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Elastic local buckling of trapezoidal plates under linear stress gradients

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Abstract. Steel plates play a pivotal role in the construction of different types of structures used in civil engineering. According to Eurocode 3, plated structures may be designed using three different approaches: the effective width method, the reduced stress method, and the finite element analysis. For the particular case of elements under stress gradients, the effective width method utilises the local buckling coefficient of a plate to calculate the effective cross-sectional area for structural elements. Since the effective width method was developed for uniform web and flange panels, Eurocode 3 and most design codes have no specific provisions for the particular case of non-rectangular panels, stating that they may conservatively be treated as rectangular panels with larger width. With the final objective of improving design rules for tapered members, this paper presented an extensive numerical analysis to evaluate the elastic local buckling behaviour of trapezoidal plates with simply supported end conditions under stress gradients. The study identifies the relative importance of several parameters that influence the local buckling coefficient, such as the tapering ratio of the panel, normalized plate length, and stress ratio. Numerical results are used to propose approximate closed-form expressions that can be used to compute the local buckling coefficient for trapezoidal plates in a direct way. The results show that the proposed formula offers a significant improvement over current Eurocode 3 and most design codes.

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1. *Introduction*

Non-uniform members, including trapezoidal steel plates, are widely used in engineering areas such as structural, mechanical, and aeronautical engineering, as shown in Fig. (1). Lee and Morrell [1] affirm that the use of tapered members saved an amount of material compared with rolled plate, and it was first proposed for economic reasons by Amirikian [2]. As it is well known, Eurocode EN-3-1-5 [3] provides three general procedures that can be used when dealing with plate buckling. These procedures are the effective width method, the reduced stress method and finite element analysis. For the case of elements under stress gradients, a reduction factor is utilised for the compressed area in order to calculate the effective width to determine the compression stress in the cross section. Moreover, this reduction factor is a function of the plate aspect ratio and the local buckling coefficient. However, Eurocode 3 only provides a single provision for tapered plates; it suggests that general design rules may be applied by assuming that the panel is rectangular with the maximum width. This means that there are no specific provisions or recommendations for the design of tapered plates in Eurocode 3 beyond this general guidance. Hence, the importance of studying elastic local buckling of tapered plates becomes apparent, which relies on several factors such as plate geometry and boundary conditions to compute the effective width of a slender cross section.

Figure 1. Trapezoidal plates in industrial buildings [4].

Based on the previous researches, the analysis of plate buckling is widely studied in various fields of engineering, such as civil and structural engineering, mechanical engineering, aerospace engineering, marine engineering, etc. It is a vital topic that finds usage in a wide range of structural designs, and other engineering applications , particularly when a lightweight design is the main objective [5]. Plate buckling occurs when a compressive load causes a sudden deflection of the plate, which leads to compressive stress exceeding a critical limit, resulting in the failure of the structure. The failure mode caused by plate buckling is often accompanied by large deflections and sudden structural failure [6, 7]. The origin and analysis of plate buckling are difficult to interpret due to the governing equations being high-order partial differential equations. This makes it challenging to accurately predict and prevent the failure of structures due to plate buckling [8, 9].

On the other hand, the critical local buckling of plates has been extensively studied using a combination of experimental, numerical, and analytical methods in many important investigations. These methods have contributed to identifying the factors affecting buckling failure and developing solutions to prevent it, considering various plate geometries, restraint conditions, loading scenarios, and material properties. A pioneering theoretical study on the buckling of tapered plates under uniform compressive loading was proposed by Pope [10]. Šapalas [11] studied the local tapered web stability under pure bending moment using a theoretical and finite element analysis utilising a COSMOS FEM code. Additionally, he conducted a thorough simulation using a large domain of the second-moment area ratio to calculate a critical load multiplier and investigate the effects of relative slenderness, steel grade, and moment of inertia of beam-ends on the local stability of tapered beams. On the experimental side, one of the initial references to the plate buckling effect on the failure of tapered members was presented by Prawel et al. [12]. Ibrahim et al.[13] conducted an experimental program utilizing three specimens to investigate the axial compressive strength of a prismatic, unstiffened, slender tapered steel web.

