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Contact interaction of multilayer slabs with an inhomogeneous base

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Abstract. The article is devoted to the development of mathematical models and new methods for solving contact problems of multilayer elements of structures with an inhomogeneous base, considering their mechanical, structural features, and to the assessment of their internal force factors. A mathematical model was developed and an analytical method was proposed for assessing the internal force factors in multilayer strip slabs on an inhomogeneous base under various loads. The solution of the problems under consideration is based on a series expansion of the reactive pressure of an inhomogeneous base in terms of orthogonal ultra-spherical Gegenbauer polynomials; the solution is reduced to the study of infinite systems of algebraic equations. Their regularity was proved and the corresponding estimates were obtained. The required number of terms of the polynomial in the expansion was established. The analysis of the results obtained made it possible to evaluate the influence of the rigidity characteristics of the filler and the inhomogeneity of the base on the distribution of internal force factors in the slabs.

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1. Introduction

The article considers mathematical models, methods, and estimates of internal force factors in contact interactions of multilayer strip-slabs lying on an elastic inhomogeneous base.

A number of structural elements, such as foundations of buildings, slabs of hydro-technical structures, road and airfield pavements, rails and sleepers of railway tracks, and other elements work jointly with the subgrade, interacting with each other under various loads. Their soil base can be homogeneous, or, more often, inhomogeneous.

Today, the problem of determining the individual characteristics of multilayer structural elements interacting with inhomogeneous bases, taking into account mechanical, geometric, and other parameters of structural elements and bases remains unsolved.

Therefore, the development of models and new analytical methods for solving contact problems of multilayer elements of structures with an inhomogeneous base, taking into account their mechanical, structural features, as well as the assessment of their internal force factors is an urgent task of structural mechanics.

It should be noted that the solution to the problems of contact interactions of various elements of structures with the foundation is considered in fundamental publications [1–5], in which the main attention is paid to the analysis of the relationship of contacting structural elements.

To date, there are a sufficient number of published studies in which various issues of the contact of structural elements with the base are considered:

- in [6], the calculation of slabs on an elastic base with a variable coefficient of one-layer and two-layer beds is given. Calculations for a two-layer slab on an elastic base with a variable bed ratio are given for different heights of the upper layer using finite element methods;
- in [7], a mathematical model of the contact interaction of two plates made of materials with different moduli of elasticity is presented taking into account physical and structural nonlinearities. To study the stress-strain state of the mechanical structure, the method of variational iteration was used, which makes it possible to reduce the partial differential equations to ordinary differential equations;
- in [8], an axisymmetric quasi-static contact problem of the thermal and mechanical interaction of a circular punch and an inhomogeneous elastic half-space is considered, the mechanical and thermophysical properties of which are taken as arbitrary functions of the depth coordinate. The numerical implementation is performed for various dependences of material properties on the depth of the half-space;
- in [9], an analytical study of the contact problem of a multilayer elastic rigid body subjected to eccentric indentation by a rigid circular plate within the framework of classical elasticity is presented. An explicit expression for the solution for an elastic field in a multilayer rigid body is presented. The numerical results of checking the methods adopted in the study and the illustrations of the inhomogeneity effect of the layered material on the elastic field are presented;
- in [10], a mathematical model and a method for solving the problem for multilayer strip-slabs on a homogeneous elastic foundation under various static loads are developed. The problem under consideration was reduced (using the Chebyshev polynomial) to solving infinite systems of algebraic equations. The regularity of infinite systems of algebraic equations is proved and the corresponding estimates are obtained;
- in [11, 12], a detailed review of the history of development of mathematical modeling and methods of elastic analysis of inhomogeneous rigid bodies and a comprehensive review of various theoretical models of elastic and viscoelastic foundations in oscillatory systems are presented;
- in [13–15], dynamic problems for the structure – foundation were jointly solved, using artificial non-reflecting boundary conditions on the boundary of the finite area of the foundations.

A seismic analysis of the dam - base system using the finite element method is considered in [14]. The results show that when solving dynamic problems, the proposed artificial boundary conditions absorb the reflections of false waves quite well.

Along with these publications, there is a number of articles [16–23], devoted to the study of internal force factors and the behavior of inhomogeneous elastic and viscoelastic systems under various influences, taking into account their characteristics, operating conditions, and interaction with the environment.

When solving specific problems, each of these approaches has its own advantages and disadvantages; nevertheless, they are used in solving practical problems.

Here we have presented just a few articles devoted to solving various problems in the joint operation of structural elements with a deformable base, which show the incompleteness of research in this direction, especially in the field of obtaining analytical solutions. Therefore, this article is devoted to the development of mathematical models and analytical methods for studying the interaction of multilayer strip-slabs lying on an elastic inhomogeneous base.

2. Methods

2.1. Mathematical models of the problem

We consider n layer strips-slabs $2l$ wide, $h_1,\ h_2,...,\ h_n$ thick, respectively, having an arbitrary character in terms of geometrical and mechanical parameters. An elastic filler is arranged between the plates, and the lower plate fits tightly to an elastic inhomogeneous base. It is considered that the strip layers are loaded with external loads of $q_1,\ q_2,...,\ q_n$ intensity, respectively, constant loads longwise the slab and arbitrary loads across it. We assume that the reaction of the elastic filler is proportional to the differences in deflections of connecting strips. The normal reactive pressures of the inhomogeneous base (Fig. 1) affect the lower strip, in addition to external loads, and the reaction of the filler of the upper strips.

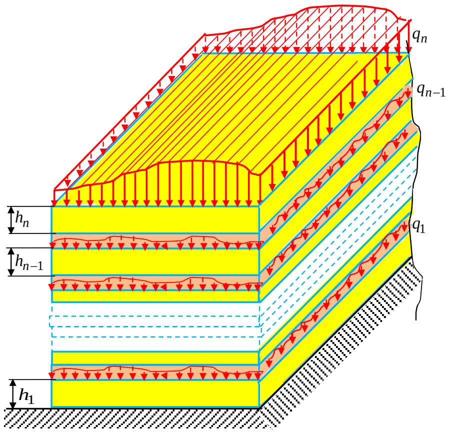

Figure 1. Design scheme of n -layer strip-slabs.

