{dt}\right), \tag{12}$$

$$\frac{d\mathbf{r}}{d\xi}=\frac{\mathbf{V}}{1-f\mathbf{n}\cdot\boldsymbol{\beta}}, \tag{13}$$

$$\frac{d\xi}{dt}=f\mathbf{n}c\frac{d\xi}{d\mathbf{r}}\left(1-\frac{d\xi}{dt}\right), \tag{14}$$

$$\frac{d\xi}{dr}=\frac{\left(1-f\mathbf{n}\cdot\boldsymbol{\beta}\right)}{\mathbf{V}}. \tag{15}$$

Differentiating (11) with respect to d/dt gives the following:

$$\frac{d^2\mathbf{r}}{dt^2}\left(1+\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{d\xi}\right)-\frac{d^2\mathbf{r}}{dtd\xi}\left(1-\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{dt}\right)=0, \tag{16}$$

and similarly, by differentiating (11) with respect to $d/d\xi$, we obtain

$$\frac{d^2\mathbf{r}}{d\xi dt}\left(1+\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{d\xi}\right)-\frac{d^2\mathbf{r}}{d\xi^2}\left(1-\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{dt}\right)=0. \tag{17}$$

Adding (16) and (17) gives

$$\left(\frac{d^2\mathbf{r}}{dt^2}+\frac{d^2\mathbf{r}}{d\xi dt}\right)\left(1+\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{d\xi}\right)-\left(\frac{d^2\mathbf{r}}{dtd\xi}+\frac{d^2\mathbf{r}}{d\xi^2}\right)\left(1-\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{dt}\right)=0. \tag{18}$$

We subsequently introduce new variables Q_t and Q_ξ, such as

$$Q_\xi=1+\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{d\xi}=\frac{1}{1-f\mathbf{n}\cdot\boldsymbol{\beta}},\quad Q_t=1-\frac{1}{c}f\mathbf{n}\frac{d\mathbf{r}}{dt}=1-f\mathbf{n}\cdot\boldsymbol{\beta}=\frac{d\xi}{dt}. \tag{19}$$

Eqs. (19) show that Q_ξ and Q_t integrals of motion, have a one-to-one correspondence, i.e.,

$$Q_\xi\, Q_t=Q_t\, Q_\xi=1. \tag{20}$$

Invariant representation of Q_ξ and Q_t in the $Q_t^+Q_t^-$ and $Q_\xi^+Q_\xi^-$

From Eqs. (19), we propose the following notation for Q_t and Q_ξ:

$$Q_t^+Q_\xi^+ = Q_t^-Q_\xi^- = 1,$$
$$1 = \frac{Q_t^-Q_\xi^-}{Q_t^+Q_\xi^+} = Q_t^+Q_\xi^+ = Q_t^-Q_\xi^- = Q_t^-Q_\xi^-Q_t^+Q_\xi^+, \tag{21}$$

where

$$Q_t^+ = 1 - \mathbf{n}\cdot\boldsymbol{\beta},\ Q_\xi^+ = \frac{1}{1-\mathbf{n}\cdot\boldsymbol{\beta}}, \tag{22}$$
$$Q_t^- = 1 + \mathbf{n}\cdot\boldsymbol{\beta},\ Q_\xi^- = \frac{1}{1+\mathbf{n}\cdot\boldsymbol{\beta}}.$$

One can see from Eqs. (22) that

$$Q_t^+Q_t^- = 1-\beta^2 \text{ and } Q_\xi^+Q_\xi^- = \frac{1}{1-\beta^2}$$

are invariants and have direct and inverse representations, and that

$$Q_t^- + Q_t^+ = 2 \text{ or } Q_\xi^- + Q_\xi^+ = 2Q_\xi^+Q_\xi^-.$$

We can express $\boldsymbol{\beta}$ in terms of $Q_t^+Q_t^-$ to obtain

$$\boldsymbol{\beta} = \mathbf{n}\sqrt{1-Q_t^+Q_t^-}. \tag{23}$$

Relations of special relativity in the $Q_t^+Q_t^-$ and $Q_\xi^+Q_\xi^-$ representations

Let us introduce the dimensionless momentum of a particle $\mathbf{P} = \mathbf{p}/mc$, and the dimensionless energy of the particle $E = \varepsilon/mc^2$.