Numerical methods have been used with profusion in the past to determine buckling loads of non-rectangular plates, particularly for skew plates [14–16]. Saadatpour et al. [17, 18] make use of Galerkin and Rayleigh-Ritz methods for the analysis of arbitrary quadrilateral plates with arbitrary boundary conditions. Differential quadrature methodology for stability analysis of straight-sided quadrilateral plates has also been employed by Karami and Malekzadeh [19], Civalek [20], and Wang et al. [21]. Eid [22] presented the analysis of a thin tapered plate using the finite difference method. Moreover, he proposed a numerical expression of the thin plates bending under a randomly distributed lateral load. Lkhenazen et al.[23] presented a buckling analysis of an isotropic plate that was subjected to in-plane patch loading. Diez et al. [4] developed a numerical analysis of trapezoidal plates subjected to uniform compression with four different boundary ends to Propose a closed formula to calculate the local buckling coefficient for trapezoidal plates. Abu-Hamd [24] developed an approximate empirical formula for tapered plate girders. This formula is based on the numerical results obtained from the FEM of a steel tapered web subjected to shear and moment. Kucukler et al.[25] suggested a simplified stiffness reduction method for the in-plane analysis of the tapered plates by dividing the web member into prismatic elements. Ziemian et al. [26] proposed a numerical technique to investigate lateral buckling of tapered beams, considering the effects of initial stress and load eccentricity. Moreover, Post-local buckling of skew and trapezoidal plates has been studied by Azhari and coworkers [27–29] and by Upadhyay and Shukla [30].

In recent years, the investigation in the field of composite plates has also provided significant results on the buckling behaviour of trapezoidal plates [31–33]. Finally, a new and interesting field of research has recently been proposed by Jing and coworkers [34] with their work on closed-form expressions to determine buckling loads of orthotropic plates.

In the literature, a few numerical results for elastic buckling of trapezoidal plates under stress gradients are available. Since Eurocode 3 directly relates plate ultimate strength with buckling coefficient, the main objective of this paper is to provide an extensive numerical analysis to evaluate the elastic local buckling behaviour of trapezoidal plates with simply supported end conditions under stress gradients. The study identifies the relative importance of several parameters that influence the local buckling coefficient, such as the tapering ratio of the panel, normalized plate length, and stress ratio. Numerical results are used to propose approximate closed-form expressions that can be used to compute the local buckling coefficient for trapezoidal plates in a direct way in order to compute the effective width of a slender cross section. The outcomes show that the Eurocode 3 formulas are more conservative in calculating the ultimate strength of slender trapezoidal plates.

2. Methods

2.1. Problem statement

A trapezoidal plate in a Cartesian coordinate system is given in Fig. 2. The plate has length a, a constant width ˝h˝ at the larger side, and a variable width ˝h1˝ at the smaller side with thickness "t". The plate is subjected to linearly varying in-plane loading in the longitudinal direction, and all its edges are simply supported in the out-of-plane direction. In other words, there is no lateral edge displacement perpendicular to the plate plane on all four edges.

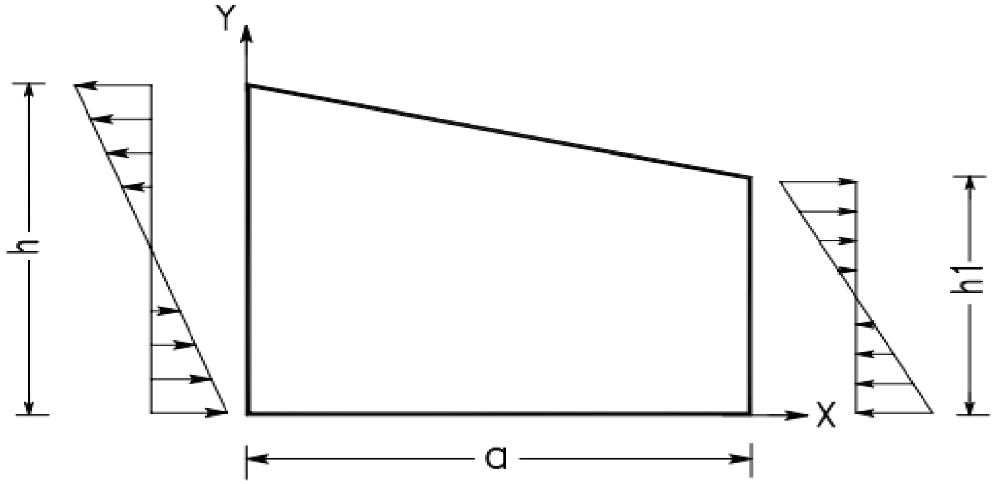

Figure 2. Geometry of a trapezoidal plate in a Cartesian coordinate system.