An n-layer slab beam cut out with a width equal to one is taken for mathematical modeling of the n-layer slab-strip deformation process. Under the above conditions, the calculation of n-layer strip-slabs is reduced to the calculation of n layer beam slabs $2l$ wide, $h_1,\ h_2,...,\ h_n$ thick, respectively, with a width equal to one (Fig. 2). If we assume that the origin of the Cartesian coordinates is in the center of symmetry of the beam slabs, then the study along the abscissa axis is performed in a segment $\left[-l;l\right]$, i.e., $-l \le x \le l,$ the ordinates of the slab deflection, $y_1,\ y_2,...,\ y_n$ are functions of the variable x, i.e., $y_i = y_i\left(x\right),\ i=1,2,...,n,$ where y_i is the deflection of the i-th beam slab.

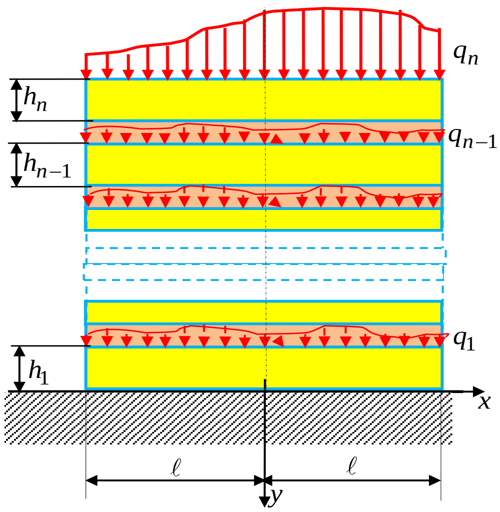

Figure 2. Design scheme of n layer beam slabs.

To simulate the deformation process of n layer beam slabs, one can write a system of differential equations for the unknown deflections of beam slabs in the following form:

$$\left.\begin{array}{l} D_n \cdot y_n^{IV} = q_n - k_{n-1}\cdot\left(y_n - y_{n-1}\right) \\ D_{n-1}\cdot y_{n-1}^{IV} = q_{n-1} + k_{n-1}\cdot\left(y_n - y_{n-1}\right) - k_{n-2}\cdot\left(y_{n-1}-y_{n-2}\right) \\ D_{n-2}\cdot y_{n-2}^{IV} = q_{n-2} + k_{n-2}\cdot\left(y_{n-1}-y_{n-2}\right) - k_{n-3}\cdot\left(y_{n-2}-y_{n-3}\right) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ D_1\cdot y_1^{IV} = q_1 + k_1\cdot\left(y_2 - y_1\right) - p \end{array}\right\}, \qquad (1)$$

where $D_i = \dfrac{E_i h_i^3}{12\left(1-\nu_i^2\right)}$; E_i, ν_i are the modulus of elasticity and Poisson's ratio of the plate material; k_i filler rigidity coefficients; $q_i = q_i\left(x\right)$ external loads of the i-th slab; $p = p\left(x\right)$ reactive normal pressure of the base.

An elastic inhomogeneous half-plane is taken as an elastic base, the deformation modulus of which changes according to the power law [4, 5]

$$E = E_m \cdot y^m, \ 0 \le m < 1,$$

where E_m is the constant; m is the inhomogeneity index.

The equation that determines the settlement $V\left(x\right)$ of an inhomogeneous base with reactive pressure $p\left(x\right)$ of the base under plane strain, according to the solution of the theory of elasticity [4], can be represented as

$$V = \frac{\theta_m}{m} \cdot \int_{-l}^{l} \frac{p\left(s\right)}{\left|x-s\right|^m} ds. \tag{2}$$

where $\theta_m = \dfrac{\left(1-\nu^2\right) \cdot \sin\dfrac{\gamma\pi}{2} \cdot \Gamma\left[1+\left(1-\gamma+m\right)/2\right] \cdot \Gamma\left[1+\left(1+\gamma+m\right)/2\right]}{\pi \cdot \left(1+m\right) \cdot E_m \cdot 2^{-1-m} \cdot \Gamma\left(m+2\right)}$; ν is the Poisson's ratio of homogeneous soil of the base; $\Gamma\left(x\right)$ is the Euler's gamma function.

The condition for the tight fit of the lower slab surface with the base is represented as

$$y_1\left(x\right) = V\left(x\right), \ -l \le x \le l. \tag{3}$$

Equation (3), as the contact conditions of the slab and the base, makes it possible to study the effect of base inhomogeneity on the internal forces of the slab.

The solution to the problem of the interaction of n-layer beam slabs with an inhomogeneous elastic base in the case of a two-way connection is reduced to solving systems of differential equations (1) and a singular integral equation (2) with condition (3). Equations (1), (2) and (3) constitute a closed system of equations for unknown functions, i.e., the number of the equations corresponds to the number of unknowns.

2.2. Solution method

To solve the problems posed (1)–(3), we use the expansion in the form [4]

$$\frac{1}{\left|x-s\right|^m} = \frac{2^{m-1} \cdot \Gamma\left(\dfrac{m}{2}\right)}{\Gamma\left(m+1\right) \cdot \cos\dfrac{m\pi}{2}} \cdot \sum_{n=0}^{\infty} \left(\frac{m}{2}+n\right) \cdot C_n^{\frac{m}{2}}\left(x\right) \cdot C_n^{\frac{m}{2}}\left(s\right), \tag{4}$$

where $C_n^{\frac{m}{2}}\left(x\right)$ is the orthogonal ultraspherical Gegenbauer polynomials [24] with weight functions

$$\rho\left(x\right) = \left(1-x^2\right)^{\left(m-1\right)/2}.$$

From expansion (4), it is seen that for singular integrals of type (2), the orthogonal Gegenbauer polynomials are eigenfunctions. This confirmation makes it possible to search for the unknown reactive pressure of an inhomogeneous base, in the following form

$$p(x) = \rho(x) \cdot \sum_{n=0}^{\infty} A_n \cdot C_n^{\frac{m}{2}}(x), \tag{5}$$

here A_n are the unknown constants.

In what follows, we use the dimensionless coordinates $"x"$ equal to the ratio of the absolute coordinate to the half-length of the beam l.