Thus, Eqs. (2) and (3) can be rewritten as

$$\mathbf{P} = \frac{\boldsymbol{\beta}}{\sqrt{1-\beta^2}}, \tag{24}$$

$$E^2 = \frac{1}{1-\beta^2} = 1 + P^2. \tag{25}$$

Substituting Eq. (23) into Eq. (24), we obtain the $\boldsymbol{\beta}$ representation of the momentum in terms of $Q_\xi^+Q_\xi^-$, i. e.,

$$\mathbf{P} = \mathbf{n}\sqrt{1-Q_t^+Q_t^-}\sqrt{Q_\xi^+Q_\xi^-} = \mathbf{n}\sqrt{Q_\xi^+Q_\xi^- - 1}, \tag{26}$$

and the corresponding representation for energy (following Eq. (25)) will be

$$E = \sqrt{Q_\xi^+Q_\xi^-}, \tag{27}$$

where $E \geq 0$.

The problem involving eigenfunctions and Q_t^+, Q_t^-, Q_ξ^+, Q_ξ^-, $Q_t^+Q_t^-$, $Q_\xi^+Q_\xi^-$, as well as $\boldsymbol{\beta} = \boldsymbol{n}\sqrt{1-Q_t^+Q_t^-}$ from d/dt

We determine the eigenvalues with respect to time t. We obtain the following:

$$\frac{d}{dt}Q_t^+ = -\mathbf{n}\frac{d\boldsymbol{\beta}}{dt} = -\mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^+Q_t^+ = q_t^+Q_t^+, \tag{28}$$

where

$$q_t^+ = -\mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^+. \tag{29}$$

Similarly, we obtain the following eigenvalues for the Q_t^-, Q_ξ^+, Q_ξ^-, $Q_t^+Q_t^-$, $Q_\xi^+Q_\xi^-$ and $\boldsymbol{\beta} = \mathbf{n}\sqrt{1-Q_t^+Q_t^-}$:

$$\frac{d}{dt}Q_t^- = \mathbf{n}\frac{d\boldsymbol{\beta}}{dt} = \mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^-Q_t^- = q_t^-Q_t^-, \tag{30}$$

$$q_t^- = \mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^-. \tag{31}$$

$$\frac{d}{dt}Q_\xi^+ = \left(1-\mathbf{n}\cdot\boldsymbol{\beta}\right)^{-2}\cdot\mathbf{n}\frac{d\boldsymbol{\beta}}{dt} = \mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^+Q_\xi^+ = q_\xi^+Q_\xi^+, \tag{32}$$

$$q_\xi^+ = \mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^+. \tag{33}$$

$$\frac{d}{dt}Q_\xi^- = -\left(1+\mathbf{n}\cdot\boldsymbol{\beta}\right)^{-2}\cdot\mathbf{n}\frac{d\boldsymbol{\beta}}{dt} = -\mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^-Q_\xi^- = q_\xi^-Q_\xi^-, \tag{34}$$

$$q_\xi^- = -\mathbf{n}\frac{d\boldsymbol{\beta}}{dt}Q_\xi^-. \tag{35}$$

$$\frac{d}{dt}\left(Q_t^+Q_t^-\right) = q_t^+Q_t^+Q_t^- + Q_t^+q_t^-Q_t^- = -2\boldsymbol{\beta}\frac{d\boldsymbol{\beta}}{dt}. \tag{36}$$