In the analysis of a trapezoidal plate, a linearly varying stress is applied to the two opposite simply supported edges at x = 0 and x = a, respectively. The loading conditions are as follows: case (1) has a uniform compression load with ψ = 1, case (2) has a trapezoidal load with ψ = 2/3, case (3) has a trapezoidal load with ψ = 1/3, case (4) has a triangular load with ψ = 0, case (5) has a unequal reverse triangular load with ψ = –1/3 ,case (6) has a unequal reverse triangular load with ψ = –2/3, and case (7) has a pure bending load with ψ = –1. For all compression and bending cases, nodal force is applied and divided according to different ψ ratios. Where ψ is the stress ratio (stress gradients) between minimum and maximum compressive stresses $\left(\psi = \sigma_2/\sigma_1\right)$ as shown in Fig. 3.

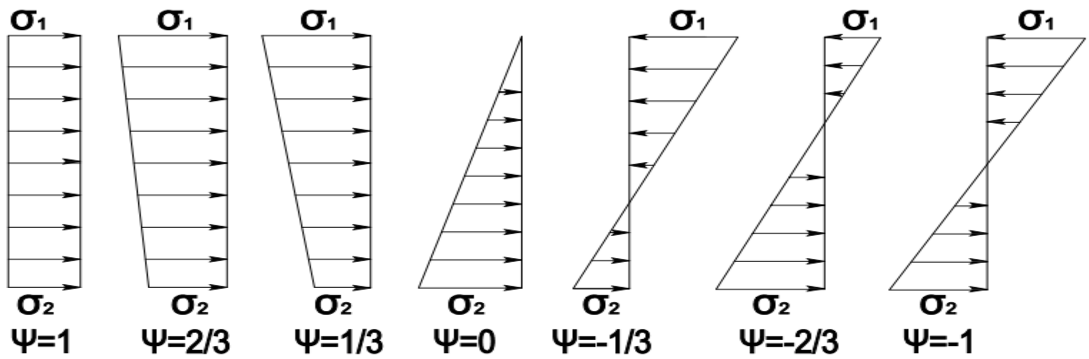

Figure 3. Example of in-plane loading conditions at x = 0 and x = a.

2.2. Finite element analysis procedure

For all study cases, the following steps were followed to develop predictive models:

1. Choosing the inputs that may affect the critical buckling coefficient;
2. The FE analysis using the ANSYS software [35] is used to execute an eigenvalue analysis to estimate the critical buckling load under normal and bending stresses. The program outputs are verified with well-known theoretical values;
3. The elastic buckling coefficient is determined for each by using Timoshenko's formula [36];
4. Construct the relations between the input variables and the predicted local buckling coefficient;
5. Conducting a regression analysis to generate predictive formulas for design purposes.

2.2.1 Influencing parameters

Based on the literature reviews [4, 37, 38], the fundamental parameters governing the predicted buckling coefficient are identified as the tapering ratio $R = h/h_1$, the normalized plate length ratio $(\alpha = a/h)$ the stress ratio between min. and max. compressive stresses $(\psi = \sigma_2/\sigma_1)$ and boundary conditions according to Mirambell et al. [39], the influence of the web depth-to-thickness ratio h/t_w is not significant, so the influence of this geometric parameter is ignored in this study. The parameter ranges and increments are shown in Table 1. The tapering ratio R ranges from 1 to 8, the normalized plate length α ranges from 0.25 to 8, and the stress gradients $\psi = \sigma_2/\sigma_1$ equal 1,2/3, 1/3,0, –1/3, –2/3, and –1 for compression and bending cases. The steel is modelled as a linearly elastic-perfectly plastic material with a Poisson ratio $\nu = 0.30$ and a modulus of elasticity E = 200 GPa.

Table 1. Different geometric parameter ranges and their increments.

Studied Parameter	Used parameter values for trapezoidal plate Compression and bending cases
Tapering ratio $R = h/h_1$	1, 1.25,1.5,1.75,2,2.25,2.5,2.75,3,4,5,6,8
Normalized plate length $\alpha = a/h$	0.25, 0.30,0.35,0.4,0.45,0.5, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80,0.85,0.90,0.95 1.00, 1.05, 1.10,1.15,1.20, 1.25,1.30, 1.35,1.40,1.45, 1.50,1.55,1.60,1.65,1.70 1.75,1.80,1.85,1.90,1.95,2.00,2.25,2.50,2.75,3.00,3.25,3.50,3.75,4,6,7,8
Stress gradients $\psi = \sigma_2/\sigma_1$	1,2/3,1/3,0, –1/3, –2/3, and –1

