



Research article

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## The method of determining the trend equation for predicting vertical displacements by the search method of nonlinear programming

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**Abstract.** Observing the settlement of buildings and structures often gives rise to the question of their prediction. This primarily requires determining the form of the time series trend equation, which would describe by a single law all sequences of the time series compiled from geodetic data. The procedure for determining the trend equation of the time series is carried out by linear and nonlinear regression analysis. However, these methods have a number of drawbacks. In this connection, the case study of changing the benchmarks values of one of the deformation marks is considered in order to analyze the possibility to apply the search method of nonlinear programming for determining the trend equation on the basis of the geodetic measurements data. A method of constructing a trend equation for a predictive model based on geodetic data is presented, including: 1) determining the trend equation by a search method of nonlinear programming using a VBA-based search program; 2) assessment of the accuracy of determining the coefficients of the trend equations by using the search method of nonlinear programming; 3) assessment of the adequacy of trend models obtained by the search method. Based on the results of the analysis and calculations, an adequate trend model in the form of a logarithmic equation was established. The correctness of the conclusions was proved by the calculations carried out in Microsoft Excel and Statistica, which coincided with the results obtained. The predicted values of the deformation benchmarks were determined.

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### 1. Introduction

An essential task under modern restrained urban conditions is to observe the deformations of the foundations of buildings and structures. According to Russian Code Specifications SP 22.13330.2016 and SP 305.1325800.2017, a set of works based on field observations of the behavior a newly erected structure and its foundation, including soil massif, is geotechnical monitoring. Thus, geotechnical monitoring should consider the "engineering structure – environment" system. The set of works on geotechnical monitoring

includes geodetic monitoring [1–4], which is a systematic control of the state of buildings and structures based on geodetic measurements.

However, it is important not only to control the deformation processes of buildings and structures but also to determine the upward or downward trend in the observed deformations for short- or long-term periods. In order to identify this trend, which is of prognostic character, it is necessary, first of all, to find the trend equation of functional relationships of all consecutive time series levels which is built on geodetic data. It is also worth mentioning that methods of linear and nonlinear regressive analysis are the main classic methods for equation construction. These methods, however, are not devoid of drawbacks. The object of study is evaluation of the possibility to apply the search method of nonlinear programming for determining the trend equation on the basis of the geodetic measurements data as illustrated by changing the benchmarks values of one of the deformation marks in the process of geodetic monitoring. The object under the monitoring: a hotel complex under construction in Krasnodar City and the existing buildings and utilities within the new construction influence zone.

In general, the problem of finding the trend equation and building the prognostic models in the process of geodetic monitoring of structural elements of buildings and constructions is topical and rather complicated. Its solution requires a complex and systematic approach with using the knowledge of geodesy, civil engineering, econometrics on the basis of applying the multifactor regressive analysis and nonlinear programming. It also necessary to observe the requirements of the existing legal and regulatory basis on the expediency of forecasting and prognostic modeling during the procedure of geodetic monitoring of buildings and constructions.

The review of regulatory documents showed that the requirements for the construction of predictive models based on geodetic data as a necessary part of geodetic monitoring are specified in Russian Guidance Documentation in Construction MDS 13-22.2009 “Guidelines for geodetic monitoring of the technical condition of high-rise and unique buildings and structures” and Russian Company Standards STO SRO-C 6054290 00043-2015 “Nuclear facilities. Geodetic monitoring of buildings and structures during construction and operation”. Thus, the existing regulatory documents allow work on predicting the development of deformations only for buildings and structures with a higher level of responsibility. No such requirements and recommendations apply to buildings with a normal level of responsibility.

Works [5–8] are devoted to the importance of finding a solution to this problem. In addition, it should be noted that there is a significant number of publications focusing on the issue of determining the trend equation and the construction of a predictive model correlating with it [9–13]. There are many works by Russian and foreign researchers devoted to prognostic modeling based on geodetic data. For example, Iu.P. Guliaev, A.P. Zaitsev, S.V. Marfenko, D.Sh. Mikhelev, O.N. Pozdysheva, B.A. Kharchenko, V.S. Khoroshilov, N.N. Kobeleva, N.V. Sucheva, B. Zhang, L. Qiu, Z. Zhou and others consider the problem of defining the trend equation of time series by linear and nonlinear regressive analysis [14–19]. Whereas, B.T. Mazurov and some other scientists apply numerical methods and dynamic prognostic models in solving the problem of building prognostic models. They argue that these methods give more accurate and effective results as compared to those of linear and nonlinear regressive analysis [20–22].

The aim of the present study is to develop a method for determining the trend equation on the geodetic measurements data by the search method of nonlinear programming. The values measured were elevation points obtained in the process of geodetic monitoring of the hotel complex being constructed in Krasnodar City.

To achieve this goal the following tasks were to be solved:

- a) performing an analysis of forming classical types of trend equations (linear, of logarithmic function and polynomials of various degrees);
- b) carrying out geodetic monitoring of the hotel complex being constructed in Krasnodar City and of the existing buildings and utilities by setting deformation marks on them and determining the points of the latter obtained for 11 monitoring cycles;
- c) building a prognostic model according to the obtained sample of deformation marks points, namely:
  - determining the trend equation;
  - assessing the accuracy of defining the trend equation coefficients;
  - evaluating the adequacy of trend models obtained by the search method;
  - forecasting the values of deformation mark points for several cycles ahead.

## 2. Methods

Linear, exponential, logarithmic functions and polynomials of various degrees are often used to build trends [11, 23, 24]. The coefficients of each of the trend equations are determined by the traditional least squares method [15], and the degree of the approximating polynomial is selected from the condition of the minimum variance of the deviations of the actually measured values from the values obtained theoretically [11]. However, the disadvantages of nonlinear regression models are the complexity of determining the type of functional dependence, as well as the complexity of determining the parameters of the model, since first, it is necessary to carry out the standard procedure for their linearization. The disadvantages of linear regression models are low adaptability and lack of ability to model nonlinear processes.

One of the problems in solving nonlinear equations is to ensure fast convergence of the computational process. This problem is discussed in [23, 25, 26].

The search method of nonlinear programming, the essence of which is an iterative search for the minimum deviations of the initial values of the time series from their experimental (theoretical) values calculated by the trend equation, is devoid of the listed disadvantages when determining the trend equation. The search for the minimum deviation of the initial values from the theoretical ones will continue until the value in the target cell – the desired deviation – decreases, i.e. in other words, the extremum of a given condition is determined, which is expressed as a specific objective function. In this case, the criteria for the termination of the program operation can be both a visual finding of the fulfillment of a given number of iterations, and compliance with the condition regarding the absence of discrepancies between the two values of the target cell at the  $n^{\text{th}}$  and  $n-1^{\text{th}}$  iterations up to a certain decimal digit.

Next, we will consider the procedure for determining the trend equation by the search method of nonlinear programming based on geodetic measurements using the example of changing the elevation values of one of the deformation control benchmarks. It should be noted here that geodetic monitoring was carried out on all the twelve deformation marks (DM1–DM12), and the construction of the trend equation and the predictive model was carried out according to the marks values of the of DM6 deformation mark.

### 2.1. Problem statement

An obligatory stage of geotechnical monitoring of a hotel complex under construction in Krasnodar and existing buildings and communications in the zone of influence of the new construction (Fig. 1) was geodetic monitoring of settlements.

