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Topology design of plane bar systems based on polygonal discretization

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Abstract. A topology formation method for frame and truss structures, which is relevant for use in computer-aided design, is proposed. A flat figure discretized into polygonal primitives is used as a basic structure. This structure is presented in the form of a finite element model and is assumed redundant. Its elements are excluded using the principle of maximizing the intensity of the force flow during the initial calculation and the shortest path of this force flow between the points that reflect force and kinematic connections. At the same time, the heuristic principle implemented in the ACO algorithms was used. Registration of geometrically variable systems is performed based on an estimate of the infinite norm of the stiffness matrix of the finite element model. For flat trusses and frames, examples of the formation of rational topologies under given force and kinematic constraints are considered. Comparison with the method of redundant structure, which implements the exclusion based on the genetic algorithm, is performed. It has been shown that the proposed technique allows one to effectively select both one and several alternative variants of the topology of bearing systems.

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1. Introduction

One of the key issues in designing plane bar systems is the formation of the initial object structure. Usually, the structure is formed notionally, while the constructor refers mainly to his/her background experience or generic solutions. Partial algorithmic presentation and mathematical implementation of this process with rapid development become possible. Certain steps have already been made in terms of selecting sustainable geometry of structures for plane bar systems. In this vein, it has been proposed to synthesize the plane bar system structure in works [1–4] based on metaheuristic algorithms, approaches are known [5–7] when the initial topology was developed using patterning the groups of geometrically invariable bars, kinematic chains and solid bodies. The following structure forming methods have been developed and improved by several authors:

- excluding elements from the basic structure, which is in certain cases deemed to be excessive [8–11];
- formation of topology on a range of produced joints via triangulation, construction of Voronoi's diagrams, etc. [12–15];
- design of topology based on transformations of system rigidity matrix and analytical equations of structure shape [16–20].

The issue of structure shape design has often been addressed in common iterative optimisation procedures for bearing structures both in normal operations and emergency actions, while genetic algorithms [21–26], in which all of the mentioned forming methods can be used have gained ground. In some works [27–29] the search for the rational topology of plane bar systems has been proposed to be made based on excluding the groups of plate elements from the continuous area, on the borders of which load and kinematical limitations are established. Such an approach is feasible in addressing one-criterion tasks upon the criterion of minimum mass for this area in a variety of plane thickness in some of its sub-areas. However, the wide application of this approach has been inhibited by the high labour input of the computational process.

Works are widespread for the topological design of frames considering the type, calculation and structure designing peculiarities. Algorithms set forth for reinforced concrete frames [30], steel trusses and [31–34], as well as wood and other bar systems [35–37] are fundamental algorithms.

The purpose of the research is to obtain topologies of bar-bearing structures that are rational in terms of material consumption. For this, an approach is used that involves the exclusion of elements from some redundant structure. The article provides the method for the design of plane bar systems based on combined heuristic strategy including the basic principle of optimisation algorithm ACO (ant colony optimization) [38] and FE analysis of the area and volume meshed on bar elements. This method was tested on the formation of flat frames and trusses topologies. The obtained results are compared with the topology optimization method based on genetic algorithms. The proposed method makes it possible to obtain guaranteed topologies that are geometrically invariable.

2. Methods

2.1. Formulation of the topology optimization problem

Let us consider a plane area in which the bar pattern structure will be formed. According to the fundamental provisions of the ACO-algorithm, the topology is formed along the shortest path of the colony between the base points. In this case, the points of application of forces and points of support are considered. Load and kinematical constraints will be input at the area boundary. The minimum bar length will be considered as the search criterion. According to the ACO algorithm basic principle, the minimum length of insects travels way is determined by the preservation of the pheromone they emit on this way that disseminates if a long or irrational way is selected. Drawing an analogy, the stress flow, which is delivered by the path of application of load to supports, will be taken as the pheromone. At the same time, considering the integral plane body modelled using plate elements, the stress flow will have various intensities similarly to the pheromone dissipation degree Fig. 1a.

a) **b)** **c)**

Figure 1. Initial data for topology design: integral body (a),
arrangement of vertical compressive stress (b), shapes for sampling, composed of bars (c).