2.2.2 Linear buckling analysis

Finite element analysis is performed using ANSYS engineering simulation software [35] with a shell element to calculate the critical buckling load. A four-node shell element (SHELL181), as shown in Fig. 4, is employed to model the tapered plate, which has six degrees of freedom at each node: translations in the x, y, and z directions and rotations about the x, y, and z-axes. In addition, SHELL181 is suitable for modelling thin to moderately thick shell structures, which enables explicit simulation of various buckling deformations. Buckling loads are obtained from eigenvalue analysis. Eigenvalue buckling analysis is also known as linear buckling analysis, where the buckling load can be estimated by using the next equation [40].

$$\left(\left[\mathrm{K_o}\right] + \lambda\left[\mathrm{K_\sigma}\right]\right)\left\{\mathrm{U}\right\} = 0, \tag{1}$$

where $\mathrm{K_o}$ and $\mathrm{K_\sigma}$ is the linear stiffness matrix and the geometric stiffness matrix, respectively; λ is the load scaling factor; $\{\mathrm{U}\}$ is the lateral displacement vector. From Eq.1, it is clear that the structure's linear stability problem is the eigenvalue problem. By solving the eigenvalue and eigenvector problems, the critical load and buckling mode shape can be determined.

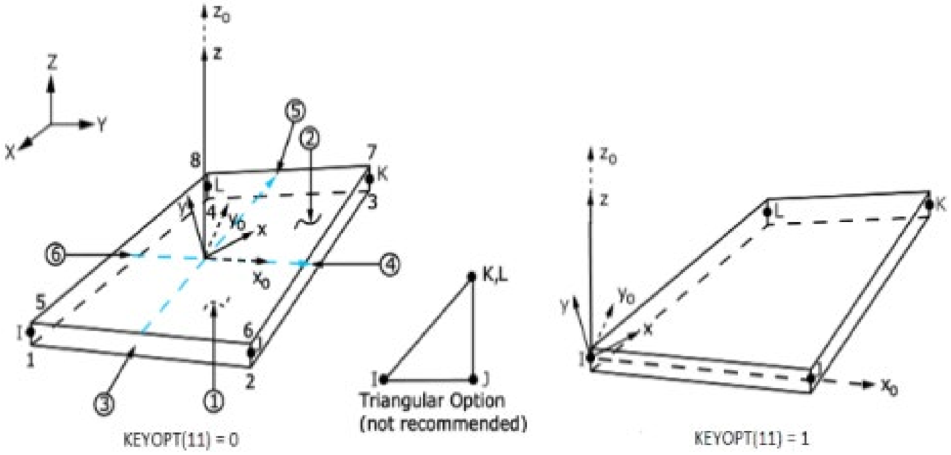

Figure 4. SHELL181 Geometry [41].

2.2.3 Validation of finite element model

To guarantee the finite element model's accuracy, a convergence test on mesh size was carried out employing a reference to the exact theoretical values of the local buckling coefficient for simply supported rectangular plates (R = 1) [38]. The mesh size is equal to 25×25 mm for all loading cases. Table 2 provides the percentage error between the FEM and the exact theoretical values of the local buckling coefficient. The normalized plate length ranged between 1 and 5. The last column of the table presents the error percentage, which varies from 0.10 % to 6.14 %. The comparison shows that the proposed boundary condition was well defined, and the results from FEM are reasonable.

Table 2. Numerical and theoretical results of the critical buckling coefficient for tapered web plates with a tapering ratio (R =1.00).

Loading Conditions	Normalized plate length $\alpha = a/h$	Theoretical results Pekoz [38]	FEM results	Error %
Case (1) (ψ = 1)	1	4.00	3.91	2.25
	2	4.00	3.947	1.325
	3	4.00	3.96	1.00
	4	4.00	3.967	0.825
	5	4.00	3.97	0.750
Case (2) (ψ = 1/3)	1	5.925	5.88	0.759
	2	5.925	5.90	0.422
	3	5.925	5.91	0.253
	4	5.925	5.916	0.152
	5	5.925	5.919	0.10
Case (3) (ψ = 0)	1	8.00	7.67	4.125
	2	8.00	7.70	3.75
	3	8.00	7.71	3.625
	4	8.00	7.723	3.465
	5	8.00	7.726	3.425
Case (4) (ψ = –1/3)	1	11.40	10.710	6.00
	2	11.40	10.745	5.70
	3	11.40	10.758	5.63
	4	11.40	10.760	5.63
	5	11.40	10.758	5.63
Case (5) (ψ = –2/3)	1	16.59	16.00	3.55
	2	16.59	15.88	4.28
	3	16.59	15.63	5.78
	4	16.59	15.57	6.14
	5	16.59	15.57	6.14

Loading Conditions	Normalized plate length $\alpha = a/h$	Theoretical results Pekoz [38]	FEM results	Error %
Case (6) (ψ = –1)	1	24.00	25.20	5.00
	2	24.00	23.87	0.54
	3	24.00	23.92	0.33
	4	24.00	23.82	0.75
	5	24.00	23.83	0.70

3. Results and Discussion

By using the validated FE model, the parametric study, which involved more than 4500 FE models, was carried out for compression and bending loading cases to investigate the influence of different parameters on the local buckling coefficient of trapezoidal plates [26]. The lowest deformed mode shapes of a trapezoidal plate for all compression and bending cases with simply supported boundary conditions are shown in Fig. 5.