$$\frac{d}{dt}\left(Q_\xi^+Q_\xi^-\right) = Q_\xi^+Q_\xi^-\left(q_\xi^- + q_\xi^+\right) = \frac{2\boldsymbol{\beta}}{\left(1-\beta^2\right)^2}\frac{d\boldsymbol{\beta}}{dt}. \tag{37}$$

$$\frac{d\boldsymbol{\beta}}{dt} = -\frac{1}{2}\mathbf{n}\left(1-Q_t^+Q_t^-\right)^{-\frac{1}{2}}\cdot\frac{d}{dt}\left(Q_t^+Q_t^-\right) = \frac{\mathbf{n}\dfrac{d\boldsymbol{\beta}}{dt}}{\sqrt{1-Q_t^+Q_t^-}}\boldsymbol{\beta}, \tag{38}$$

where the eigenvalue of the velocity modulus is determined by the expression

$$\beta = \frac{\mathbf{n}\dfrac{d\boldsymbol{\beta}}{dt}}{\sqrt{1-Q_t^+Q_t^-}}. \tag{39}$$

Substituting the values from Eq. (39) into Eqs. (36) and (37), then taking into account that $\boldsymbol{\beta} = \mathbf{n}\beta$, we obtain:

$$\frac{d}{dt}\left(Q_t^+Q_t^-\right) = -2\left(1-Q_t^+Q_t^-\right)^{\frac{3}{2}}, \tag{40}$$

$$\frac{d}{dt}\left(Q_\xi^+Q_\xi^-\right) = 2\left(Q_\xi^+Q_\xi^- - 1\right)^{\frac{3}{2}}\left(Q_\xi^+Q_\xi^-\right)^{\frac{1}{2}}. \tag{41}$$

Variables ξ, $\frac{\mathbf{n}\cdot\mathbf{r}}{c}$ and t in the $Q_t^+Q_t^-$ representation. We now represent Eq. (23) by the following form:

$$\boldsymbol{\beta} = \frac{1}{c}\frac{d\mathbf{r}}{dt} = \frac{1}{c}\frac{d\mathbf{r}}{d\left(Q_t^+Q_t^-\right)}\frac{d\left(Q_t^+Q_t^-\right)}{dt} = \mathbf{n}\sqrt{1-Q_t^+Q_t^-}. \tag{42}$$

Substituting Eq. (40) into (42), and integrating over the $d\left(Q_t^+Q_t^-\right)$ from 0 to $Q_t^+Q_t^-$, we obtain

$$\frac{\mathbf{n}\cdot\mathbf{r}}{c}=\frac{1}{2}\ln\left(1-Q_t^+Q_t^-\right). \tag{43}$$

From Eq. (8), we now express dt as follows:

$$dt=d\xi+f\mathbf{n}\frac{d\mathbf{r}}{c}. \tag{44}$$

It is known that $d\xi=Q_t dt$, $Q_t=1-f\mathbf{n}\cdot\boldsymbol{\beta}$, and hence, substituting them into Eq. (44), we obtain

$$\left(1-Q_t\right)dt=f\mathbf{n}\frac{d\mathbf{r}}{c},\ \text{or}\ \boldsymbol{\beta}dt=\frac{d\mathbf{r}}{c}. \tag{45}$$

Substituting $\boldsymbol{\beta}$ from Eq. (23) and $d\mathbf{r}$ rom Eq. (42) into Eq. (45), we obtain

$$dt=\frac{1}{2}\frac{d\left(1-Q_t^+Q_t^-\right)}{\left(1-Q_t^+Q_t^-\right)^{\frac{3}{2}}}. \tag{46}$$