If the designations P and M are introduced for the sum of all vertical forces and the sum of their moments relative to the middle of the beam slabs, respectively, then the equilibrium equations for the beam slabs take the form

$$\int_{-1}^{1} p(x)\,dx = \frac{P}{l}; \ \int_{-1}^{1} x \cdot p(x)\,dx = \frac{M}{l^2}. \tag{6}$$

After satisfying the equilibrium equations, i.e., substituting (5) into (6) and taking into account the orthogonality of the Gegenbauer polynomials, the value of the first two coefficients of series (5) is determined in the following form

$$A_0 = P \cdot \left(l \cdot \left\| C_0^{m/2} \right\| \right)^{-1}, \ A_1 = M \cdot m \cdot \left(l^2 \cdot \left\| C_1^{m/2} \right\| \right)^{-1}, \tag{7}$$

where $\left\| C_0^{m/2} \right\|$, $\left\| C_1^{m/2} \right\|$ are the values of the norm of the orthogonal Gegenbauer polynomial determined by the following formula [24]

$$\left\| C_j^{\nu} \right\| = \frac{\pi \cdot 2^{1-2\nu} \cdot \Gamma(2\nu + j)}{\Gamma^2(\nu) \cdot j! \cdot (\nu + j)}. \tag{8}$$

If the remaining coefficients of series (5) are assumed zero, then we have the case of a rigid beam slab. The terms of series (5), starting from the values $n = 2$, represent a correction that differs from the distribution of reactive pressure for absolutely rigid beam slabs.

Substituting (5) into (2), and considering expansion (4) and the orthogonality of the Gegenbauer polynomials, for the base settlement, we obtain the following relationship

$$V = \alpha_m \cdot \sum_{n=0}^{\infty} A_n \cdot \left(\frac{m}{2} + n \right) \cdot \left\| C_n^{m/2} \right\| \cdot C_n^{\frac{m}{2}}(x), \tag{9}$$

where $\alpha_m = \dfrac{\theta_m \cdot 2^{m-1} \cdot \Gamma^2\left(\frac{m}{2}\right)}{\Gamma(m+1) \cdot \cos\frac{m\pi}{2}}$.

The resulting expression (9) allows us to determine the base settlement, i.e., the vertical displacement of the point of the inhomogeneous base.

3. Results and Discussion

Now, using the above results, we solve specific problems.

3.1. Problem

A two-layer beam slab interacting with an inhomogeneous elastic base is considered. The system of differential equations for the deflections of beam slabs (1) takes the form

$$\left.\begin{aligned} \frac{D_2}{l^4} y_2^{IV} &= q_2 - k_1(y_2 - y_1) \\ \frac{D1}{l^4} y_1^{IV} &= q_1 + k_1(y_2 - y_1) - p \end{aligned}\right\} \tag{10}$$

The general solution of the system of differential equations (10) is represented in the following form

$$\begin{aligned} y_1 = \frac{l^4}{D_1 + D_2} \cdot \Bigg\{ &\sum_{i=1}^{4} C_i \cdot x^{4-i} + f_q(x) - f_p(x) - \\ &-D_2 \cdot \left[\sum_{i=1}^{4} B_i \cdot u_i(\alpha x) + \psi_q(x) + \varphi_p(x) \right] \Bigg\}, \end{aligned} \tag{11}$$

$$\begin{aligned} y_2 = \frac{l^4}{D_1 + D_2} \cdot \Bigg\{ &\sum_{i=1}^{4} C_i \cdot x^{4-i} + f_q(x) - f_p(x) + \\ &+ D_1 \cdot \left[\sum_{i=1}^{4} B_i \cdot u_i(\alpha x) + \psi_q(x) + \varphi_p(x) \right] \Bigg\}, \end{aligned} \tag{12}$$

where C_i, B_i are the integration constants determined from the boundary conditions of the problem under consideration:

$$\left.\begin{aligned} u_1(x) = chx \cdot \cos x; \quad u_2(x) = chx \cdot \sin x + shx \cdot \cos x; \\ u_3(x) = shx \cdot \sin x; \quad u_4(x) = chx \cdot \sin x - shx \cdot \cos x \end{aligned}\right\}; \tag{13}$$

$$f_q^{IV}(x) = q_1(x) + q_2(x); \tag{14}$$

$$f_p^{IV}(x) = p(x); \tag{15}$$

$$\psi_q(x) = \frac{1}{4\alpha^3} \int_0^x \left[\frac{q_1(z)}{D_2} - \frac{q_2(z)}{D_1} \right] \cdot u_4\left[\alpha(x-z)\right] dz; \tag{16}$$

$$\varphi_p(x) = \frac{1}{4\alpha^3} \int_0^x \frac{p(z)}{D_1} \cdot u_4\left[\alpha(x-z)\right] dz. \tag{17}$$

Substituting (5) in (15) and (17), and performing the appropriate operations, we can write the following

$$f_p(x) = \sum_{n=0}^{\infty} A_n \cdot f_n(x), \tag{18}$$

$$\varphi_p(x) = D_1^{-1} \cdot \sum_{n=0}^{\infty} A_n \cdot \varphi_n(x), \tag{19}$$

where

$$f_n(x) = \frac{m(m+2)(m+4)(m+6)\left(1-x^2\right)^{\frac{m+7}{2}} C_{n-4}^{\frac{m}{2}+4}(x)}{n(n-1)(n-2)(n-3)(n+m)(n+m+1)(n+m+2)(n+m+3)} \quad n > 3, \tag{20}$$

$$f_n'(x)=\frac{m(m+2)(m+4)\left(1-x^2\right)^{\frac{m+5}{2}}C_{n-3}^{\frac{m}{2}+3}(x)}{n(n-1)(n-2)(n+m)(n+m+1)(n+m+2)}\quad n>2, \tag{21}$$

$$f_n''(x)=\frac{m(m+2)}{n(n-1)(n+m)(n+m+1)}\cdot\left(1-x^2\right)^{\frac{m+3}{2}}C_{n-2}^{\frac{m}{2}+2}(x)\quad n>1, \tag{22}$$

$$f_n'''(x)=\frac{m}{n(n+m)}\cdot\left(1-x^2\right)^{\frac{m+1}{2}}C_{n-1}^{\frac{m}{2}+1}(x)\quad n>0, \tag{23}$$

$$f_n^{IV}(x)=\left(1-x^2\right)^{\frac{m-1}{2}}C_n^{\frac{m}{2}}(x), \tag{24}$$

$$\varphi_n(x)=\frac{1}{4\alpha^3}\int_0^x u_4\left[\alpha(x-z)\right]\cdot\left(1-z^2\right)^{\frac{m-1}{2}}C_n^{\frac{m}{2}}(z)\,dz. \tag{25}$$