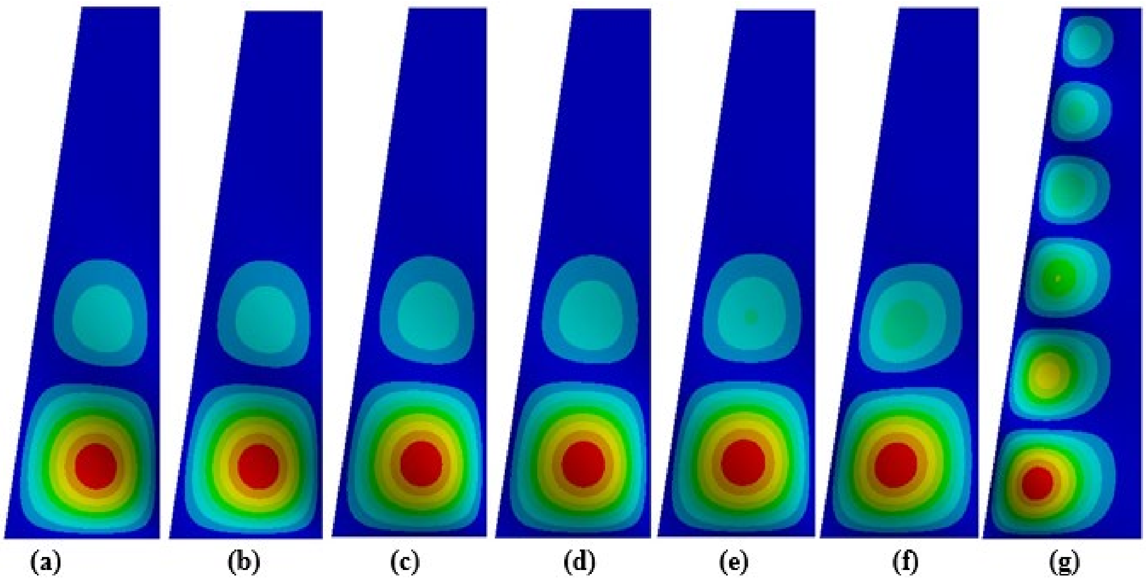

Figure 5. The lowest buckling mode shapes of trapezoidal plates for different loading cases: a) case (1) (ψ = 1) b) case (2) (ψ = 2/3) c) case (3) (ψ = 1/3) d) case (4) (ψ = Zero) e) case (5) (ψ = –1/3) f) case (6) (ψ = –2/3) g) case (7) (ψ = –1) respectively.

Fig. 6 displays the relationship between the plate buckling coefficient and normalized plate length (α) for several tapering ratios (R) for different loading cases. It can be observed when the normalized plate length (α) is more than 1.00; the K values reduce with increasing (α) and R values. For α less than 1.00, buckling behaviour is exactly the opposite, where k values increase with decrease α and decrease with increase R because the buckling waves are shorter when α values less than 1.00 compared with values more than 1.00. Moreover, for all R values, K declines with increases α up to 5.00, at which point the rate of decrease sharply declines. Typically, the applied stress along the loaded edges of plates ranges from uniform compression case to pure bending moment case. The uniform compression case is considered the most critical loading, and it has the lowest elastic local buckling stress. It can be noted that if the condition $\sigma_1 > 0$ is not satisfied, local buckling will not happen because the plate is subjected to only tension stress [42].