Subsequently, integrating this over $d\left(1-Q_t^+Q_t^-\right)$ from 0 to $Q_t^+Q_t^-$ results in

$$t=1-\frac{1}{\sqrt{\left(1-Q_t^+Q_t^-\right)}}. \tag{47}$$

Finally, substituting Eqs. (43) and (47) into Eq. (7) results in

$$\xi=1-\frac{1}{\sqrt{\left(1-Q_t^+Q_t^-\right)}}-f\frac{1}{2}\ln\left(1-Q_t^+Q_t^-\right); \tag{48}$$

then, differentiating Eq. (48) with respect to d/dt gives a relationship between $Q_t^+Q_t^-$ and Q_t as follows:

$$\frac{d\xi}{dt}=1-f\sqrt{1-Q_t^+Q_t^-}=1-f\mathbf{n}\cdot\boldsymbol{\beta}=Q_t. \tag{49}$$

The relationship between $Q_t^+Q_t^-$ and Q_t. Eq. (49) demonstrates the following relationship between $Q_t^+Q_t^-$ and Q_t:

$$\sqrt{1-Q_t^+Q_t^-}=f\left(1-Q_t\right). \tag{50}$$

By considering Eq. (50), we can represent the following formulas:

$$\boldsymbol{\beta}=\mathbf{n}\sqrt{1-Q_t^+Q_t^-}=f\mathbf{n}\left(1-Q_t\right), \tag{51}$$

$$\frac{d\boldsymbol{\beta}}{dt}=\boldsymbol{\beta}\sqrt{1-Q_t^+Q_t^-}=\mathbf{n}\left(1-Q_t\right)^2, \tag{52}$$

$$\frac{\mathbf{n}\cdot\mathbf{r}}{c}=\frac{1}{2}\ln\left(1-Q_t^+Q_t^-\right)=\frac{1}{2}\ln\left[\left(1-Q_t\right)^2\right], \tag{53}$$

$$t=1-\frac{1}{\sqrt{1-Q_t^+Q_t^-}}=1-\frac{1}{f\left(1-Q_t\right)}, \tag{54}$$

$$\xi=1-\frac{f}{\left(1-Q_t\right)}-f\frac{1}{2}\ln\left[\left(1-Q_t\right)^2\right]. \tag{55}$$

Eigenfunctions and eigenvalues of Q_t and Q_ξ. We now take the derivative of ξ from Eq. (55) with respect to time, i. e.,

$$\frac{d\xi}{dt} = -f\frac{Q_t}{\left(1-Q_t\right)^2}\frac{dQ_t}{dt}, \tag{56}$$

and substitute $\frac{d\xi}{dt} = Q_t$ into Eq. (56) to obtain

$$\frac{dQ_t}{dt} = -f\left(1-Q_t\right)^2 Q_\xi Q_t, \tag{57}$$

where

$$q_t = -f\left(1-Q_t\right)^2 Q_\xi = -f\left(Q_\xi - 2 + Q_t\right). \tag{58}$$

Using the relationship

$$\frac{d\left(Q_t Q_\xi\right)}{dt} = \frac{dQ_t}{dt}Q_\xi + Q_t\frac{dQ_\xi}{dt} = 0, \tag{59}$$

we can derive $\frac{dQ_\xi}{dt}$ as follows:

$$\frac{dQ_\xi}{dt} = -\frac{dQ_t}{dt}\left(Q_\xi\right)^2, \tag{60}$$

$$\frac{dQ_\xi}{dt} = f\left(1-Q_t\right)^2\left(Q_\xi\right)^2 = f\left(1-Q_t\right)^2 Q_\xi Q_\xi, \tag{61}$$

and finally,

$$q_\xi = f\left(1-Q_t\right)^2 Q_\xi = f\left(Q_\xi - 2 + Q_t\right). \tag{62}$$

Eqs. (58) and (62) show that

$$q_t + q_\xi = 0. \tag{63}$$

Differentiating Eq. (55) with respect to $\frac{d}{d\xi}$, we obtain the following:

$$\frac{dQ_t}{d\xi} = -f\frac{\left(1-Q_t\right)^2}{Q_t}Q_\xi Q_t, \tag{64}$$

where

$$p_t = -f\frac{\left(1-Q_t\right)^2}{Q_t}Q_\xi = -f\left(Q_\xi - 1\right)^2. \tag{65}$$