Let us split the area into shapes so that the obtained bar construction has applied forces and supports in bar nodes at the area boundary. Wherein, the patterns shown in Fig. 1 will be used as splitting shapes. Wherein, the stability of geometrical shape should be provided. The obtained bar pattern will contain some possible stress flow paths of various intensities. Thus, we obtain one criterion for the optimisation of the gross-force task.

$$L_s\left(Y\right)\rightarrow \min, \qquad (1)$$

where L_s is the overall bar length; Y is the set of bar groups, consecutively excluded from the initial framework formed in splitting the plane area.

2.2. Boundary conditions of the task

In respect to the task of obtaining the bearing structure pattern, the following constraints should be obtained:

1. Stability of geometrical shape of the framework at each stage of its formation. For relatively simple splitting and small areas to evaluate this criterion the procedure for evaluating the FE model stiffness matrix determinant can be used effectively. The system is invariable provided that the matrix is positive-definite. If a time-consuming task is solved, then the algorithm will be relevant based on the material solidification hypothesis (links weakening). Considering the results of this work the following analytical signs of stability of the geometrical shape of the system is used:

$$\det\left[K\right] \neq 0; \tag{2}$$

$$\alpha_{cond} = cond_{\infty}\left[K\right]/e^{m}, \tag{3}$$

where $\left[K\right]$ is the global stiffness matrix of the finite element model; $cond_{\infty}\left[K\right] = \left\|\left[K\right]\right\|_{\infty} \times \left\|\left[K\right]^{-1}\right\|_{\infty}$ is the matrix of condition number $\left[K\right]$; $\left\|\bullet\right\|_{\infty}$ is the endless matrix norm symbol; m is the number of discs in the system.

The value $\alpha_{cond} < 10^{3}$ is responsible for the stability of the object's geometrical shape; if $\alpha_{cond} > 10^{100}$ then calculating is interrupted due to obtaining $\det\left[K\right] = 0.$ If the determinant of the stiffness matrix is zero, then the topology is geometrically variable. Intermediate results can be interpreted as instant instability of the examined system.

1. The structure calculations are made by the linear finite element (FE) analysis. At the same time, if the frame construction is searched, only bar elements of the bent should be used in the framework (three degrees of freedom in the node), and if a truss is synthesized, then truss finite elements are used (two degrees of freedom in the node).

2. The stiffness of all bars is described correspondingly and accepted the same for all members. The value of the assigned stiffness and loads should enable to perform effective arranging of groups of bars by the intensity of stresses arising in them.

2.3. Algorithm of design topology for plane bar systems

The following sequence of actions is performed:

1. Select primitives, in which the area will be split. Select the shape of the splitting area. The rectangular area is the first preference. At the same time, it is recommended to use splitting shapes of similar type and size. Split the area to obtain a bar pattern.

2. Overlay load and kinematical limitations on the obtained plane model. Forces and stiffness values are set correspondingly. Calculate the stress-strain state of the object to obtain initial forces distribution. In calculating the stresses, bending and compression strains are considered.

3. Rank the stress values. Split the range of stress values into 15–20 intervals. Create a list of groups of bars occurring in each range.

4. Take attempts to exclude any groups of bars with low stresses (making less than 15–20 % from maximal) and groups with the least number of bars. Upon excluding each group, the stability of the geometrical shape of the system is checked (see constraint one). If the system becomes alterable, then group exclusion is not allowed. It has been established empirically that in the first implementation of this stage the number of excluded groups of bars should not exceed 3. Late in framework forming, it is possible to try excluding one or several groups with relatively high stress. If the number of groups in which exclusion the system does not pass into a geometrically invariable state equals zero, then one may talk about the ultimate pattern.

In excluding the groups in externally statistically indeterminate systems with supports in a form of rigid restraints, it may prove that the framework degenerates: splits into parts, each of which has shortest paths from the load to its support Fig. 1b. For solving this problem, the fundamental propositions of graph theory are used. Let us present the sampled bar structure as a directed graph $G\left(B,N\right),$ consisting of several vertices N and edges B. In this case, it is considered that the vertex is a nodal joint, and the edge is a bar. The presence of at least one path in the graph from one supporting node to another is checked. If there is only one such path, the exclusion of the group of bars passing through the supporting nodes is not allowed. The condition of path existence is formalised by the construction of the incident matrix and reachability matrix for the graph $G(B,N)$.