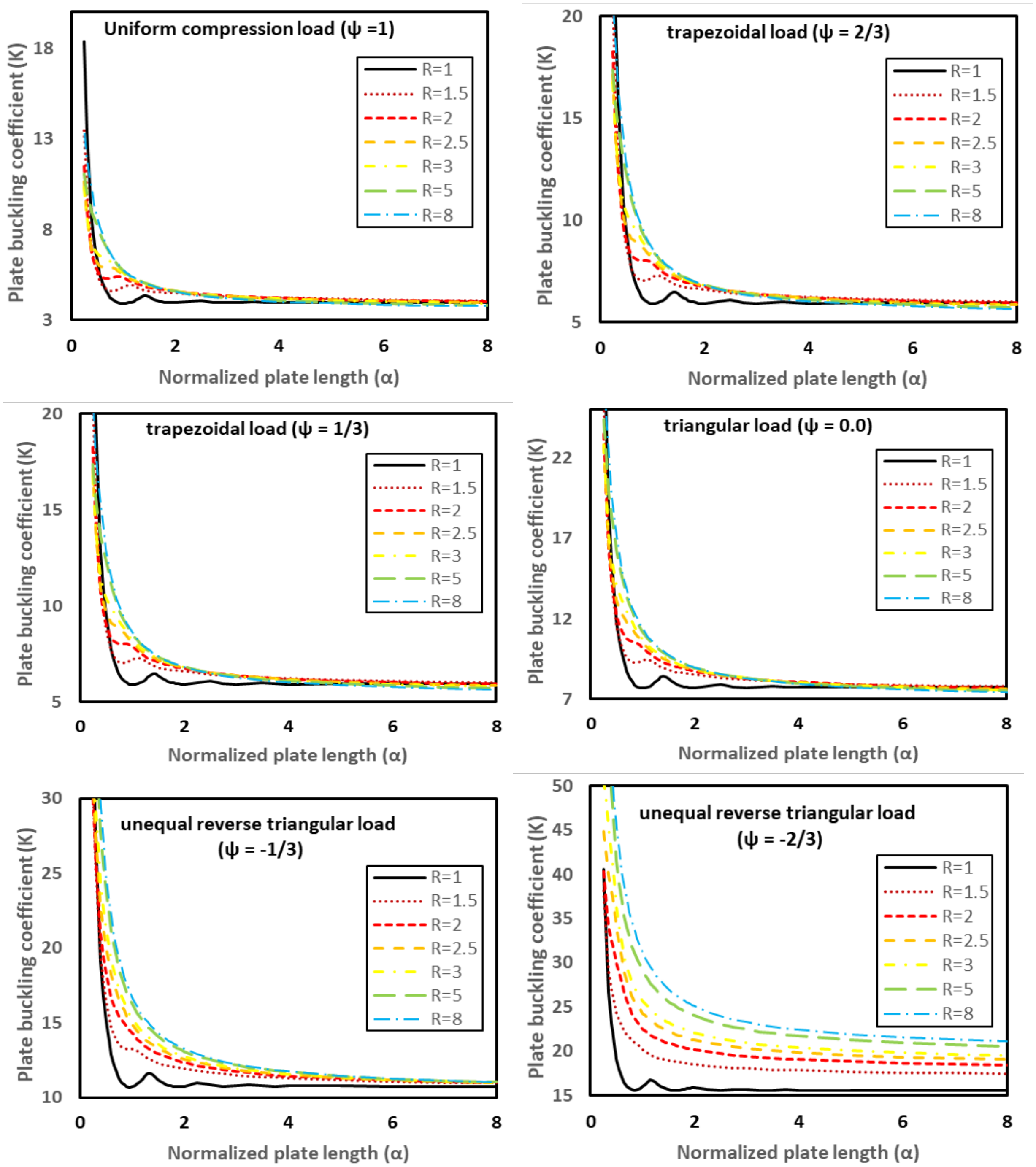

Figure 6. Plate buckling coefficient against normalized plate length (α) for several tapering ratios (R) for case (1) (ψ = 1), case (2) (ψ = 2/3), case (3) (ψ = 1/3), case (4) (ψ = 0), case (5) (ψ = –1/3),
and case (6) (ψ = –2/3) respectively.

Fig. 7 presents the relationship between the plate buckling coefficient and normalized plate length (α) for several tapering ratios (R) for the pure bending load (ψ = –1). It can be observed that when the normalized plate length (α) is greater than 2.00, the predicated plate buckling coefficient values for all tapering ratios (R) tend to reach 23.90 for simply supported edges. For the range of α < 2.00 (hatched in Fig. 7), the critical buckling coefficient k values are noticeably lower than expected, especially for higher tapering ratios. Because the interaction with the induced shear stresses developed by additional shear force results from the vertical component of the flange force [43], so, it can be ignoring these deviations because they aren't in the practical application domain of these tapered plates.