Using the following identity

$$\frac{d\left(Q_t Q_\xi\right)}{d\xi} = \frac{dQ_t}{d\xi}Q_\xi + Q_t\frac{dQ_\xi}{d\xi} = 0, \tag{66}$$

we find that $\frac{dQ_\xi}{d\xi}$ to be

$$\frac{dQ_\xi}{d\xi} = f\frac{\left(1-Q_t\right)^2}{Q_t}Q_\xi Q_\xi = p_\xi Q_\xi, \tag{67}$$

where

$$p_\xi = f\frac{\left(1-Q_t\right)^2}{Q_t}Q_\xi = f\left(Q_\xi - 1\right)^2, \tag{68}$$

$$p_t + p_\xi = 0. \tag{69}$$

Integrals of motion

From Eq. (5) we can introduce the dimensionless integral of motion $\gamma = \gamma_0/mc$, and then using Eq. (50) we obtain

$$\gamma = \frac{(1 - f\mathbf{n}\cdot\boldsymbol{\beta})}{\sqrt{1-\beta^2}} = \sqrt{\frac{1 - f\mathbf{n}\cdot\boldsymbol{\beta}}{1 + f\mathbf{n}\cdot\boldsymbol{\beta}}} = \frac{Q_t}{\sqrt{Q_t^+ Q_t^-}} = \frac{Q_t}{\sqrt{2Q_t - (Q_t)^2}} = \frac{1}{\sqrt{2Q_\xi - 1}}. \tag{70}$$

Expressing Q_t and Q_ξ in terms of the motion integral γ, we obtain:

$$Q_t = \frac{2\gamma^2}{1+\gamma^2}, \; Q_\xi = \frac{1+\gamma^2}{2\gamma^2}, \tag{71}$$

where

$$1 - Q_t = \frac{1-\gamma^2}{1+\gamma^2}. \tag{72}$$

Substituting Eq. (72) into the obtained definition for laboratory time t within Eq. (54), the laboratory coordinate $\mathbf{n}\cdot\mathbf{r}/c$ in Eq. (53), and space-time coordinate ξ in Eq. (55), we obtain the dependence of these parameters on the integral of motion γ:

$$t = 1 - \frac{1}{f(1-Q_t)} = 1 - f\frac{1+\gamma^2}{1-\gamma^2}, \tag{73}$$

$$\frac{\mathbf{n}\cdot\mathbf{r}}{c} = \frac{1}{2}\ln\left[(1-Q_t)^2\right] = \frac{1}{2}\ln\left[\left(\frac{1-\gamma^2}{1+\gamma^2}\right)^2\right], \tag{74}$$

$$\xi = t - f\frac{\mathbf{n}\cdot\mathbf{r}}{c} = 1 - f\frac{1+\gamma^2}{1-\gamma^2} - f\frac{1}{2}\ln\left[\left(\frac{1-\gamma^2}{1+\gamma^2}\right)^2\right]. \tag{75}$$

The dependences of energy and momentum on Q_ξ and γ. By considering Eqs. (21) and (50), the particle energy, as shown in Eq. (27) dependent on both Q_ξ and γ, takes the following form:

$$E = \sqrt{Q_\xi^+ Q_\xi^-} = \frac{Q_\xi}{\sqrt{2Q_\xi - 1}} = Q_\xi \gamma = \frac{1}{2\gamma}(1+\gamma^2). \tag{76}$$

In the absence of an initial velocity of the particle ($\boldsymbol{\beta} = 0$, $\gamma = 0$), the particle's energy is $E = 1$, i. e., equal to mc^2.