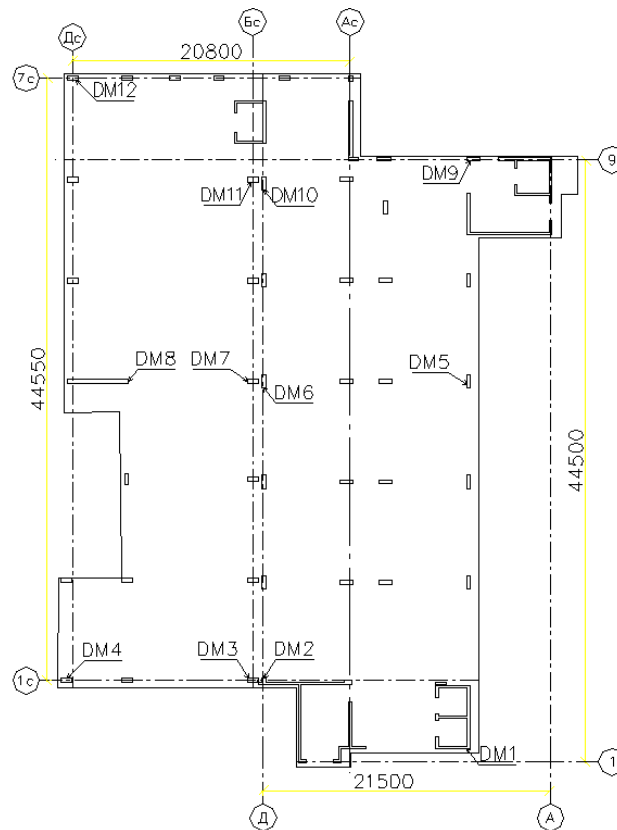
According to the project, the hotel complex under construction has one underground floor intended for parking, 8 aboveground floors in 1-9/A-D axes (hotel) and 1 aboveground floor in 1s-7s/As-Ds axes (conference hall). The shape of the building is complex in terms of its overall axial dimensions of 35.8 × 50.55 m. The structural scheme of the building is frame: the main supporting structures are monolithic reinforced concrete columns, walls and ribbed floors. In 1-9/A-D axes in the area of the walls, the foundation is designed in the form of a reinforced concrete slab with a thickness of 700 mm. The foundations of the columns in this part of the building are reinforced concrete belts located in the digital axes. In 1s-7s/As-Ds axes, the foundation is 500 mm thick slab.

The erection of the underground part of the building was carried out in a pit about 4.9 m deep. The project provides for the implementation of work to consolidate the foundation soils. To fasten the walls of the pit, a fence made of steel piles with a plank fence was arranged along its northern and western sides. The piles were immersed in 4–6 m long soil-cement elements and joined at the upper ends by a steel I-beam.

Let us consider the procedure for determining the trend equation by the search method according to the deformation control benchmarks when determining the draft of such a hotel complex. Observation of sedimentation was carried out by the method of class II geometric leveling in a conventional system of heights. Table 1 shows the deformation control benchmarks obtained during 11 observation cycles.

**Table 1. The deformation control benchmarks obtained from the geodetic monitoring.**

Cycle No.	1	2	3	4	5	6	7	8	9	10	11
Date	25.09	28.10	27.11	28.12	27.01	25.02	28.03	27.04	29.05	27.06	27.07
Benchmark No.	Benchmarks, m										
DM1	0.704	0.702	0.700	0.699	0.699	0.699	0.699	0.697	0.697	0.697	0.697
DM2	0.914	0.911	0.912	0.911	0.910	0.909	0.906	0.905	0.904	0.904	0.904
DM3	0.884	0.881	0.880	0.880	0.879	0.879	0.880	0.878	0.877	0.876	0.876
DM4	0.842	0.840	0.842	0.841	0.840	0.842	0.839	0.840	0.839	0.839	0.838
DM5	0.878	0.875	0.872	0.871	0.870	0.870	0.870	0.869	0.868	0.868	0.868
<b>DM6</b>	<b>0.895</b>	<b>0.892</b>	<b>0.889</b>	<b>0.888</b>	<b>0.886</b>	<b>0.886</b>	<b>0.885</b>	<b>0.885</b>	<b>0.884</b>	<b>0.884</b>	<b>0.883</b>
<b>DM7</b>	<b>0.882</b>	<b>0.878</b>	<b>0.876</b>	<b>0.876</b>	<b>0.873</b>	<b>0.874</b>	<b>0.874</b>	<b>0.873</b>	<b>0.873</b>	<b>0.871</b>	<b>0.871</b>
DM8	0.930	0.928	0.927	0.927	0.927	0.926	0.926	0.925	0.925	0.924	0.923
<b>DM9</b>	<b>0.852</b>	<b>0.848</b>	<b>0.846</b>	<b>0.845</b>	<b>0.845</b>	<b>0.844</b>	<b>0.844</b>	<b>0.843</b>	<b>0.842</b>	<b>0.841</b>	<b>0.838</b>
<b>DM10</b>	<b>0.849</b>	<b>0.845</b>	<b>0.843</b>	<b>0.843</b>	<b>0.841</b>	<b>0.842</b>	<b>0.841</b>	<b>0.840</b>	<b>0.838</b>	<b>0.837</b>	<b>0.837</b>
DM11	0.872	0.868	0.866	0.866	0.868	0.865	0.864	0.863	0.862	0.862	0.862
DM12	0.852	0.849	0.849	0.849	0.849	0.848	0.847	0.847	0.848	0.846	0.845

**Figure 1. The building plan with deformation control benchmarks locations.**

The values in Table 1 show the greatest changes in DM6, DM7, DM9 and DM10. The data in Table 1 for these four benchmarks can be presented in the form of a time series, in which the marks of the benchmark ( $x_i$ ) are the levels of the series, and the cycle numbers ( $y_i$ ) are the time marks (Table 2).

**Table 2. Time series.**

$x_i$	1	2	3	4	5	6	7	8	9	10	11	Benchmark
	895.00	892.00	889.00	888.00	886.00	885.50	884.50	885.00	884.00	884.00	883.00	DM6
	882.00	878.00	876.00	876.00	873.00	874.00	874.00	873.00	873.00	871.00	871.00	DM7
$y_i$	852.00	848.00	846.00	845.00	845.00	844.00	844.00	843.00	842.00	841.00	838.00	DM9
	849.00	845.00	843.00	843.00	841.00	842.00	841.00	840.00	838.00	837.00	837.00	DM10

Building a predictive model needs to [14]:

1. determine the trend equation;
2. evaluate the accuracy of the coefficients of the trend equations;
3. evaluate the adequacy of trend models obtained by the search method;
4. predict the values of the deformation benchmarks for several cycles ahead.

### 2.2. General information about the search method of nonlinear programming

As V.I. Mitskevich, D. Himmelblau [26], and other authors [27–29] note, the search method of nonlinear programming has the following advantages:

1. clarity and simplicity of the trend equation;
2. the uniformity of the applied calculation method, regardless of the desired form of the trend equation;
3. the ability to simulate both linear and nonlinear processes;
4. no need to use derivatives;
5. sufficient efficiency in preparing the problem for solution.

There are the following types of search methods:

1. Hook-Jeeves' direct search;
2. Nelder and Mead's method (search by deformable polyhedron);
3. Rosenbrock's method;
4. Davis-Svenn-Campy's method (DSC);
5. Powell's method;
6. Method of random search.

Each of the above methods, both individually and in various combinations, makes it possible to determine the extremum of a given objective function. The choice of the method is determined by the degree of relevance (adequacy to the nature of the problem being solved) and the effectiveness of solving the problem. We shall consider the procedure for determining the extremum of the objective function by a combination of the DSC and Powell algorithms.

Let the function  $f(x)$  be unimodal and have a unique value  $x^*$  such that  $f(x^*)$  is the minimum of  $f(x)$ .

Here is this algorithm (Fig. 2):

*Step 1.* An initial arbitrary value of the element  $x^{(0)}$  is set.

*Step 2.* The value of the objective function  $f(x^{(0)})$  is calculated.

*Step 3.* The variable  $x^{(0)}$  is changed by the selected value  $\Delta x$  and  $x^{(1)} = x^{(0)} + \Delta x$  is set.

*Step 4.* The value of the objective function  $f(x^{(1)})$  is calculated.