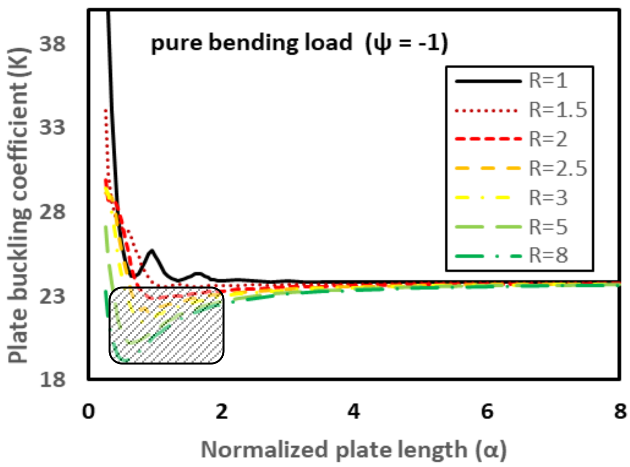

Figure 7. Plate buckling coefficient of pure bending load (ψ = –1) against normalized plate length (α) for several tapering ratios (R).

3.1. Proposed formulas for the elastic local buckling coefficients of trapezoidal plates under stress gradients

Regression analysis is employed for the output results to propose a prediction formula to calculate the critical buckling coefficient of trapezoidal plate subjected to compression and bending cases. Regression analysis with MATLAB is performed to estimate the relation between the output (K) and inputs, including stress ratio (ψ), normalized length (α), and tapering ratios (R). Moreover, regression analysis can measure the validity of predicted values with the actual dataset using many tools, such as the coefficient of correlation (R), mean square error (MSE), and standard deviation (SD).

Eqs. (2) to (4) show the elastic local buckling coefficient formulas for trapezoidal plates under stress gradients.

For case (1) [uniform compression load (ψ = 1.00)].

$$K_{UC} = 6.31 + \frac{0.87}{\alpha^{1.53}} - \frac{2.81}{R^{0.28}} - \frac{0.29}{\alpha R}, \tag{2}$$

where K_{UC} is the elastic local buckling coefficient of a uniform compression load with ψ = 1.00.

For case (2) to case (6) [compression and bending cases (1.00 < ψ < –1.00)].

$$K_{CB} = 1 + 1.82K_{UC} - 5.75\psi - 0.82K_{UC}\psi + 4.75\psi^2, \tag{3}$$

where K_{CB} is the elastic local buckling coefficient of a compression and bending cases with 1.00 < ψ < –1.00.

For case (7) [pure bending load (ψ = –1.00)]

$$K_{PB} = 23.90, \tag{4}$$

where, K_{PB} is the elastic local buckling coefficient of a pure bending load with ψ = –1.00.

3.2. Validation of Proposed Formulas

The proposed formulas for the elastic buckling coefficient of trapezoidal plates are verified with FE results. Fig. 8 compares the predicted local buckling coefficient from the proposed formula K_{UC} with the actual local buckling coefficient from the FE modeling for a uniform compression load (ψ =1.00). The mean (μ) and standard deviation (σ) for the ratio $K_{FE}/K_{\mathrm{Prop.}}$ are 1.00 and 0.09, respectively, where the mean is the average of two or more numbers. The standard deviation is a statistical parameter that measures the dispersion of a set of numbers from its mean. The standard deviation is defined as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}{n-1}}, \tag{5}$$

where n is the total number of data set, y_i is data point and $\overline{y}$ is the average of the data set.

The coefficient of determination $R^2 = 0.988$ indicates a very strong fit since it is close to 1. In general, good predictions of the elastic buckling coefficient were obtained from the proposed formula.

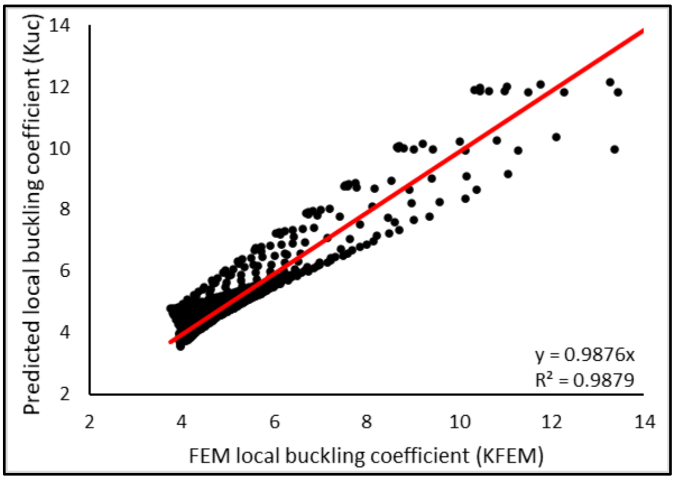

Figure 8. Validating the predicated local buckling coefficient from the proposed formula K_{UC} against the actual local buckling coefficient from the FE modelling for uniform compression load (ψ =1.00).