5. Calculate the system obtained at stage 4 by the FE analysis to obtain a new stress arrangement. Then continue with stage 4.

2.4. Forming trusses topology

In designing the truss patterns there are stages implemented for task solving similar to that of the rigid frames with the following differences:

- in splitting the area, the truss bar elements are used, only stresses from axial forces are used. The splitting grid should be formed in such a way so that one side of the spiting primitive is not smaller in size than any structural element, that is the structural element of the future framework should contain one finite element of the calculation model.
- in implementing the exclusion attempts at each stage of framework formation, only one group of bars should be excluded, in which the priority is first given to small stress values, then to the number of bar elements in the group. The Search condition for paths in this graph $G(B,N)$ is not applied.

2.5. FE analysis for topology design

To determine the components of the stress-strain state of elements in the redundant topology of the rod system, a linear static calculation is used. For the design of flat frames, bar elements are used with three degrees of freedom each in the node. This is a vertical, horizontal displacement and rotation in the plane of the frame. For the topology of trusses, a bar element with two degrees of freedom (linear displacements in a plane) is used. In the topology, all bars can be excluded or groups of boundary elements can be formed that cannot be excluded. If boundary elements are present in the topology, then their stiffnesses are set for 2-3 orders of magnitude higher than for the rest of the bars. This technique makes it possible to avoid their exclusion in the search process.

The FE mesh for trusses must be formed so that one finite element corresponds to one bar of the truss. There is no such restriction for frame structures. The size of the FE mesh significantly affects the amount of computation during the search. When a group of elements is excluded from the topology, the FE model is rebuilt. In this case, the element is excluded not by creating zero stiffnesses for it, but by removing the topology matrix of the FE model. The reason for this is related to the estimation of the determinant of the stiffness matrix.

3. Results

3.1. Design frame topology for the symmetrically loaded area. Example 1

Let us consider the design of the framework on the given rectangular area at the given load and the kinematical limitations shown in Fig. 2a. Sample this area to elementary triangles composed of plane bar finite elements (Fig. 2b). Force P is applied in the middle of the span. P force and stiffness value is set in due form: P = 500 kN, the bar cross-section area A = 0.4 m^2, central moment of inertia lx = 1.3E10^{-4} m^4. The X -axis is perpendicular to the longitudinal axis Z of the bar and the plane of the frame (Fig. 2c). Consider the axial force values as one of the criteria to estimate the possible exclusion of bar groups. For all bars in this and the following examples, a material with an Young's modulus E = 70000 MPa, and density of 27 kN/m^3 was assigned.

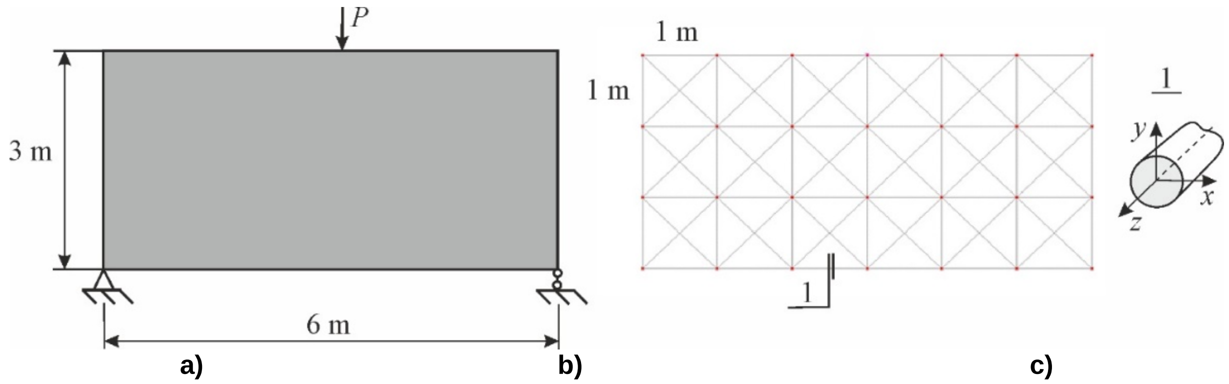


Figure 2. Baseline data for frame structure design.