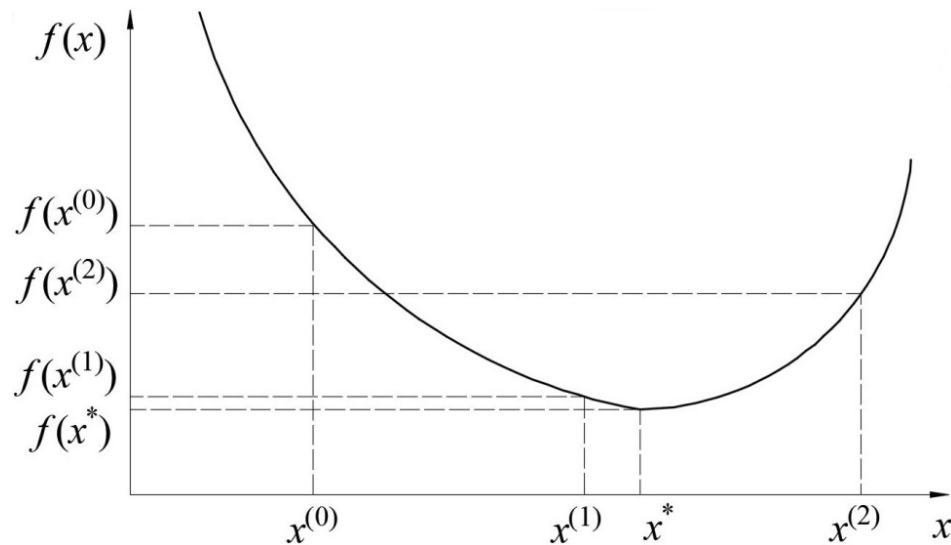
*Step 5.* If  $f(x^{(0)}) > f(x^{(1)})$ , then the variable  $x^{(1)}$  is changed by the selected value  $\Delta x$  and  $x^{(2)} = x^{(0)} + 2\Delta x$  or  $x^{(2)} = x^{(1)} + \Delta x$  is set. If  $f(x^{(0)}) \leq f(x^{(1)})$ , then  $x^{(2)} = x^{(0)} - \Delta x$ .

*Step 6.* The value of the objective function  $f(x^{(2)})$  is calculated.

Step 7. The variable  $x^*$  is determined corresponding to the value of the minimum of the objective function  $f(x)$ :

$$x^* = x^{(1)} + \frac{\Delta x \left[ f(x^{(0)}) - f(x^{(2)}) \right]}{2 \left[ f(x^{(0)}) - 2f(x^{(1)}) + f(x^{(2)}) \right]}. \quad (1)$$

Step 8. The value of the objective function  $f(x^*)$  is calculated. Thus, the minimum of the objective function  $f(x)$  is achieved.



**Figure 2. Geometric interpretation of minimization of the function  $f(x)$  by the Powell–DSC method.**

The above sequence of calculating the minimum  $f(x)$  is described in general form in [27, 28]. In expression (1) the second term is a correction that is introduced into the variable  $x^{(1)}$  to achieve the minimum of the objective function. Obviously, it is possible to determine the corresponding corrections for the variables  $x^{(0)}$  and  $x^{(2)}$ , which was noted in [27].

### 3. Results and Discussion

#### 3.1. Determining the trend equation by the search method of nonlinear programming in the VBA-based program

The problem of development of automated programs for processing the results of geodetic measurement has always been topical [28]. To determine the regularity of the change in the mark values of the deformation benchmarks DM6, DM7, DM9 and DM10 during automated calculations, a VBA-based program was compiled [29], which operates as a Macro in Microsoft Excel. The program implements the algorithm of the nonlinear programming search method described above. In turn, the unknown coefficients of the analyzed trend equations are also directly selected by the search method.

Thus, the following coefficients and analytical equations were obtained by the search method of nonlinear programming, presented in Table 3.

**Table 3. Types of the trend equation for settlement in DM6, DM7, DM9 and DM10, obtained by the search method.**

Type of trend equation	Trend equation
DM6	
linear	$y = -1.0409x + 893.1550$
logarithmic	$y = -5.0020 \cdot \ln(x) + 894.8680$
second degree polynomial (quadratic)	$y = 0.1463x^2 - 2.7962x + 896.9580$
third degree polynomial (cubic)	$y = -0.0218x^3 + 0.5379x^2 - 4.7585x + 899.3330$
DM7	
linear	$y = -0.8818x + 879.8818$
logarithmic	$y = -4.2298 \cdot \ln(x) + 881.3210$
second degree polynomial (quadratic)	$y = 0.1026x^2 - 2.1126x + 882.5486$
third degree polynomial (cubic)	$y = -0.0324x^3 + 0.6865x^2 - 5.0387x + 886.0827$
DM9	
linear	$y = -1.0618x + 850.6564$
logarithmic	$y = -4.7837 \cdot \ln(x) + 851.8969$
second degree polynomial (quadratic)	$y = 0.031x^2 - 1.4338x + 851.4719$
third degree polynomial (cubic)	$y = -0.0468x^3 + 0.8732x^2 - 5.6543x + 856.5719$
DM10	
linear	$y = -1.0273x + 847.5727$
logarithmic	$y = -4.6821 \cdot \ln(x) + 848.8589$
second degree polynomial (quadratic)	$y = 0.0583x^2 - 1.7265x + 849.0877$
third degree polynomial (cubic)	$y = -0.0258x^3 + 0.5233x^2 - 4.0569x + 851.9090$

The form of the analytical trend equations selected by the search method can be any at the operator's choice.

Further, from the obtained trend equations, the best one is selected, which characterizes the maximum change in the observed process, the settlement of deformation benchmarks.

Let us consider this and subsequent problems of choosing the best trend equation using the example of DM6. For this, empirical methods of analyzing the patterns of development of the investigated time series were used [30], such as:

- method of analysis of variance;
- calculation and analysis of the root mean square error of approximation;
- the least squares criterion of empirical and theoretical values of the levels of the time series.

The method of analysis of variance is based on the comparison of variances. The method is as follows: the total variance of the time series is divided into two parts: the variation due to the trend  $V_{f(t)}$  and the random variation  $V_{\varepsilon}$ . In this case, the variation due to the tendency is defined as the difference between the total  $V_{\text{total}}$  and random variations  $V_{\varepsilon}$  [31, 32].

The total variation is defined as the sum of the squares of the deviations of the initial values of the levels of the series  $(y_i)$  from the mean level of the initial time series  $(\bar{y})$ . The random variation is calculated as the sum of the squares of the deviations of the initial values of the time series  $(y_i)$  from the theoretical ones obtained by the trend equation  $(\hat{y})$  [30, 31].

On the basis of these indicators of variation, the total number of elements of the initial time series  $n$  and the number of parameters of the trend equation  $l$ , various types of variance are determined: total, variance of the random component and variance of the trend.

Further, a hypothesis was put forward and passed a test whether the considered trend equation is suitable or not suitable for describing the trend of the initial time series. The hypothesis was tested on the basis of the Fischer–Snedecor F-test [30, 32]. For a given significance level  $\alpha$  and the number of degrees of freedom ( $k_1 = l - 1$ ,  $k_2 = n - l$ ), the F-test is determined from the table of critical points of the Fisher–Snedecor distribution. If the calculated F-test value  $F_p$  is greater than  $F_t$ , then the trend equation is consistent with the initial data and is suitable for reflecting the trend of the initial time series [30, 32].

The choice of the best trend equation is also indicated by the value of the root mean square error of approximation [30]  $\sigma_{er}$ . The smaller the error value, the better the function describes the trend in the original series.

The application of the criterion of the least sum of squares of deviations of the initial and theoretical values of the levels of the time series also suggests that the trend is best described by the trend, which corresponds to the smallest value of the sum of squares of deviations.

Table 4 shows the formulas for the calculation and the results of the analysis of the trend equations obtained by the search method.

**Table 4. Results of the analysis of the trend equations obtained by the search method for DM6.**

Parameter	Calculation formula	Linear equation	Logarithmic equation	Second degree polynomial	Third degree polynomial
Results of analysis of variance					
Total variation $V_{tot}$ , mm <sup>2</sup>	$\sum_{i=1}^n (y_i - \bar{y})^2$		141.409		
Random variation $V_\varepsilon$ , mm <sup>2</sup>	$\sum_{i=1}^n (y_i - \hat{y}_i)^2$	22.225	2.493	3.868	0.944
Variation due to the trend $V_{f(t)}$ , mm <sup>2</sup>	$V_{\text{обш}} - V_\varepsilon$	119.184	138.916	137.541	140.465
Total variance $\sigma^2_{total}$ , mm <sup>2</sup>	$\frac{V_{\text{обш}}}{n-1}$		14.141		
Dispersion of the random component $\sigma^2_\varepsilon$ , mm <sup>2</sup>	$\frac{V_\varepsilon}{n-l}$	2.469	0.277	0.484	0.135
Dispersion of the trend $\sigma^2_{f(t)}$ , mm <sup>2</sup>	$\frac{V_{f(t)}}{l-1}$	119.184	138.916	68.771	46.822
Fischer–Snedecor F-test $F_p$	$\frac{\sigma^2_{f(t)}}{\sigma^2_\varepsilon}$	48.264	501.570	142.232	347.174
Calculation and analysis of the root mean square error of approximation					
Mean square error $\sigma_{er}$ , mm	$\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-l-1}}$	1.667	0.558	0.743	0.397
The least squares criterion of empirical and theoretical values of the levels of the time series					
Sum of squares of deviations, mm <sup>2</sup>	$\sum (y_i - \hat{y}_i)^2 \rightarrow \min$	22.225	2.493	3.868	0.944

The works [10, 11] note that the choice of the best trend equation is also carried out according to the values of the correlation coefficient, determination coefficient  $R^2$  and standard approximation error  $\sigma_t$ . The definitions of these values of the factors for each of the trend equations obtained by the search method are shown. Also, the value of the normalized R-square, the relative approximation error  $E_t$  and the average approximation error  $\bar{A}$  was additionally calculated. The results are shown in Table 5.