Following the particle momentum as given by Eq. (26), we can substitute Eqs. (21) and (50) to obtain

$$\mathbf{P} = \mathbf{n}\sqrt{Q_\xi^+ Q_\xi^- - 1} = \mathbf{n}\sqrt{\frac{Q_\xi^2}{(2Q_\xi - 1)} - 1} = f\mathbf{n}(Q_\xi - 1)\gamma = \frac{f\mathbf{n}}{2\gamma}(1-\gamma^2). \tag{77}$$

Subtracting the longitudinal component of momentum (as given in $\mathbf{n}\cdot\mathbf{P}$, Eq. (77)) from the energy E (see Eq. (76)), we obtain the integral of motion in the form

$$E - \mathbf{n}\cdot\mathbf{P} = \gamma^+, \tag{78}$$

where $\gamma^+ = \sqrt{Q_t^+ / Q_t^-}$, $\gamma^+\gamma^- = 1$.

Adding the energy E given by Eq. (76) and the longitudinal component of the momentum $\mathbf{n}\cdot\mathbf{P}$ into Eq. (77), we obtain the inverse integral of motion:

$$E + \mathbf{n}\cdot\mathbf{P} = \gamma^-. \tag{79}$$

We then use Eqs. (78) and (79) to conclude that

$$E - f\mathbf{n}\cdot\mathbf{P} = \gamma. \tag{80}$$

Assuming that the momentum **P** of the system is equal to the sum of the longitudinal and transverse components, we can see from Eqs. (25), (78) and (79) that the transverse component of the momentum is zero for this $\mathbf{P}_\perp = 0$ system.

For a freely moving particle with a sufficiently small change in energy and momentum, the particle velocity is determined by the following expression:

$$\frac{dE}{d\mathbf{P}} = \frac{d}{d\mathbf{P}}\left(\sqrt{1+P^2}\right) = \frac{\mathbf{P}}{E} = \boldsymbol{\beta} = f\mathbf{n}\left(1-Q_t\right) = f\mathbf{n}\left(\frac{1-\gamma^2}{1+\gamma^2}\right). \quad (81)$$

Acceleration of an uncharged relativistic particle in a force field and intensity of its radiation

We now investigate the dynamics of a relativistic charged particle in a force field (following Eq. (6)), i. e.,

$$\mathbf{n}\frac{d\mathbf{P}}{dt} = \frac{d}{dt}\left(\sqrt{Q_\xi^+ Q_\xi^- - 1}\right) = \sqrt{\left(Q_\xi^+ Q_\xi^- - 1\right)Q_\xi^+ Q_\xi^-}\, P = P_1 P, \quad (82)$$

where

$$P_1 = \sqrt{\left(Q_\xi^+ Q_\xi^- - 1\right)Q_\xi^+ Q_\xi^-} = P\sqrt{Q_\xi^+ Q_\xi^-} = PE;$$

$$P = \sqrt{Q_\xi^+ Q_\xi^- - 1}, \quad (83)$$

and

$$\mathbf{n}\cdot\mathbf{P}_1 = \sqrt{\left(Q_\xi^+ Q_\xi^- - 1\right)Q_\xi^+ Q_\xi^-} = P\sqrt{Q_\xi^+ Q_\xi^-} = PE. \quad (84)$$

Differentiating Eq. (82) with respect to time and using Eq. (25), we obtain the radiative friction acting on an uncharged particle as follows:

$$\mathbf{F}_{rad} = \frac{d^2\mathbf{P}}{dt^2} = \left(2+3P^2\right)P^2\mathbf{P}. \quad (85)$$

The radiation intensity of an uncharged particle is determined from the total energy (see Eq. (27)), and without considering the radiative friction force in Eq. (85), the radiation intensity takes the form

$$I = -\frac{dE}{dt} = -\frac{d}{dt}\left(\sqrt{Q_\xi^+ Q_\xi^-}\right) = -\left(Q_\xi^+ Q_\xi^- - 1\right)^{\frac{3}{2}} = -P^3 = -\frac{f}{8\gamma^3}\left(1-\gamma^2\right)^3, \quad (86)$$

where $I \geq 0$.