Identifying the explicit form of the function $f_n(x)$ at $n<4,$ we use the following formulas [24, 25]

$$C_n^{\nu}(x)=\sum_{j=0}^{\left[\frac{n}{2}\right]}(-1)\frac{2^{n-2j}\cdot\Gamma(\nu+n-j)}{\Gamma(\nu)\cdot j!\cdot(n-2j)!}\cdot x^{n-2j}, \tag{26}$$

$$\left(1-x^2\right)^{\frac{m-1}{2}}=\sum_{k=0}^{\infty}c_k\cdot x^{2k}, \tag{27}$$

where

$$c_k=\frac{\Gamma\left(\frac{1-m}{2}+k\right)}{\Gamma\left(\frac{1-m}{2}\right)\cdot k!}. \tag{28}$$

Introducing (5), (26), (27) into (15) and performing the appropriate mathematical operations, we obtain an explicit form for the function $f_n(x)$ at:

$$f_0(x)=\sum_{k=0}^{\infty}c_k\cdot\frac{x^{2k+4}}{(2k+1)(2k+2)(2k+3)(2k+4)}; \tag{29}$$

$$f_1(x)=\sum_{k=0}^{\infty}c_k\cdot m\cdot\frac{x^{2k+5}}{(2k+2)(2k+3)(2k+4)(2k+5)}; \tag{30}$$

$$f_2(x)=\sum_{k=0}^{\infty}c_k\cdot\left[\frac{m(m+2)}{2!}\cdot\frac{x^{2k+6}}{(2k+3)(2k+4)(2k+5)(2k+6)}-\right.$$
$$\left.-\frac{m}{2}\cdot\frac{x^{2k+4}}{(2k+1)(2k+2)(2k+3)(2k+4)}\right]; \tag{31}$$

$$f_3(x)=\sum_{k=0}^{\infty}c_k\cdot\left[\frac{m(m+2)(m+4)}{3!}\cdot\frac{x^{2k+7}}{(2k+4)(2k+5)(2k+6)(2k+7)}-\right.$$
$$\left.-\frac{m(m+2)}{2!}\cdot\frac{x^{2k+5}}{(2k+2)(2k+3)(2k+4)(2k+5)}\right]. \tag{32}$$

Substituting (18) and (19) into (11) and (12), we obtain the following expressions for the deflections of beam slabs:

$$y_1=\frac{l^4}{D_1+D_2}\cdot\left\{\sum_{i=1}^{4}C_i x^{4-i}+f_q(x)-\sum_{n=0}^{\infty}A_n\cdot f_n(x)-\right.$$
$$\left.-D_2\cdot\left[\sum_{i=1}^{4}B_i\cdot u_i(\alpha x)+\psi_q(x)+\frac{1}{D_1}\cdot\sum_{n=0}^{\infty}A_n\cdot\varphi_n(x)\right]\right\}, \tag{33}$$

$$y_2=\frac{l^4}{D_1+D_2}\cdot\left\{\sum_{i=1}^{4}C_i x^{4-i}+f_q(x)-\sum_{n=0}^{\infty}A_n\cdot f_n(x)+\right.$$
$$\left.+D_1\left[\sum_{i=1}^{4}B_i\cdot u_i(\alpha x)+\psi_q(x)+\frac{1}{D_1}\cdot\sum_{n=0}^{\infty}A_n\cdot\varphi_n(x)\right]\right. . \tag{34}$$

Expressions (33) and (34), which determine the deflections of beam slabs, have a general character corresponding to an arbitrary law of distribution of external loads. In specific external loads with specific distribution laws, it will be possible to find the corresponding deflections of the beam slabs that satisfy the corresponding boundary conditions, i.e. the constant coefficients of integration B_i, C_i determined from the boundary conditions of the problems under consideration. It should be noted here that coefficients A_n in formulas (9), (33), and (34) are unknown. The contact conditions (3) are used to determine these unknown coefficients A_n. The deflection of the lower beam slabs satisfying the boundary conditions (33) and the settlement of the base (9) are determined by condition (3). Further, the equality obtained is multiplied by $\left(1-x^2\right)^{\frac{m-1}{2}}C_n^{\frac{m}{2}}(x)$, and then integrated between –1 and 1. As a result of integration with respect to unknown coefficients A_n, an infinite system of algebraic equations with infinite unknowns is obtained. An infinite system of algebraic equations is solved by the reduction method (the use of the reduction method will be justified strictly mathematically). Certain coefficients A_n are substituted in (5), (9), (33), (34) and the regularities of the reactive pressure, the settlement of the base, and the deflections of the beam slabs are found. Further, with determined deflections of the beam slabs, it is possible to define the change patterns of the internal forces of the slabs corresponding to a change in the inhomogeneity of the base.

3.2. Problem

Consider a two-layer beam slab loaded with uniformly distributed external loads, i.e.

$$q_1(x)=q_2(x)=q=const.$$

In this case, due to the symmetry property of the load in series (5), only even polynomials are taken into account

$$p(x)=(1-x)^{\frac{m-1}{2}}\cdot\sum_{n=0}^{\infty}A_{2n}\cdot C_{2n}^{\frac{m}{2}}(x). \tag{35}$$

From the equation of equilibrium (6), we obtain

$$A_0=4q\cdot\left\|C_0^{m/2}\right\|^{-1}. \tag{36}$$

Expression (3) describing the base settlement takes the following form

$$V(x) = \alpha_m \cdot \sum_{n=0}^{\infty} A_{2n} \cdot \left(\frac{m}{2} + 2n \right) \cdot C_{2n}^{\frac{m}{2}}(x), \tag{37}$$

expressions (14), (16) and (17), respectively, take the forms

$$f_q(x) = 2q \cdot \frac{x^4}{24}, \tag{38}$$

$$\psi_q(x) = \frac{1}{4\alpha^3} \cdot \frac{D_1 - D_2}{D_1 \cdot D_2} \cdot q \cdot \frac{1}{\alpha} \left[1 - u_1(\alpha x) \right], \tag{39}$$

$$\varphi_p(x) = \frac{1}{D_1} \cdot \sum_{n=0}^{\infty} A_{2n} \cdot \varphi_{2n}(x), \tag{40}$$

where

$$\varphi_{2n}(x) = \frac{1}{4\alpha^3} \int_0^x u_4 \left[\alpha (x - z) \right] \cdot \left(1 - z^2 \right)^{\frac{m-1}{2}} \cdot C_{2n}^{\frac{m}{2}}(z)\, dz. \tag{41}$$