**Table 5. Calculation of the coefficients of regression statistics.**

Coefficient	Linear equation	Logarithmic equation	Second degree polynomial	Third degree polynomial
Correlation coefficient (multiple $R$ )	0.9181	0.9911	0.9862	0.9967
Determination coefficient $R^2$	0.8428	0.9824	0.9726	0.9933
Normalized $R$ -square	0.8254	0.9804	0.9696	0.9926
Standard approximation error $\sigma_t$ , mm	1.5714	0.5263	0.6953	0.3672
Relative approximation error $E_t$ , %	0.1772	0.0593	0.0784	0.0414
Average approximation error $\bar{A}$ , %	0.1382	0.0457	0.0530	0.0291

The closer the determination coefficient to 1,000 and the smaller the error rates are, the better the equation describes the original time series.

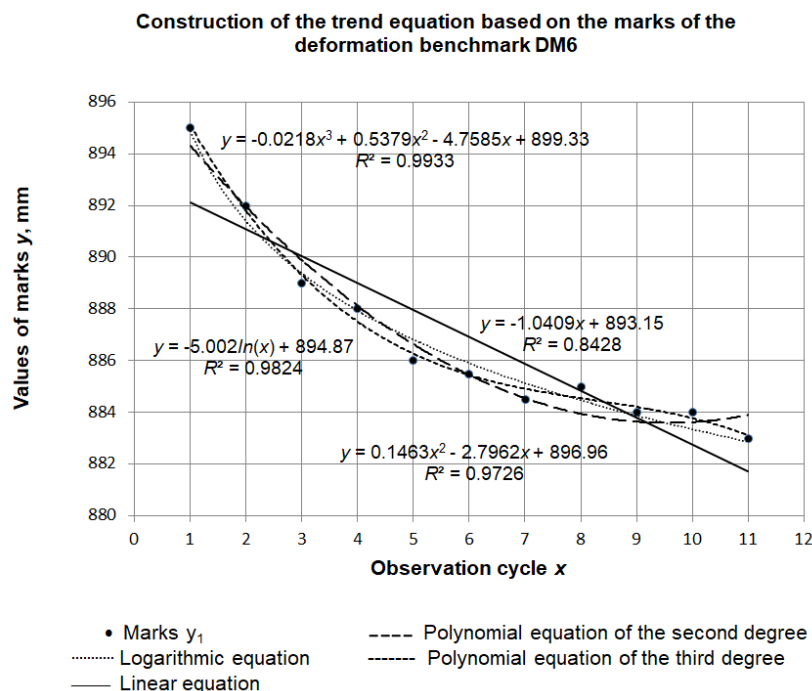
### 3.2. Validation of the Microsoft Excel calculations

The validation of the calculations performed and the obtained trend equations by the values of the deformation mark marks was carried out in *Microsoft Excel* using the built-in function of this program (Fig. 3).

Fig. 3 shows that the coefficients of the considered trend equations calculated by the search method and the equations obtained by the built-in function of *Microsoft Excel* completely coincide. In addition, the values of the determination coefficients  $R^2$ , calculated in *Microsoft Excel* (Fig. 3), also coincide with the values of such a coefficient for the trend equations obtained by the search method of nonlinear programming.

The built-in functions of *Microsoft Excel* were also used to calculate the indicators of regression statistics and analysis of variance data on the example of a linear trend model. The protocol for solving the problem, drawn up in *Microsoft Excel*, confirmed the correctness of the calculations performed to determine the indicators of analysis of variance and indicators of regression statistics for the linear trend equation obtained by the search method.

We should note that *Microsoft Excel* does not provide for a similar calculation protocol for equations of the logarithmic form and polynomials of different degrees. The use of the compiled program to determine the trend equation by the search method of nonlinear programming allows calculating the necessary parameters to estimate the best trend equation in *Microsoft Excel*.



The results of regression and variance analyses with the built-in *MS Excel* package “Data analysis” for the linear trend model are shown in Fig. 4.

	A	B	C	D	E	F	G	H	I
1	Results output								
2									
3	Regression statistics								
4	Multiple R	0.9181							
5	R-square	0.8428							
6	Normalized R-square	0.8254							
7	Standard error	1.5714							
8	Observations	11							
9									
10	Variance analysis								
11		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance of F</i>			
12	Regression	1	119,184	119,184	48,264	6.70544E-05			
13	Remainder	9	22,225	2,469					
14	Total	10	141,409						
15									
16		<i>Coefficients</i>	<i>Standard error</i>	<i>t-statistics</i>	<i>P-values</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
17	Y intersection	893.155	1.016	878.91001	1.627E-23	890.8557254	895.4533655	890.8557254	895.4533655
18	Variance X1	-1.0409	0.150	-6.947195	6.705E-05	-1.379851617	-0.701966565	-1.379851617	-0.701966565

**Figure 4. The results of regression and variance analyses with the *MS Excel* package “Data analysis” for the linear trend model.**

The results given in Fig. 4 can only be seen in *MS Excel* for the linear trend equation.

### 3.3. Validation of the calculations in *Statistica*

In order to additionally verify the calculations performed and the trend equations obtained from the deformation benchmarks, we carried out the equivalent calculations of coefficients for the linear and logarithmic trend equations as well as polynomial equations of the second and third degree in the software *Statistica* with adduction of corresponding statistics which characterizes the reliability of both the regression models and their particular indicators. The data obtained are given below (Fig. 5–9).

```
. regress y x
```

Source	SS	df	MS	Number of obs	=	
Model	119.184091	1	119.184091	F(1, 9)	=	48.26
Residual	22.225	9	2.46944444	Prob > F	=	0.0001
Total	141.409091	10	14.1409091	R-squared	=	0.8428
				Adj R-squared	=	0.8254
				Root MSE	=	1.5714

	y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
	x	-1.040909	.1498316	-6.95	0.000	-1.379852 - .7019666
	_cons	893.1545	1.016207	878.91	0.000	890.8557 895.4534

**Figure 5. Indicators of the linear regression trend model obtained with the software *Statistica*.**

```
. generate lnX=log(x)
. regress y x x2
```

Source	SS	df	MS	Number of obs	=	
Model	137.541026	2	68.7705128	F(2, 8)	=	142.23
Residual	3.86806527	8	.483508159	Prob > F	=	0.0000
Total	141.409091	10	14.1409091	R-squared	=	0.9726
				Adj R-squared	=	0.9658
				Root MSE	=	.69535

	y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
	x	-2.796154	.2924786	-9.56	0.000	-3.470611 -2.121697
	x2	.1462704	.0237388	6.16	0.000	.0915287 .2010121
	_cons	896.9576	.7636361	1174.59	0.000	895.1966 898.7185

**Figure 6. Indicators of the linear regression trend model of a second degree polynomial obtained with the software *Statistica*.**

```
. regress y x x2 x3
```

Source	SS	df	MS	Number of obs	=	11
Model	140.465035	3	46.8216783	F(3, 7)	=	347.17
Residual	.944055944	7	.134865135	Prob > F	=	0.0000
				R-squared	=	0.9933
				Adj R-squared	=	0.9905
Total	141.409091	10	14.1409091	Root MSE	=	.36724

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x	-4.758547	.4488666	-10.60	0.000	-5.819948	-3.697146
x2	.5378788	.0850325	6.33	0.000	.3368088	.7389488
x3	-.021756	.0046724	-4.66	0.002	-.0328045	-.0107076
_cons	899.3333	.6503738	1382.79	0.000	897.7954	900.8712

**Figure 7. Indicators of the linear regression trend model of a third degree polynomial obtained with the software Statistica.**

```
. regress y lnx
```

Source	SS	df	MS	Number of obs	=	11
Model	138.916424	1	138.916424	F(1, 9)	=	501.57
Residual	2.49266693	9	.276962992	Prob > F	=	0.0000
				R-squared	=	0.9824
				Adj R-squared	=	0.9804
Total	141.409091	10	14.1409091	Root MSE	=	.52627

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnx	-5.002145	.2233522	-22.40	0.000	-5.507403	-4.496887
_cons	894.8681	.3891958	2299.27	0.000	893.9877	895.7485

**Figure 8. Indicators of the logarithmic regression trend model obtained with the software Statistica.**

To make the comparison of the results obtained by the search method and by the software for more convenient comparing the results *MS Excel* and *Statistica* more convenient, we developed the following table.