Upon system calculation, rank the obtained stress components by splitting the obtained values into 16 equal intervals. A different number of bars may occur in each of such intervals. These data are presented in Fig. 3, via a colour-coded indication. The grey lines show the bar's excluded at the current stage of the search.

In implementing attempts to exclude bars for each iteration, one or several groups of bars were excluded. At the same time, to determine what groups are to be excluded from the available, the criterion of Eq. (1) was used. The intermediate results at the third iteration of the structure search are shown in Fig. 3b. Eventually, at the last iteration, the configuration shown in Fig. 3c is obtained. The result of structure design accurately coincided with the precise analytical solution by Goldstein [39], obtained in solving the task of the optimal search for elastic lines of bar systems based on the calculus of variations.

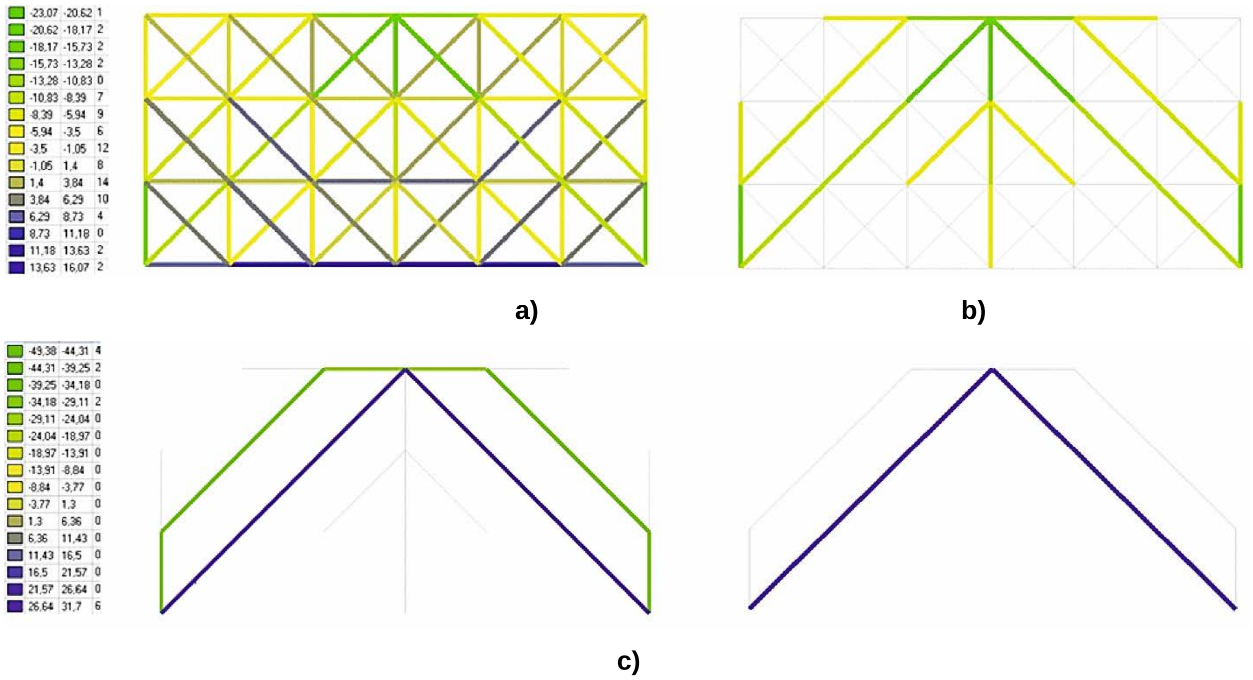
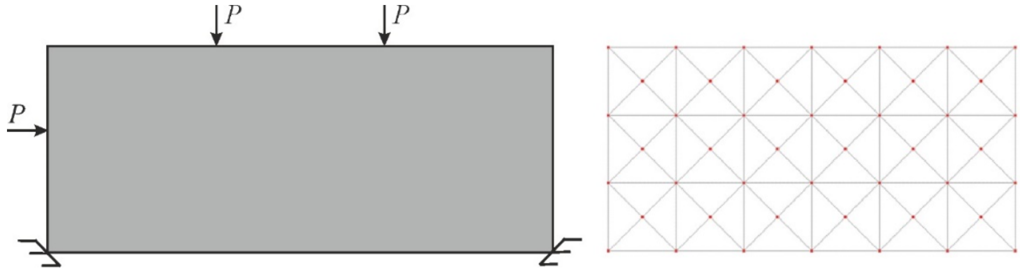


Figure 3. Results of single-span system design loaded by one force in the span centre: the first and second columns in the color legends show the boundaries of the intervals for the values of axial forces (tonnes). The third column is the number of bars in this interval.