In this case, the deflections of the slabs that satisfy the boundary conditions of the problem under consideration, take the following forms:

$$\begin{aligned} y_1 = \frac{l^4}{D_1 + D_2} \Bigg\{ & q \cdot C \cdot x^2 + C_4 + 2q \cdot \frac{x^4}{24} - \frac{D_1 - D_2}{D_1} \cdot \frac{q}{24} - \\ & - \frac{D_2}{D_1} \sum_{n=0}^{\infty} A_{2n} \cdot \left[\phi_{1,2n} \cdot u_1(\alpha x) + \phi_{2,2n} \cdot u_3(\alpha x) + \varphi_{2n}(x) \frac{D_1}{D_2} \cdot f_{2n}(x) \right] \end{aligned} \tag{42}$$

$$\begin{aligned} y_2 = \frac{l^4}{D_1 + D_2} \cdot \Bigg\{ & q \cdot C \cdot x^2 + C_4 + 2q \cdot \frac{x^4}{24} + \frac{D_1 - D_2}{D_1} \cdot \frac{q}{24} + \\ & + \sum_{n=0}^{\infty} A_{2n} \cdot \left[\phi_{1,2n} \cdot u_1(\alpha x) + \phi_{2,2n} \cdot u_3(\alpha x) + \varphi_{2n}(x) - f_{2n}(x) \right] \Bigg\} \end{aligned} \tag{43}$$

where

$$C = -\frac{1}{2} - \frac{2}{m+1} \cdot \left\| C_0^{m/2} \right\|^{-1} \tag{44}$$

$$\phi_{1,2n} = \frac{b^{-1}}{8\alpha^3} \cdot \int_0^1 \left\{ 2u_1(\alpha) \cdot u_1[\alpha(1-z)] + u_4(\alpha) \cdot u_2[\alpha(1-z)] \right\} \cdot (1 - z^2)^{\frac{m-1}{2}} C_{2n}^{\frac{m}{2}}(z)\, dz \tag{45}$$

$$\phi_{2,2n} = \frac{b^{-1}}{8\alpha^3} \cdot \int_0^1 \left\{ 2u_3(\alpha) \cdot u_1[\alpha(1-z)] - u_2(\alpha) \cdot u_2[\alpha(1-z)] \right\} \cdot (1 - z^2)^{\frac{m-1}{2}} C_{2n}^{\frac{m}{2}}(z)\, dz \tag{46}$$

The integration factor C_4 can be eliminated by proceeding to relative displacement.

Expressions (37) and (42) are substituted into (3), and then the obtained equalities are multiplied by

$$(1 - x^2)^{\frac{m-1}{2}} \cdot C_{2k}^{\frac{m}{2}}(x), \quad k = 1, 2, 3, \ldots$$

and are integrated between –1 and 1.

In this case, considering the orthogonality of the ultraspherical Gegenbauer polynomials, we obtain the following infinite system with respect to unknown coefficients A_{2n} :

$$a_{2k}+\sum_{n=1}^{\infty} a_{2n,2k}\cdot A_{2n}=A_{2k}\cdot\alpha_m\cdot\left(\frac{m}{2}+2k\right)\cdot\left\|C_{2k}^{m/2}\right\| \qquad k=1,2,3,\ldots \tag{47}$$

where

$$\begin{gathered}a_{2k}=\frac{l^4}{D_1+D_2}\int_{-1}^{1}\left\{q\cdot\left(\overline{C_2}\cdot x^2+\frac{x^4}{24}\right)-\frac{D_2}{D_1}A_0\cdot\left[\phi_{1,0}\cdot(u_1(\alpha x)-1)+\right.\right.\\ \left.\left.+\phi_{2,0}\cdot u_3(\alpha x)+\varphi_0(x)+\frac{D_1}{D_2}\cdot(f_0(x)-f_0(0))\right]\right\}\cdot(1-x^2)^{\frac{m-1}{2}}\cdot C_{2k}^{\frac{m}{2}}(x)dx,\end{gathered} \tag{48}$$

$$\begin{gathered}a_{2n,2k}=-\frac{l^4}{D_1+D_2}\cdot\frac{D_2}{D_1}\cdot\int_{-1}^{1}\left\{\phi_{1,2n}\cdot[u_1(\alpha x)-1]+\phi_{2,2n}\cdot u_3(\alpha x)+\right.\\ \left.\left.+\varphi_{2n}(x)+\frac{D_1}{D_2}\cdot(f_{2n}(x)-f_{2n}(0))\right]\right\}\cdot(1-x^2)^{\frac{m-1}{2}}\cdot C_{2k}^{\frac{m}{2}}(x)d.\end{gathered} \tag{49}$$

By integrating (48), (49) by parts, it is possible to eliminate the singularities of the integrals and bring them to the following convenient form:

$$\begin{gathered}a_{2k}=\frac{l^4}{D_1+D_2}\cdot\int_{-1}^{1}\left\{2q\cdot\left(\overline{C}_2+\frac{x^2}{4}\right)-A_0\cdot\frac{D_2}{D_1}\cdot\left[\phi_{1,0}\cdot u_1''(\alpha x)+\right.\right.\\ \left.\left.+\phi_{2,0}\cdot u_3''(\alpha x)+\varphi_0''(x)+\frac{D_1}{D_2}\cdot f_0''(x)\right]\right\}\cdot\alpha_{2k}\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx,\end{gathered} \tag{50}$$

$$\begin{gathered}a_{2n,2k}=-\frac{l^4}{D_1+D_2}\cdot\frac{D_2}{D_1}\cdot\int_{-1}^{1}\left\{\phi_{1,2n}\cdot u_1''(\alpha x)+\phi_{2,2n}\cdot u_3''(\alpha x)+\right.\\ \left.\left.+\varphi_{2n}''(x)+\frac{D_1}{D_2}\cdot f_{2n}''(x)\right]\right\}\cdot\alpha_{2k}\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)d.\end{gathered} \tag{51}$$

where

$$\alpha_{2k}=\frac{m\cdot(m+2)}{2k\cdot(2k-1)(2k+m)(2k+m+1)} \tag{52}$$

Thus, the considered problem is reduced to the study of infinite systems of algebraic equations with infinite unknowns for the coefficients A_{2n}.