**Table 6. Aggregate table of calculations of trend equations with statistics for DM6.**

Indicator	Search method	MS Excel	Statistica
For a linear trend equation:			
- equation form	$y = -1.0409x + 893.1550$	$y = -1.0409x + 893.15$	$y = -1.0409x + 893.1545$
- determination coefficient $R^2$	0.8428	0.8428	0.8428
- standard error of approximation $\sigma_t$ ,	1.5714	1.5714	1.5714
- normalized R-square	0.8254	0.8254	0.8254
- Fisher's test, $F_p$	48.264	48.264	48.26
For a logarithmic trend equation:			
- equation form	$y = -5.0020 \cdot \ln(x) + 894.8680$	$y = -5.002 \cdot \ln(x) + 894.87$	$y = -5.0020 \cdot \ln(x) + 894.8681$
- determination coefficient $R^2$	0.9824	0.9824	0.9824
- standard error of approximation $\sigma_t$ ,	0.5263	-	0.5263
- normalized R-square	0.9804	-	0.9804
- Fisher's test, $F_p$	501.570	-	501.57

Indicator	Search method	MS Excel	Statistica
For a second degree polynomial trend equation:			
- equation form	$y = 0.1463x^2 - 2.7962x + 896.9580$	$y = 0.1463x^2 - 2.7962x + 896.96$	$y = 0.1463x^2 - 2.7962x + 896.9576$
- determination coefficient $R^2$	0.9726	0.9726	0.9726
- standard error of approximation $\sigma_t$ ,	0.6953	-	0.6954
- normalized R-square	0.9696	-	0.9658
- Fisher's test, $F_p$	142.232	-	142.23
For a third degree polynomial trend equation:			
- equation form	$y = -0.0218x^3 + 0.5379x^2 - 4.7585x + 899.3330$	$y = -0.0218x^3 + 0.5379x^2 - 4.7585x + 899.33$	$y = -0.0218x^3 + 0.5379x^2 - 4.7585x + 899.3333$
- determination coefficient $R^2$	0.9933	0.9933	0.9933
- standard error of approximation $\sigma_t$ ,	0.3672	-	0.3672
- normalized R-square	0.9926	-	0.9905
- Fisher's test, $F_p$	347.174	-	347.17

The coincidence of the statistical indicators obtained by the search method with the corresponding indicators obtained in the software MS Excel и Statistica for trend equations prove the confidence of the calculations performed.

### 3.4. Assessment of the accuracy of the coefficients of the trend equations by the search method of nonlinear programming

Ch.N. Zheltko [29] described the general procedure for assessing the accuracy by the search method. The work [27] presents gradually the accuracy assessment procedure by the search method using the example of assessing the accuracy of the coordinates of a determined point in a geodetic triangle.

The estimation of the accuracy of the trend line by the search method of nonlinear programming is carried out through determining the coefficients of the normal equations of the unknowns. In the considered example, the unknowns are the coefficients of the determined trend equation. For a linear trend model, the unknowns are the coefficients  $k$  and  $b$ . Taking the algorithm given in [27] as a basis, the following calculations were performed. The value of the sum of squared deviations for the linear trend model is taken as the function  $F_{\min}$ . Then, alternately setting the increments  $\Delta k_1$ ,  $\Delta b_1$  into the corresponding determined coefficients of the linear trend model, new values of the sum of squares of deviations are calculated: the functions  $Fk_1$ ,  $Fb_1$  are obtained, respectively. This made it possible to calculate the changes in the function  $\Delta F_1 = Fk_1 - F_{\min}$  and  $\Delta F_2 = Fb_1 - F_{\min}$ .

Further, the increments  $\Delta k_1$  and  $\Delta b_1$  are simultaneously specified in the corresponding coefficients of the linear trend equation and the new value of the function  $Fkb_1$ , is calculated, which made it possible to calculate the change in the function  $\Delta F_{12} = Fkb_1 - F_{\min}$ .

The determination of the coefficients of the normal equations of the unknowns is carried out by the formulas:

$$N_{11} = \frac{\Delta F_1}{\Delta k_1^2}; N_{22} = \frac{\Delta F_2}{\Delta b_1^2}; N_{12} = \frac{\Delta F_{12} - \Delta F_1 - \Delta F_2}{2\Delta k_1 \Delta b_1}. \quad (2)$$

The inverse weight matrix of the determined coefficients of the linear trend equation has been compiled:

$$Q_K = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}^{-1} = \begin{bmatrix} q_{k_1} & q_{k_1 b_1} \\ q_{b_1 k_1} & q_{b_1} \end{bmatrix}. \quad (3)$$

The mean square errors of the determined coefficients of the linear trend equation were calculated:

$$m_{k_1} = \mu \sqrt{q_{k_1}}; \quad m_{b_1} = \mu \sqrt{q_{b_1}}, \quad (4)$$

here  $q$  is the inverse weight of the estimated value;  $\mu$  is the root mean square error (RMS) of the unit of weight, calculated by the formula

$$\mu = \sqrt{\frac{F_{\min}}{n-k}}, \quad (5)$$

where  $n$  and  $k$  are the number of performed and required measurements, respectively. It should be noted that the number of measurements performed is 11, and the number of required measurements for the linear trend model is 2.

A similar calculation procedure was applied to other types of trend models. The results of evaluating the accuracy of determining the coefficients constructed by the trend equations by the search method are presented in Table 7.

**Table 7. Results of evaluating the accuracy of determining the coefficients constructed by the trend equations by the search method.**

Type of trend equation	The inverse weight matrix of the determined coefficients of the linear trend equation, $Q_k$	RMS of the unit of weight $\mu$	RMS of the determined coefficients of the linear trend equation $m_i$
Linear equation	$\begin{bmatrix} 0.009 & -0.055 \\ -0.055 & 0.418 \end{bmatrix}$	1.571	$m_{k_1} = 0.150$ $m_{b_1} = 1.016$
Logarithmic equation	$\begin{bmatrix} 0.180 & -0.286 \\ -0.286 & 0.546 \end{bmatrix}$	0.526	$m_{k_1} = 0.223$ $m_{b_1} = 0.389$
Second degree polynomial	$\begin{bmatrix} 0.001 & -0.016 & 0.036 \\ -0.016 & 0.204 & -0.491 \\ 0.036 & -0.491 & 1.399 \end{bmatrix}$	0.695	$m_{c_1} = 0.025$ $m_{d_1} = 0.314$ $m_{e_1} = 0.822$
Third degree polynomial	$\begin{bmatrix} 0.0002 & -0.003 & 0.015 & -0.018 \\ -0.003 & 0.054 & -0.277 & 0.348 \\ 0.015 & -0.277 & 1.494 & -2.013 \\ -0.018 & 0.348 & -2.013 & 3.136 \end{bmatrix}$	0.367	$m_{b_1} = 0.0047$ $m_{d_1} = 0.085$ $m_{c_1} = 0.449$ $m_{e_1} = 0.650$

It is shown that the obtained value of the mean square error in determining the coefficients of the linear trend equation corresponds to the value of the standard error of the coefficients calculated in *Microsoft Excel* as part of regression analysis. Determining the value of the standard error for equations of a different kind in *Microsoft Excel*, as noted earlier, is not possible, in contrast to the search method of nonlinear programming.