3.2. Design frame topology for an asymmetrically loaded area. Example 2

Synthesize the frame construction topology on the rectangular area with limiting conditions shown in Fig. 4a. Split this area into triangle primitives consisting of frame bar elements, see Fig. 4b. The value of forces and geometric characteristics of bars are taken the same as in Example 1.



a) b)

Figure 4. Load diagram for frame topology design.

Solutions are searched similarly to Example 1. The process of topology design was performed in 9 iterations presented in Fig. 5. At the same time, at each iteration, several attempts were taken to exclude the groups with the preliminary evaluation of the possibility of such an exclusion.

Considering the kinematical limitations of Example 2, shown in Fig. 4, a situation is possible when the least length structure is discrete for each support. So, to obtain the ultimate framework shown in Fig. 6, an additional knockout criterion is used according to the arrangement of the existing continuous path between bearing summits in the oriented unweighted graph $G(B, N)$. It should be emphasized that it was not necessary to use the additional criterion of continuous path existence in the topology graph between bearing nods in Example 1, as parts of the framework in abruption of topology between supports turned out to be sub static.

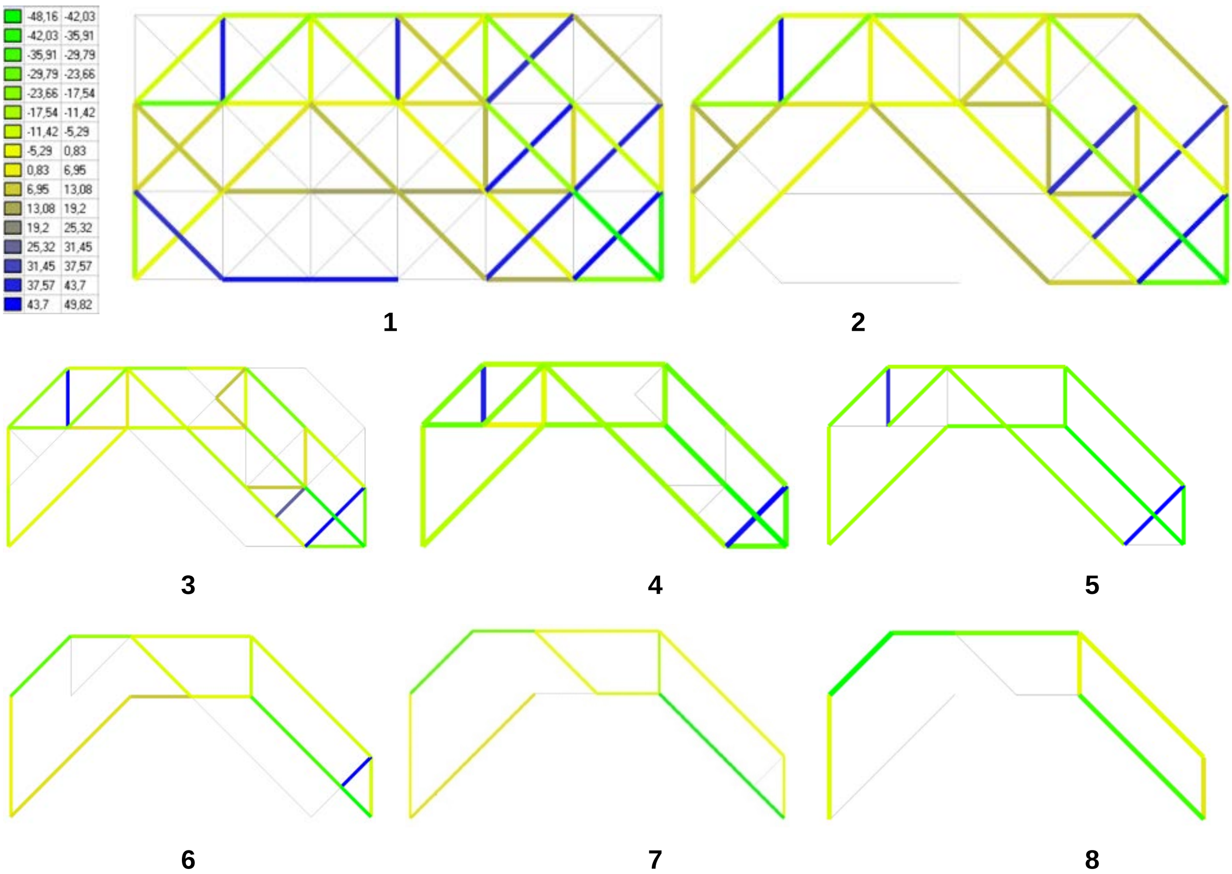


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Figure 6. Stages of frame topology design: the colours shown the levels of axial forces in the bars.

3.3. Truss structure design. Example 3

Let us consider an example of a rectangular area design (Fig. 7). The supports are taken as the hinged-fixed. The area has dimensioned *a* = 4.064 m (160 in.), *b* = 0.762 m (30 in.). The value of point forces and geometric characteristics of truss bar elements were accepted from Example 1. Solution search stages are shown in Fig. 8.