Infinite systems of algebraic equations (47) can be solved by the reduction method. Based on the infinity reduction method, systems are limited by the first r equations corresponding to the first r unknowns. These conditions exactly correspond to the restriction in series (35) by the first r terms with unknown coefficients A_i at $i=0,\ 1,\ 2,\ldots,r$.

3.3. Substantiation of the solution method

It is known that the reduction method is applicable only for regular infinite systems of algebraic equations.

Therefore, to apply the reduction method, it is necessary to prove the regularity or quasi-regularity of the system of infinite equations. To prove the regularity of system (47), the coefficients a_{2k} and $a_{2n,2k}$ are determined by formulas (50) and (51).

Research is performed as follows.

First, we estimate the coefficient a_{2k}. To estimate it, we take into account the following inequalities [24]:

$$\left|\int_a^b f_1(x)\cdot f_2(x)dx\right| \le \left\{\int_a^b f_1^2(x)dx\right\}^{1/2}\cdot\left\{\int_a^b f_2^2(x)dx\right\}^{1/2} \tag{53}$$

$$(1-x^2)^{\alpha} \le (1-x^2)^{\beta}, \quad \alpha \ge \beta, \quad -1 \le x \le 1 \tag{54}$$

Expressions (53) are called Cauchy-Bunyakovsky inequality.

Applying inequalities (53), (54) in (50), we obtain the following estimate

$$\left|a_{2k}\right| \le \alpha\cdot\alpha_{2k}\cdot\left\|C_{2k-2}^{m/2+2}\right\| \tag{55}$$

where

$$\alpha = \frac{l^4}{D_1+D_2}\cdot\left\{\int_{-1}^{1}\left[q\cdot\left(2\bar{C}_2+\frac{x^2}{2}\right)-A_0\cdot\frac{D_2}{D_1}\cdot\left(\phi_{1,0}\cdot u_1''(\alpha x)+\right.\right.\right.$$

$$\left.\left.\left.+\phi_{2,0}\cdot u_3''(\alpha x)+\varphi_0''(x)+\frac{D_1}{D_2}\cdot f_0''(x)\right)\right]^2 dx\right\}^{\frac{1}{2}},$$

Substituting (8) and (52) into (55), we can see that

$$\left|a_{2k}\right| < \infty \text{, at } k = 1,2,3,... \tag{56}$$

Next, it is necessary to analyze the following infinite numerical series, consisting of coefficients $a_{2n,2k}$:

$$\sum_{n=1}^{\infty}\left|a_{2n,2k}\right|, \quad k = 1,2,3,... \tag{57}$$

If conditions (56) are met for the infinite system (47) and the numerical series (57) converges and has a sum less than one for any, then the infinite system (47) will be quasiregular

$$\lim_{k\to\infty}\sum_{n=1}^{\infty}\left|a_{2n,2k}\right| = 0 \tag{58}$$

Based on these results, the numerical series (57) is investigated. For this, the common terms of series (57) defined by formula (51) are rewritten in the following form

$$\phi_{2n}''(x)\, a_{2n,2k} = \frac{l^4}{D_1+D_2}\cdot\frac{D_2}{D_1}\cdot\alpha_{2k}\cdot\left(u_{1,2k}\cdot\phi_{1,2n}+u_{3,2k}\cdot\phi_{2,2n}+\varphi_{2n,2k}+f_{2n,2k}\right) \tag{59}$$

where

$$u_{1,2k} = \int_{-1}^{1} u_1''(\alpha x)\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx; \tag{60}$$

$$u_{3,2k} = \int_{-1}^{1} u_3''(\alpha x)\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx; \tag{61}$$

$$\phi_{2n,2k} = \int_{-1}^{1} \phi''_{2n}(x) \cdot (1-x^2)^{\frac{m+3}{2}} \cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx; \tag{62}$$

$$f_{2n,2k} = \int_{-1}^{1} f''_{2n}(x) \cdot (1-x^2)^{\frac{m+3}{2}} \cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx. \tag{63}$$

With (13), and considering the inequalities (53), (54) in (58) and (59), we can obtain the following estimates

$$\left|u_{1,2k}\right| \le a_1 \cdot \left\|C_{2k-2}^{m/2+2}\right\|; \ \left|u_{3,2k}\right| \le a_3 \cdot \left\|C_{2k-2}^{m/2+2}\right\|^{1/2} \tag{64}$$

where

$$a_1 = \left\{\int_{-1}^{1} 4\alpha^4 \cdot u_3^2(\alpha x)dx\right\}^{1/2}; \qquad a_3 = \left\{\int_{-1}^{1} 4\alpha^4 \cdot u_1^2(\alpha x)dx\right\} \tag{65}$$

Now it is necessary to estimate the coefficients $\phi_{1,2n}$ and $\phi_{2,2n}$, determined by formulas (45) and (46), respectively. Integration by parts the coefficients are reduced to the following form

$$\begin{gathered}\phi_{1,2n} = \frac{b^{-1}}{8\alpha^3} \cdot (-2\alpha^2) \cdot \alpha_{2n} \cdot \int_0^1 \{2u_1(\alpha) \cdot u_3[\alpha(1-z)] + \\ + u_4(\alpha) \cdot u_4[\alpha(1-z)]\} \cdot (1-z^2)^{\frac{m+3}{2}} \cdot C_{2n-2}^{\frac{m}{2}}(z)dz,\end{gathered} \tag{66}$$

$$\begin{gathered}\phi_{2,2n} = \frac{b^{-1}}{8\alpha^3} \cdot 2\alpha b \cdot \alpha_{2n} \cdot C_{2n-2}^{\frac{m}{2}}(0) - \frac{b^{-1}}{8\alpha^3} \cdot 2\alpha^2 \cdot \alpha_{2n} \cdot \int_0^1 \{2u_3(\alpha) \cdot u_3[\alpha(1-z)] - \\ - u_2(\alpha) \cdot u_4[\alpha(1-z)]\} \cdot (1-z^2)^{\frac{m+3}{2}} \cdot C_{2n-2}^{\frac{m}{2}}(z)dz.\end{gathered} \tag{67}$$