Thus, the analysis of the data in Tables 4–7 showed that the best trend equations obtained by the search method of nonlinear programming in this example are equations of the logarithmic form and the third-degree polynomial, while the worst trend equation is linear.

However, the possibility of using a trend model for analysis and forecasting can be determined only after establishing its adequacy, i.e. correspondence of the model to the investigated process [14].

### 3.5. Assessment of the adequacy of trend models obtained by the search method

A trend model is considered adequate if it correctly reflects the systematic components of the time series. This requirement is equivalent to the requirements for the residual component [30]:

1. randomness;
2. compliance with the normal distribution law;
3. independence of values (no autocorrelation).

Let us conduct a study on the adequacy of the obtained trend models of the logarithmic form and the third-degree polynomial.

To test the randomness of the residual component, the run test and the peak test are used [30, 32].

Deviations of the initial values from the theoretical ones were determined:  $\varepsilon_i = y_i - \hat{y}_i$  for each level of the initial time series.

By arranging the obtained series  $\varepsilon_i$  in ascending order of values, the median  $\varepsilon_m$  of the obtained new series  $\varepsilon'_i$  is determined. Since the number of measurements is  $n = 11$ , the median is the median value.  $\varepsilon_i$  should be compared with  $\varepsilon_m$  and a sequence should be formed [14]. If  $\varepsilon_i > \varepsilon_m$ , then “+” is put, if  $\varepsilon_i < \varepsilon_m$  then “-” is put, with  $\varepsilon_i = \varepsilon_m$  – the value is omitted. A sequence of consecutive “+” or “-” is a series.

Further, the following hypothesis was tested: if deviations from the trend are random, then their alternation should be random [32]. For the sequence  $\varepsilon_i$  to be random, the length of the longest series  $K_{\max}$  should not be too long, and the number of series  $v$  should not be too small. The residual sequence is recognized as random if the following two inequalities are met at  $\alpha = 0.05$ :

$$K_{\max} < [3.3(\lg n + 1)], \quad (6)$$

$$v > \left[ \frac{1}{2} (n + 1 - 1.96\sqrt{n-1}) \right], \quad (7)$$

where  $n$  is the number of points in a row.

Square brackets on the right side of inequalities (6) and (7) denote an integer value.

The series criterion calculation is presented in:

- Table 8 for a linear trend model;
- Table 9 for a logarithmic trend model;
- Table 10 for a trend model of the second-degree polynomial;
- Table 11 for a trend model of the third-degree polynomial.

**Table 8. Calculation of the criterion for the series of the linear trend model.**

$y_i$	$f(t)$	$\varepsilon_t$	$\varepsilon'_t$	$\varepsilon_m$	$\varepsilon_t > \varepsilon_m$	$\varepsilon_t < \varepsilon_m$
895.00	892.11	2.89	-1.95		+	
892.00	891.07	0.93	-1.41		+	
889.00	890.03	-1.03	-1.37			-
888.00	888.99	-0.99	-1.03			-
886.00	887.95	-1.95	-0.99			-
885.50	886.91	-1.41	0.17	0.17		-

$y_i$	$f(t)$	$\varepsilon_t$	$\varepsilon_t'$	$\varepsilon_m$	$\varepsilon_t > \varepsilon_m$	$\varepsilon_t < \varepsilon_m$
884.50	885.87	-1.37	0.21			-
885.00	884.83	0.17	0.93			
884.00	883.79	0.21	1.25		+	
884.00	882.75	1.25	1.30		+	
883.00	881.70	1.30	2.89		+	

For the considered linear trend model,  $K_{\max} = 5$ ,  $\nu = 3$ . Thus, according to (1)  $K_{\max} < 6$ , according to (2)  $\nu > 2$ .

For the considered linear trend model, inequalities (1) and (2) are met. This means that the hypothesis about the randomness of deviations of the time series levels from the linear trend is confirmed.

**Table 9. Calculation of the criterion for the series of the logarithmic trend model.**

$y_i$	$f(t)$	$\varepsilon_t$	$\varepsilon_t'$	$\varepsilon_m$	$\varepsilon_t > \varepsilon_m$	$\varepsilon_t < \varepsilon_m$
895.00	894.87	0.13	-0.82		+	
892.00	891.40	0.60	-0.63		+	
889.00	889.37	-0.37	-0.41			-
888.00	887.93	0.07	-0.37			-
886.00	886.82	-0.82	0.07			-
885.50	885.91	-0.41	0.12	0.12		-
884.50	885.13	-0.63	0.13			-
885.00	884.47	0.53	0.13		+	
884.00	883.88	0.12	0.53			
884.00	883.35	0.65	0.60		+	
883.00	882.87	0.13	0.65		+	

For the considered logarithmic trend model,  $K_{\max} = 5$ ,  $\nu = 3$ . Thus, according to (1)  $K_{\max} < 6$ , according to (2)  $\nu > 2$ .

For the considered logarithmic trend model, inequalities (1) and (2) are met. This means that the hypothesis about the randomness of deviations of the time series levels from the logarithmic trend equation is confirmed.

**Table 10. Calculation of the criterion for the series of the trend model of the second-degree polynomial.**

$y_i$	$f(t)$	$\varepsilon_t$	$\varepsilon_t'$	$\varepsilon_m$	$\varepsilon_t > \varepsilon_m$	$\varepsilon_t < \varepsilon_m$
895.00	894.31	0.69	-0.90		+	
892.00	891.95	0.05	-0.89			
889.00	889.89	-0.89	-0.63			-
888.00	888.11	-0.11	-0.11			-
886.00	886.63	-0.63	-0.05			-
885.50	885.45	0.05	0.05	0.05		
884.50	884.55	-0.05	0.05			-
885.00	883.95	1.05	0.36		+	
884.00	883.64	0.36	0.38		+	
884.00	883.62	0.38	0.69		+	
883.00	883.90	-0.90	1.05			-

For the considered trend model of the second-degree polynomial,  $K_{\max} = 3$ ,  $\nu = 4$ . Thus, according to (1)  $K_{\max} < 6$ , according to (2)  $\nu > 3$ .

For the considered trend model of the second-degree polynomial, inequalities (1) and (2) are met. This means that the hypothesis about the randomness of deviations of the time series levels from the trend of the second degree polynomial is confirmed.

**Table 11. Calculation of the criterion for the series of the trend model of the third-degree polynomial.**

$y_i$	$f(t)$	$\varepsilon_t$	$\varepsilon'_t$	$\varepsilon_m$	$\varepsilon_t > \varepsilon_m$	$\varepsilon_t < \varepsilon_m$
895.00	895.09	-0.09	-0.42			-
892.00	891.79	0.21	-0.31		+	
889.00	889.31	-0.31	-0.27			-
888.00	887.51	0.49	-0.21		+	
886.00	886.27	-0.27	-0.12			-
885.50	885.45	0.05	-0.09	-0.09	+	
884.50	884.92	-0.42	0.05			-
885.00	884.55	0.45	0.21		+	
884.00	884.21	-0.21	0.22			-
884.00	883.78	0.22	0.45		+	
883.00	883.12	-0.12	0.49			-

For the considered trend model of the third-degree polynomial,  $K_{\max} = 1$ ,  $\nu = 11$ . Thus, according to (1)  $K_{\max} < 6$ , according to (2)  $\nu > 3$ .

For the considered trend model of the third-degree polynomial, inequalities (1) and (2) are met. This means that the hypothesis about the randomness of deviations of the time series levels from the trend of the third degree polynomial is confirmed.

According to the test for adequacy according to the series criterion, all considered trend models are recognized as adequate.

To check the peaks criterion, we determine the total number of peak points  $p$  of the  $\varepsilon_t$ , series, meeting the following condition [9, 20].

$$\varepsilon_{t-1} < \varepsilon_t < \varepsilon_{t+1} \quad \text{OR} \quad \varepsilon_{t-1} > \varepsilon_t > \varepsilon_{t+1}.$$

For the linear model, the number of turning points is 3, for the logarithmic model – 9, for the second-degree polynomial – 8, for the third-degree polynomial – 9.