Initially, braces in the middle truss area were excluded with axial force values [–0.34; 6.4] (see scheme I on Fig. 8). In attempting to exclude additionally one more group of three rods at this stage, for which the value of the force belongs to the interval [–0.34; –7.32], the system becomes geometrically invariable. Upon calculating the framework obtained at stage I, perform finite element analysis and new force distribution to be analysed at stage II. At the second stage in the provision of stability of geometrical shape condition and the minimum length of the output, structure bars only one group of the two bars of side plates was excluded with the axial forces in the interval [27.84; 34.69]. In the attempt to exclude more groups, these limitations were not put into effect. Further, the calculation was performed and axial force distribution at stage III was obtained. At this stage, the exclusion of any single group resulted in the violation of limitations, so efforts were taken to exclude 2 and more bar groups. This resulted in the exclusion of two groups. The first group consists of five bars including end and middle pickets and end panel upper strake bars. Axial forces for these bars belong to the range [–7.77; 0.43] (see scheme I Fig. 8). The second group consists of two bars of middle panel lower core with the axial forces range of [8.62; 16.81]. This resulted in the topology shown in Fig. 9.

4. Discussion

The solution obtained in Fig. 9, accurately coincides with Kirsch truss structural design result (Fig. 10a) [8, 11], where based on the genetic algorithm 19 topology variants were obtained. The three of such variants are shown in Fig. 10b-d. The analysis of all the 19 topologies [8] shows that the solution given in Fig. 9 is the only one of all the solutions according to the geometrically invariable topology. The rest of the solutions are either geometrically unstable or have large length bars. The absence of bars was simulated by setting fragility elements but not the full removal from the topology; hence system alterability was not recorded. The use of Eqs. (2) and (3) in the proposed algorithm ensures the registration and removal from consideration of such topologies.

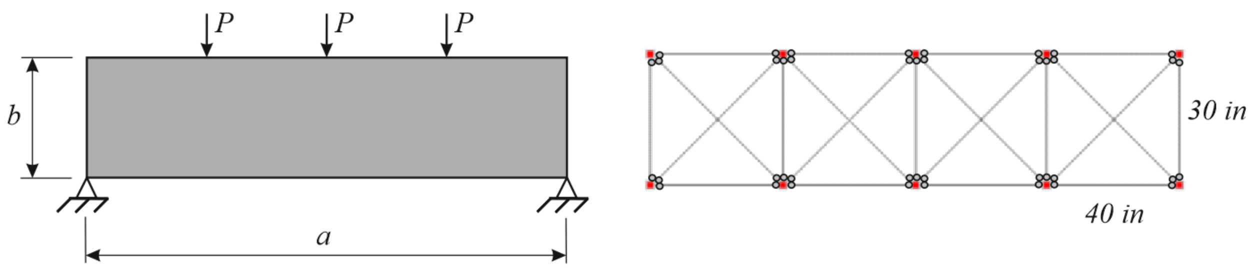

Figure 7. Rectangular area and its sampling for truss topology formation.