Taking inequalities (53) and (54) in (66), (67), we can obtain the following estimates

$$\left|\phi_{1,2n}\right| \le \phi_1 \cdot \alpha_{2n} \cdot \left\|C_{2k-2}^{m/2+2}\right\|^{1/2}; \ \left|\phi_{2,2n}\right| \le \phi_2 \cdot \alpha_{2n} \cdot \left\|C_{2k-2}^{m/2+2}\right\|^{1/2} \tag{68}$$

where

$$\phi_1 = \frac{b^{-1}}{4\alpha} \cdot \left\{\int_0^1 \left[2u_1(\alpha) \cdot u_3(\alpha x) + u_4(\alpha) \cdot u_4(\alpha x)\right]^2 dx\right\}^{1/2} \tag{69}$$

$$\phi_2 \le \frac{1}{4\alpha^2} + \frac{b^{-1}}{4\alpha} \cdot \left\{\int_0^1 \left[2u_3(\alpha) \cdot u_3(\alpha x) - u_2(\alpha) \cdot u_4(\alpha x)\right]^2 dx\right\}^{1/2} \tag{70}$$

It is necessary to estimate the coefficient $\phi_{2n,2k}$ determined by formula (62). To do this, first, the function $\phi''_{2n}(x)$ participating in the integral (62), with (41), and by integrating by parts, is reduced to the following form

$$\varphi''_{2n} = \alpha_{2n} \cdot \left[(1-x^2)^{\frac{m+3}{2}} \cdot C_{2n-2}^{\frac{m}{2}+2}(x) - u_1(\alpha x) \cdot C_{2n-2}^{\frac{m}{2}+2}(0) - 4\alpha^4 \cdot \varphi_{2n}(x)\right]. \tag{71}$$

Substituting (71) into (62), we obtain

$$\phi_{2n,2k} = \alpha_{2n} \cdot \left(J_{1,2n,2k} + J_{2,2n,2k} + J_{3,2n,2k}\right) \tag{72}$$

where

$$J_{1,2n,2k}=\int_{-1}^{1}(1-x^2)^{\frac{m+3}{2}}\cdot C_{2n-2}^{\frac{m}{2}+2}(x)\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2n-2}^{\frac{m}{2}+2}(x)dx; \quad (73)$$

$$J_{2,2n,2k}=C_{2n-2}^{\frac{m}{2}+2}(0)\cdot\int_{-1}^{1}u_1(\alpha x)\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx; \quad (74)$$

$$J_{3,2n,2k}=-4\alpha^4\int_{-1}^{1}\phi_{2n}(x)\cdot(1-x^2)^{\frac{m+3}{2}}\cdot C_{2k-2}^{\frac{m}{2}+2}(x)dx. \quad (75)$$

Applying (53), (54) in (73), (74), (75) we obtain the following estimates

$$\left|J_{1,2n,2k}\right|\le 4\alpha^3\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}, \quad (76)$$

$$\left|J_{2,2n,2k}\right|\le \overline{a}_1\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}, \quad (77)$$

$$\left|J_{3,2n,2k}\right|\le 8\alpha^4\cdot\left|\varphi_{2n}(1)\right|\cdot\left\|C_{2k-2}^{m/2+2}\right\| \quad (78)$$

where

$$\varphi_{2n}(1)=\frac{1}{4\alpha^3}\cdot\int_{0}^{1}u_4[\alpha(1-z)]\cdot(1-z^2)^{\frac{m+3}{2}}\cdot C_{2n-2}^{m/2+2}(z)dz \quad (79)$$

Applying (53), (54) in (79), we obtain

$$\left|\varphi_{2n}(1)\right|\le\frac{1}{4\alpha^3}\cdot\left\{\int_{-1}^{1}u_4^2[\alpha(1-z)]dz\right\}^{1/2}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2} \quad (80)$$

Substituting (80) into (78), we obtain the following estimate

$$\left|J_{3,2n,2k}\right|\le a_4\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2} \quad (81)$$

where

$$a_4=2\alpha\left\{\int_{-1}^{1}u_4^2(\alpha x)dx\right\}^{1/2}. \quad (82)$$

Taking into account (76), (77), (77) in (72), we obtain the following estimates

$$\left|\varphi_{2n,2k}\right|\le\alpha_{2n}\cdot\left(4\alpha^3+\overline{a}_1+a_4\right)\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}. \quad (83)$$

Substituting (22) into (63), then applying (53), (54), we obtain the following estimates

$$\left|f_{2n,2k}\right|\le\alpha_{2n}\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}. \quad (84)$$

From equality (59), taking into account (64), (68), (83), (84), we obtain

$$\left|a_{2n,2k}\right|\le a\cdot\alpha_{2n}\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}\cdot\alpha_{2k}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2} \quad (85)$$

where

$$a=\frac{l^4}{D_1+D_2}\cdot\frac{D_2}{D_1}\cdot\left(\phi_1\cdot a_1+\phi_2\cdot a_3+\overline{a}_1+a_4+1\right). \quad (86)$$

Introducing (85) into (57), we obtain

$$\sum_{n=1}^{\infty}\left|a_{2n,2k}\right| \le a\cdot\alpha_{2k}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}\cdot\sum_{n=1}^{\infty}\alpha_{2n}\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2}. \qquad (87)$$

In this inequality, the numerical series

$$\sum_{n=1}^{\infty}\alpha_{2n}\cdot\left\|C_{2n-2}^{m/2+2}\right\|^{1/2} \qquad (88)$$

is absolutely convergent. This can be verified by substituting (52) and (8) in (88). If we denote the sums of series (88) by $S,$ then inequality (88) takes the form

$$\sum_{n=1}^{\infty}\left|a_{2n,2k}\right| \le S\cdot a\cdot\alpha_{2k}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2},\ \ k=1,2,3,... \qquad (89)$$

Taking into account (8), (10), it can be shown that

$$\lim_{k\to\infty}\alpha_{2k}\cdot\left\|C_{2k-2}^{m/2+2}\right\|^{1/2}=0. \qquad (90)$$

Equality (90) shows that

$$\sum_{n=1}^{\infty}\left|a_{2n,2k}\right|\to 0\text{, at }k\to\infty. \qquad (91)$$

The proved limit values (91) state that conditions (56) and (91) are satisfied in the problem under consideration. As follows from the fulfillment of these conditions, the infinite system of algebraic equations (47) is quasiregular. Due to the quasiregularity of the system of infinite algebraic equations (47), it is possible to apply the reduction method to its solution.