Let us calculate the mathematical expectation of the number of peak points in a row and the variance using the formulas [9].

$$\bar{p} = \frac{2}{3}(n-2) = \frac{2}{3}(11-2) = 6,$$

$$\sigma^2 = \frac{16n-29}{90} = \frac{16 \cdot 11 - 29}{90} = 1.63.$$

The resulting number of turning points of the series must exceed the integer value of the equation  $\left[ \bar{p} - 1.96\sqrt{\sigma^2} \right]$ , which, for the considered series  $\varepsilon_t$ , is equal to 3. If this criterion is met, then the investigated trend model is considered adequate.

Thus, among the studied types of trend models, according to the criterion of peaks, the linear trend model is inadequate. The best indicators of adequacy according to the criterion of peaks in the logarithmic trend model and the third-degree polynomial.

Compliance with the normal distribution law. In this case, the hypothesis is tested that deviations from the trend obey the law of normal distribution. For this, as a rule, the correspondence of the distribution of the residual component to the normal law is performed using the *RS* test.

The ratio of the range of variations  $R$  to the standard deviation  $S$  is calculated for each type of trend model under consideration.



The range of variation was calculated using the formula [30, 32]

$$R = \varepsilon_{\max} - \varepsilon_{\min}, \quad (8)$$

where  $\varepsilon_{\max}$ ,  $\varepsilon_{\min}$  are the maximum and minimum values in the series  $\varepsilon_i$ , respectively.

The standard deviation was calculated by the formula

$$S = \sqrt{\frac{\sum \varepsilon_i^2}{n-1}}. \quad (9)$$

Table 12 shows the results of testing the hypothesis about the normal distribution of the residual component using the *RS* test.

**Table 12. Results of the RS test calculation.**

Type of trend model	Range of variations <i>R</i>	Standard deviation <i>S</i>	<i>RS</i> test
Linear model	4.89	1.49	3.28
Logarithmic model	1.47	0.49	3.00
Second degree polynomial	1.95	0.62	3.15
Third degree polynomial	0.91	0.31	2.94

For the number of elements in the series  $n = 11$ , the value of the *RS* test for all trend equations falls within the interval between the critical boundaries from 2.67 to 3.68, which means that the hypothesis about the normal distribution is accepted for all considered trend models.

Similar conclusions were made after checking the correspondence of the distribution of the residual component to the normal law using the indices of asymmetry and kurtosis.

**Independence of values (no autocorrelation).** To check the autocorrelation in the levels of the series, various criteria are used, including the Durbin–Watson d-test, the calculated value of which is determined by the formula [30, 32]:

$$d = \frac{\sum_{i=2}^n (\varepsilon_i - \varepsilon_{i-1})^2}{\sum_{i=1}^n \varepsilon_i^2}. \quad (10)$$

According to the results of calculations for the logarithmic equation, the d-test was 1.782, for the third-degree polynomial – 3.585.

Further, according to special tables, the critical values of the Durbin–Watson d-test are determined for a given number of observations, the number of independent variables of the model and the level of significance [31].

Comparing the calculated values of the d-criterion of trend models with tabular data, the following results were obtained:

- the logarithmic trend model does not have autocorrelation, which means it can be recognized as adequate;
- the trend model of the third-degree polynomial has autocorrelation and cannot be recognized as adequate.

Thus, according to the results of the performed assessment of the adequacy of the two types of trend models obtained by the search method of nonlinear programming, the logarithmic trend model was recognized the best.

### 3.6. Comparison of predicted deformation benchmarks with their actual values

To confirm the conclusions about the best form of the trend model obtained by the search method, the predicted values of the marks of the deformation benchmark DM6 were compared with the actual ones measured during observations of the stability of the building.

According to the requirements of STO SRO-S 6054290 00043-2015, according to geodetic monitoring data, forecasting can be carried out only to a depth of no more than 20–25 % of the length of the time series. Thus, Table 13 shows the actual marks of the deformation benchmarks and the predicted marks of this benchmark obtained for 12–14 observation cycles according to the studied trend models.

**Table 13. Comparison of the predicted and actual values.**

Names of elements of compared models		Cycle No.		
		12	13	14
Actual marks $H_{\text{calc}}$ , m		0.8800	0.8816	0.8810
Predictive marks $H_{\text{predict}}$ , m	Linear model	0.8807	0.8796	0.8786
	Logarithmic model	0.8824	0.8820	0.8817
	Second degree polynomial	0.8845	0.8853	0.8865
	Third degree polynomial	0.8821	0.8809	0.8784
Prediction errors $\Delta$ , mm	Linear model	-0.7	+2.0	+2.4
	Logarithmic model	-2.4	-0.4	-0.7
	Second degree polynomial	-4.5	-3.7	-5.5
	Third degree polynomial	-2.1	+0.7	+2.6

Table 13 shows that the logarithmic trend model has the smallest forecast error.

After correcting the obtained predictive marks [30] according to STO SRO-S 6054290 00043-2015 of the deformation benchmark DM6, considering the actual state of marks according to the data of 12–14 cycles, new (corrected) trend equations were determined. Once again, the predicted value of the mark was extrapolated one cycle ahead for all the trend equations. The obtained predicted value of the mark, considered deformation mark, was compared with the actual mark for cycle 15. The logarithmic trend equation again showed the smallest predictive error.

Thus, the validity of the earlier conclusion that the logarithmic trend model obtained by the search method of nonlinear programming is the best form of the trend model for the example under consideration is confirmed.

With a view to comparing the research results with similar scientific publications devoted to the application of nonlinear programming methods for solving geodetic problems, it should be noted that, despite their large number and variability, few of them highlight the possibility of combinative convergence of Powell and DSC algorithms for solving nonlinear programming problems. Thus, a consistent application of the Powell and DSC methods, which is close in nature to this work, declared as a single "parabola method" and aimed at solving nonlinear optimization problems, is given in the scientific works of A.V. Zubova and N.N. Eliseeva [33, 34], Ch.N. Zheltko [29]. However, in this case, there is no possibility of direct comparison of research results due to the different nature of applied and scientific problems, the solution of which was aimed at a combination of the above search methods of nonlinear programming.

#### 4. Conclusion

The authors developed a method for constructing a trend equation for a predictive model using a search method of nonlinear programming based on geodetic data, including:

- determining the trend equation by the search method of nonlinear programming in the VBA-based program;
- assessing the accuracy of the coefficients of the trend equations by the search method of nonlinear programming;
- assessing the adequacy of trend models obtained by the search method.

This technique was used to calculate the construction of a predictive model based on geodetic data using a search method of nonlinear programming for various types of trend equations. The obtained calculation results were confirmed by calculations in *Microsoft Excel*, using the example of a linear trend model. In addition, the computation results were confirmed by the calculations performed in the *Statistica* program for all the four trend equations.

The use of the search method of nonlinear programming to assess the accuracy of determining the coefficients of the equations of trend models made it possible to obtain the inverse weight matrix of the vector of coefficients, the root-mean-square error of the unit of weight and the root-mean-square error of each determined coefficient of the trend equation.

Comparison of the predicted values of the marks of the deformation benchmark with the actual marks is carried out. The correctness of the choice of the trend equation in the form of a logarithmic model for the considered example has been confirmed.