I			II			III		
-77,07	-70,09	4	-75	-68,14	4	-106,07	-97,87	2
-70,09	-63,12	0	-68,14	-61,29	0	-97,87	-89,68	0
-63,12	-56,14	0	-61,29	-54,43	0	-89,68	-81,49	0
-56,14	-49,17	0	-54,43	-47,58	0	-81,49	-73,3	2
-49,17	-42,19	0	-47,58	-40,72	0	-73,3	-65,11	0
-42,19	-35,22	0	-40,72	-33,87	2	-65,11	-56,92	0
-35,22	-28,24	2	-33,87	-27,01	0	-56,92	-48,72	0
-28,24	-21,27	6	-27,01	-20,15	6	-48,72	-40,53	0
-21,27	-14,29	0	-20,15	-13,3	0	-40,53	-32,34	2
-14,29	-7,32	0	-13,3	-6,44	0	-32,34	-24,15	0
-7,32	-0,34	3	-6,44	0,41	1	-24,15	-15,96	0
-0,34	6,63	2	0,41	7,27	2	-15,96	-7,77	2
6,63	13,61	0	7,27	14,13	0	-7,77	0,43	5
13,61	20,58	0	14,13	20,98	0	0,43	8,62	0
20,58	27,56	2	20,98	27,84	2	8,62	16,81	2
27,56	34,53	2	27,84	34,69	2	16,81	25	2

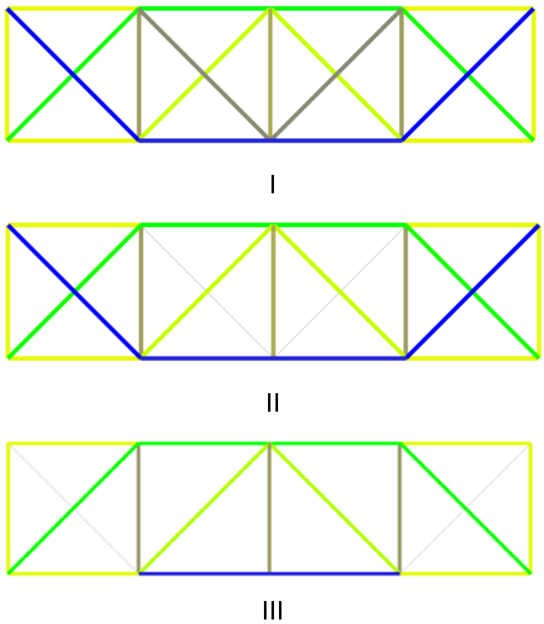

Figure 8. Truss topology formation process: I-III – search steps.

Figure 9. Resulting truss topology.

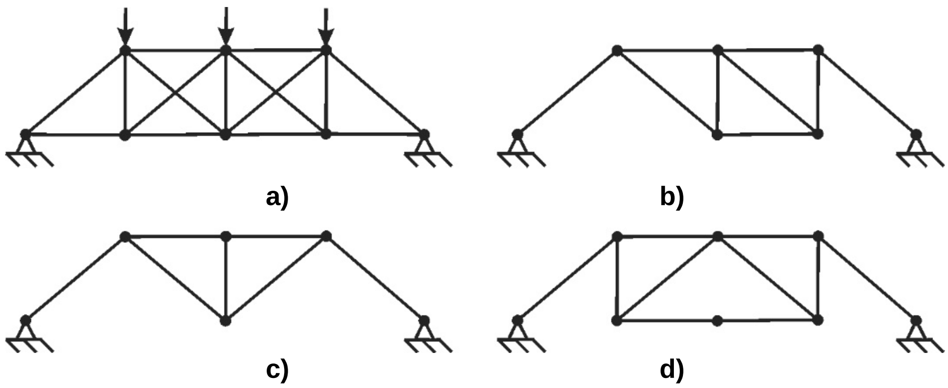

Figure 10. Some solutions for optimisation of Kirsch truss topology.

5. Conclusion

The method of plane bar systems rational topology design has been developed based on sampling the rectangular area to bar primitives. Basic principles of ant colony imitation and search for ways in oriented graphs have been taken as a basis. The efficiency of the proposed method has been proved based on the coincidence of the obtained results with the common solutions for standard test tasks. Such an approach for frameworks design can be recommended to be used in forming layout arrangements for frame buildings and structural systems.

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