Generalization of the obtained results

To generalize the above results to electrons accelerated by the transverse electromagnetic field of an incident laser pulse on the frontal surface of the target and estimate the temperature of fast electrons, similar to the authors of Ref. [16], we substitute the expression for the amplitude of an electron oscillating in a field of a plane monochromatic wave, i. e.,

$$P^2 = Q_\xi^+ Q_\xi^- - 1 = \frac{q^2 E_0^2}{m^2 c^2 \omega^2}, \quad (87)$$

and obtain a formula for the kinetic energy of an electron oscillating in the transverse field of an incident light wave as follows,

$$\bar{K}_e = m_e c^2\left(\sqrt{1+\left(\frac{q^2 E_0^2}{m^2 c^2 \omega^2}\right)^2} - 1\right) = m_e c^2\left(\sqrt{1+\frac{I\lambda^2}{1.37\cdot 10^{18}}} - 1\right), \quad (88)$$

where m_e, g, is the electron mass; c, km/s, is the speed of light; E_0, V/m, is the amplitude of the electric field of the incident electromagnetic wave; ω, s^{-1}, is the carrier frequency; I, W/cm^2, is the intensity of the incident wave; λ, μm, is the wavelength.

A similar formula has been used to theoretically estimate the temperature of fast electrons on the frontal surface of the target and to analyze experimental results [17 – 20].

Further, it is of interest to use this method to describe the dynamics of a relativistic particle in stationary electrical, magnetic and electromagnetic fields [1 – 23], and to study the Doppler effect of a particle displacement in high-intensity fields.

Conclusion

In this study, we have obtained the direct and inverse integrals of the particle motion and demonstrated the advantage of using these approaches to investigate the dynamics of relativistic particle dynamics. It was demonstrated that the relativistic root can be represented by the form $\sqrt{1-\beta^2}=\sqrt{Q_t^+Q_t^-}$, and further calculation of the arbitrary partial in Q_t^+ and Q_t^- considerably simplifies calculations in relativistic particle dynamics. The total energy of a particle is related to its momentum (25) through $Q_\xi^+Q_\xi^-$ also simplifies the calculations since the total energy and momentum can be represented as $E=\sqrt{Q_\xi^+Q_\xi^-}$ and $\mathbf{P}=\mathbf{n}\sqrt{Q_\xi^+Q_\xi^- -1}$, respectively. It was shown that the energy (76) and momentum (77) could be separately expressed in terms of the integral of motion γ, and their difference gave the direct integral of motion (78), while the sum did the inverse integral of motion (79). Additionally, a relationship between $Q_t^+Q_t^-$ and Q_t was obtained. A detailed analysis is given for the space-time coordinate ξ and its dependence on $Q_t^+Q_t^-$, Q_t and the integral of motion γ.

Furthermore, we demonstrated that as $|\mathbf{V}| \to c$, $t \to 0$, to interpret the dependence of the physical quantities t (47) and $\mathbf{n}\cdot\mathbf{r}\,/\,c$ (43) on $|\boldsymbol{\beta}|$, it is necessary to apply the following gauge transformations, $-t \to t$ and $-\mathbf{n}\cdot\mathbf{r}\,/\,c \to \mathbf{n}\cdot\mathbf{r}\,/c$, its dependence on $Q_\xi^+Q_\xi^-$. The radiation intensity of a particle in the far-field was obtained, and its dependence of γ on $Q_\xi^+Q_\xi^-$ was shown. The results appeared from the special theory of relativity. This approach can also be generalized for tensor use, allowing a more detailed description of the dynamics of relativistic particles in a medium.

The proposed approach has all the limitations of the special theory of relativity.

Further, the generalization of this approach to the Lagrangian and Hamiltonian formalism is significant.

The future scope of this work will be to investigate the spectral-angular characteristics of particle radiation and particle dynamics in a force field in the presence of radiative friction forces.

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