## References

1. Bryn, M.J., Afonin, D.A., Bogomolova, N.N. Geodetic Monitoring of Deformation of Building Surrounding an Underground Construction. *Procedia Engineering*. 2017. 189. Pp. 386–392. DOI: 10.1016/J.PROENG.2017.05.061
2. Gura, D.A., Shevchenko, G.G., Gura, A.Y. Development research methodology elastic deformation total station. *Journal of Engineering and Applied Sciences*. 2016. 11 (13). Pp. 2885–2888. DOI: 10.3923/JEASCI.2016.2885.2888
3. Mustafin M.G., Kazantsev A.I., Valkov V.A. Monitoring of Deformation Processes in Buildings and Structures in Metropolises. *Procedia Engineering*. 2017. No. 189. Pp. 729–736.
4. Gubaidullina, R.A. Features of using the method of aerial topographic survey in deformation monitoring. Trends in the development of science and education. *SRC L-Journal*. 2018. 45. Pp. 386–391. (rus)
5. Li, D., Yan, C. Building deformation prediction based on ground surface settlements of metro-station deep excavation. *Advances in Civil Engineering*. 2018. 2018. DOI: 10.1155/2018/6050353
6. Shardakov, I., Barayakh, A., Yepin, V., Tsvetkov, R., Glot, I. Control of surface subsidence based on building deformation monitoring data. *MATEC Web of Conferences*. 2019. 265. Pp. 05026. DOI: 10.1051/MATECONF/201926505026
7. Huang, M.Q., Zhou, L., Qi, L.X., Huang, H.Y., Tang, M.Y., Shi, Y.J. Multi-factor building deformation analysis and prediction model based on multi-source data. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences - ISPRS Archives*. 2020. 42(3/W10). Pp. 193–198. DOI: 10.5194/ISPRS-ARCHIVES-XLII-3-W10-193-2020
8. Degtyarev, G.V., Dats'O, D.A. The seasonal regulation basin dam basis deformation forecast. *IOP Conference Series: Materials Science and Engineering*. 2019. 698(2). DOI: 10.1088/1757-899X/698/2/022013
9. Bazilevskiy, M. Analytical dependences for some adequacy criteria of deming regression model. *Proceedings of Irkutsk State Technical University*. 2016. 117 (10). Pp. 81–89. DOI: 10.21285/1814-3520-2016-10-81-89
10. Makarova, I.L., Ignatenko, A.M. Determination coefficient for nonlinear regression. *Yevraziyskoye Nauchnoye Obyedineniye*. 2018. 3 (37). Pp. 161–163. (rus)
11. Skvortsov, V.V. Study methods of the tendency of the temporary rows in economic researches. *Social-Economic Phenomena and Processes*. 2008. 2(10). Pp. 75–78. (rus)
12. Espinosa-Garza, G., Loera-Hernández, I. Proposed model to improve the forecast of the planned value in the estimation of the final cost of the construction projects. *Procedia Manufacturing*. 2017. 13. Pp. 1011–1018. DOI: 10.1016/J.PROMFG.2017.09.103
13. Huang, Z., Tu, Y., Meng, S., Bagge, N., Nilimaa, J., Blanksvärd, T. Validation of a numerical method for predicting shear deformation of reinforced concrete beams. *Engineering Structures*. 2019. 197. DOI: 10.1016/J.ENGSTRUCT.2019.109367
14. Gulyaev YU.P. Prognozirovaniye deformatsiy sooruzhenij na osnove rezul'tatov geodezicheskikh nablyudenij [Forecasting deformations of structures based on the results of geodetic observations]. Novosibirsk: SGGa, 2008. 256 p.
15. Zajcev A.K., Marfenko S.V., Mihelev D.SH. i dr. Geodezicheskie metody issledovaniya deformatsiy sooruzhenij [Geodetic methods for studying deformations of structures]. Nedra, Moskva, 1991. 272 p.
16. Pozdysheva, O.N. Forecasting of deformations of building structures according to the results of geodetic observation. 2017. (No 02 (56). Part 2). Pp. 56–58. DOI: 10.23670/IRJ.2017.56.058. (rus)
17. Kharchenko, B.A. Analysis and prediction of the buildings deformations using the geodetic data are based on the program product the Mathcad-15. *Interexpo Geo-Sibir*. 2015. 1 (1). Pp. 21-27 (rus)
18. Khoroshilov, V.S., Kobeleva, N.N., Sycheva, N.V. Mathematical modeling of the high-rise buildings deformation development process in Moscow (Voststania square). *IOP Conference Series: Materials Science and Engineering*. 2019. 698 (4). Pp. 044004. DOI: 10.1088/1757-899X/698/4/044004.
19. Zhang, B., Qiu, L., Zhou, Z. Prediction Analysis for Building Deformation Based on Multiple Linear Regression Model. *IOP Conference Series: Earth and Environmental Science*. 2020. 455(1). DOI:10.1088/1755-1315/455/1/012047.
20. Bronshtein, V.I., Groshev, M.E. Numerical modeling of the stress-strain state of high dams. *Hydrotechnical construction*. 2002. No. 6. Pp. 2–11. (rus)
21. Salazar, F., Morán, R., Toledo, M.Á., Oñate, E. Data-Based Models for the Prediction of Dam Behaviour: A Review and Some Methodological Considerations. *Archives of Computational Methods in Engineering*. 2015. 24(1). Pp. 1–21. DOI: 10.1007/S11831-015-9157-9.
22. Mazurov B.T., Pankrushin V.K., Seredovich V.A. Matematicheskoe modelirovaniye i identifikatsiya napryazhenno-deformirovannogo sostoyaniya geodinamicheskikh sistem v aspekte prognoza prirodnykh i tekhnogennykh katastrof [Mathematical modeling and identification of the stress-strain state of geodynamic systems in the aspect of predicting natural and man-made disasters]. *Vestnik SGGa*. 2004. No. 9. Pp. 30–35. (rus)
23. Lebega, O., Vytvitsky, Y. Use of correlation-regression analysis for estimation of prospects of natural gas extraction of shale rocks. *EUREKA: Social and Humanities*. 2017. 4 (4). Pp. 37–43. DOI: 10.21303/2504-5571.2017.00383.
24. Seredovich, V.A., Ekhigiator-Irughe, R., Ekhigiator, O.M., Oriakhi, H. Prognoz deformatsiy s ispol'zovaniem funktsiy pokazatel'nogo mnogochlena [Deformation prediction using exponential polynomial functions]. *Interexpo Geo-Sibir*. 2012. VIII Mezhdunar. nauch. kongr.: Mezhdunar. nauch. konf. «Geodeziya, geoinformatika, kartografiya, markshejderiya»: sb. materialov v 3 t. (Novosibirsk, 10-20 aprelya 2012 g.). -Novosibirsk: SGGa, 2012. T. 1. Pp. 148-155.
25. Degtyarev, V.G., Khodakovskiy, V.A. Probabilistic and deterministic methods to find solutions of nonlinear equations and systems. *System analysis and analytics*. 2017. 1 (2). Pp. 4-12 (rus)
26. David M. Himmelblau Applied nonlinear programming. Mir, Moscow, 1975. 536 p. (rus)
27. Shevchenko, G.G. Using search methods for leveling and assessing the accuracy of elementary geodetic constructions. *Geodezia i Kartografiya*. 2019. 952 (10). Pp. 10–20. DOI: 10.22389/0016-7126-2019-952-10-10-20
28. Shevchenko, G.G., Bryn, M.J., Afonin, D.A., Gura, D.A. Experimental researches in defining deformations by free station method and results processing by search method. *Lecture Notes in Civil Engineering*. 2020. 50. Pp. 163–175. DOI: 10.1007/978-981-15-0454-9\_17
29. Zheltko, Ch.N., Shevchenko, G.G., Gura, D.A., Pastukhov, M.A. Universal'naya programma opredeleniya trekhmernykh koordinat tochek cherez obrabotku izmerenij gorizontalnykh, vertikal'nykh uglov i rasstoyanij poiskovym sposobom [Universal program for determining three-dimensional coordinates of points through the processing of measurements of horizontal, vertical angles and distances by the search method]. Certificate of state registration of a computer program: copyright holder: Kuban State Technological University. Reg. No. RU 2015617205 dated 03.07.15. Rospatent, Moscow, 2015. (rus)

30. Sadovnikova, N.A., Shmojllova, R.A. Analiz vremennyh ryadov i prognozirovanie [Time series analysis and forecasting]. Universitet «Sinergiya», Moscow, 2016. 152 p. (rus)
31. Krasina, I.B., Danovich, L.M. Statistical methods for processing of experimental data. Publishing house of the Kuban State Technological University, Krasnodar, 2015. 244 p. (rus)
32. Tatarenko, S.I. (comp.) Metody i modeli analiza vremennyh ryadov [Methods and models of time series analysis]. Publishing house of Tambov State Technical University, Tambov, 2008. 32 p. (rus)
33. Zubov, A.V., Eliseeva, N.N. Software suite for determining the tilts of tower-type structures according to terrestrial laser scanning. Geodezia i Kartografiya. 2020. 961 (7). Pp. 2–7. DOI: 10.22389/0016-7126-2020-961-7-2-7
34. Eliseeva, N.N., Zubov, A.V. The application of search methods for solving optimization problems. Topical Issues of Rational Use of Natural Resources 2019. 2019. 1. Pp. 346–352. DOI: 10.1201/9781003014577-43.

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