3.4. Problem

Let a two-layer beam slab lying on an elastic inhomogeneous foundation be loaded with uniformly distributed external loads with the following characteristics of the soil material of the base

$$q(x)=q=const.$$

We take the following mechanical and geometrical parameters of the base and slabs:

For soil of the base:

$$\nu_0=0.3,\ E_0=5\cdot10^2\ \mathrm{kg/cm^2}.$$

For slabs:

$$l=5\,m,\ \ h_1=h_2=0.45\,m,\ \ \nu_1=\nu_2=0.617$$
$$E_1=E_2=1.25\cdot10^5\ kg/cm^2.$$

First, we determine the numerical values by solving the system (47), corresponding to the specific values of the coefficient of inhomogeneity m of the base and the values of the coefficient of rigidity k of the fillers. Solving the system (47), it is possible to determine the deflections of the slabs and the base, and the internal forces of the slab and the pressure of the inhomogeneous base according to the known formulas.

When solving this problem, we restrict ourselves to the first four terms in series (35). Then the system of infinite equations (47) turns into a system of three equations with three unknown coefficients A_2, A_4, A_6. The coefficient A_0 is considered known and its value is calculated by formula (36).

The numerical values of the coefficients A_0, A_2, A_4, A_6 for different values of the filler rigidity coefficient k and the base inhomogeneity coefficient $m,$ are given in Table 1.

Results of solving algebraic equations corresponding to different values of the filler rigidity (k) and the coefficient of inhomogeneity of the base (m).

Table 1. A table of solutions of a system of algebraic equations.

$k\left(kg/sm^3\right)$	m	A_0/q	A_2/q	A_4/q	A_6/q
0.25	0.25	0.675059	–0.145148	0.097165	–0.008346
0.25	0.50	0.599070	–0.127346	0.065661	0.005514
0.25	0.75	0.543265	–0.115487	0.044763	0.002671
2.5	0.25	0.675059	–0.145148	0.097165	–0,008316
2.5	0.50	0.599070	–0.127346	0.065661	0.005514
2.5	0.75	0.543265	–0.115487	0.044763	0,002671
250	0.25	0.675059	–0.145148	0.097163	–0.008346
250	0.50	0.599070	–0.127346	0.065661	0.005514
250	0.75	0.543265	–0.115487	0.044763	0.002671

Analysis of the results given in Table 1 shows that:

- the change in the values of the filler rigidity coefficients does not substantially affect the change in the solution of system (47), it also does not affect the change in the reactive pressure of the base, determined by formula (35);
- the change in the values of the inhomogeneity coefficients of elastic base substantially affects the change in the solution of system (47), i.e., the absolute values of the solution to system (47) decrease due to an increase in the base inhomogeneity, which leads to a decrease in the reactive pressure of the base;
- the calculations of the internal force factors of the strip-slab can be limited to the first 4 terms of the series (35).

The maximum numerical values of the bending moments of the first $M_1(x)$ and second slabs $M_2(x)$ at $x=0,$ are given in Table 2.

The results of the maximum values of bending moments of the slabs corresponding to the values of the filler rigidity coefficient (k) and the base inhomogeneity coefficient (m).

Table 2. Table of values of bending moments of plates.

$k\left(kg/sm^3\right)$	m	The greatest value of bending moments of slabs, at $x=0$	
		$M_1(x)/\left(ql^2\right)$	$M_2(x)/\left(ql^2\right)$
0.25	0.25	1.084615	1.067307
	0.50	1.039137	1.031462
	0.75	0.994724	0.992371
2.5	0.25	1.039906	1.036302
	0.50	0.974743	0.974401
	0.75	0.922241	0.922186
25	0.25	0.923714	0.923609
	0.50	0.799507	0.798897
	0.75	0.699268	0.699260

Analysis of the results given in Table 2 shows that:

- an increase in the value of the filler rigidity coefficient (at a constant value of the base inhomogeneity), leads to a substantial decrease in the value of bending moments;
- with an increase in the values of the coefficients of inhomogeneity of the base (with a constant value of the filler coefficient), the values of bending moments are substantially reduced;

- with an increase in the values of the filler rigidity coefficient and the values of the coefficients of inhomogeneity of the base, the values of bending moments substantially decrease (down to 36 %);
- with an increase in the values of the filler rigidity coefficient, the values of the bending moments of the slabs approach each other.

The results obtained show the effectiveness of the technique in determining the internal force factors in multilayer slabs.

The results obtained show the effectiveness of the proposed model and the developed technique for solving contact problems on the interaction of multilayer slabs with elastic inhomogeneous bases.

It should be noted that the results obtained, when the coefficient of inhomogeneity (m) of the base tends to zero, are similar to the results obtained in [10] for a homogeneous base.

4. Conclusion

1. A mathematical model was developed to assess the internal force factors of multilayer strip-slabs on an elastic inhomogeneous base under various static loads.

2. To assess the internal force factors of multilayer strip-slabs interacting with an inhomogeneous base, an analytical method for solving the problem was proposed, based on the approximation of orthogonal polynomials.

3. A theoretically substantiated possibility of using the proposed method for solving contact problems on the interaction of multilayer slabs with elastic inhomogeneous bases is given.

4. The required number of terms of the Gegenbauer polynomial was established to obtain a result with satisfactory accuracy, and the efficiency of the method for solving the problem was shown by the example of solving test problems.

5. Analysis of the results obtained made it possible to establish that:

- an account of the rigidity characteristics of the filler leads to a redistribution of internal forces in the slabs;
- an account of the inhomogeneity of the base leads to a decrease in internal forces in the slabs;
- an account of the inhomogeneity of the base leads to a redistribution of the reactive pressures of the base.

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