Fig. 9 presents the comparison between the predicated local buckling coefficient from the proposed formula K_{CB} and the actual local buckling coefficient from the FE modelling for compression and bending cases (1.00 < ψ < –1.00). The majority of predicted values for different subsets exhibit a favourable and robust ability to forecast, as they mostly fall within an error range of –20 % to +20 %, and predictions with an error exceeding 20% tend to fall in the underestimate region, indicating a safer outcome. The mean (μ) and standard deviation (σ) for the ratio $K_{FE} / K_{Prop.}$ are 1.04 and 0.145, respectively. The coefficient of determination $R^2 = 0.95$ indicates a very strong fit since it is close to 1. In general, good predictions of the elastic buckling coefficient were obtained from the proposed formula.

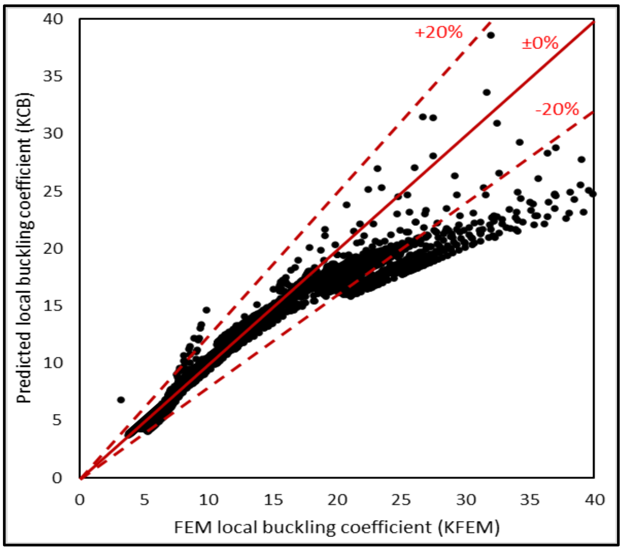

Figure 9. Validating the predicated local buckling coefficient from the proposed formula K_{CB} against the actual local buckling coefficient from the FE modelling for compression and bending cases (1.00 < ψ < –1.00).

3.3. Comparison with previous studies

The elastic local buckling coefficient obtained from the proposed formula for a uniform compression load case (Eq. (2)) was compared with the buckling coefficient predicted by Diez et al.[4]. Diez et al. provides the following formulas to estimate the elastic buckling coefficient of trapezoidal plates under a uniform compression load case. The closed-form expression does not give a value for buckling coefficient of trapezoidal plates as a function of R, α, and θ. It provides approximations to minimum and stationary values of buckling coefficient as a function of plate angle θ.

$$\bar{k}_{T_{min}} = 4\left(1+tan\,\theta\right); \tag{6}$$

$$\bar{k}_{T_{st}} = 4+7.5\,tan\,\theta, \tag{7}$$

where θ is plate angle, $\bar{k}_{T_{min}}$ is minimum value of buckling coefficient, $\bar{k}_{T_{st}}$ is stationary value of buckling coefficient.

Table 4 displays the comparison between the buckling coefficient obtained from the proposed formula (Eqs.2) and the buckling coefficient obtained from Diez formula for uniform compression load. For all analyses, plate width "h" is equal 1000 mm, plate thickness is equal 10 mm , stress gradients ψ = 1, nine values of plate tapering ratio R, and normalized plate length α range from (0.45 to 5). It can be seen that the Diez formula are conservative in calculating the buckling coefficient of trapezoidal plates under uniform compression. The mean value of the $K_{FE} / K_{Prop.}$ ratio is 1.02 with a corresponding coefficient of variation (COV) of 0.07 and the mean value of the K_{FE} / K_{Diez} ratio is 0.75 with a corresponding coefficient of variation (COV) of 0.17. In addition, it should be noted that the Diez formula is not applicable and has limitations to its use, being restricted within a certain range of conditions for normalized plate length α.

Table 4. Comparison of the buckling coefficient obtained from the proposed formula and the Diez formulas for uniform compression load case.

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
G 1	1.00	1.00	1.00	3.91	4.66	4.00	0.84	0.98
		1.00	1.05	3.92	4.58	4.00	0.86	0.98
		1.00	1.10	3.95	4.52	4.00	0.87	0.99
		1.00	1.15	3.99	4.45	4.00	0.90	1.00
		1.00	1.20	4.05	4.40	4.00	0.92	1.01
		1.00	1.25	4.12	4.35	4.00	0.95	1.03
		1.00	1.30	4.20	4.31	4.00	0.97	1.05
		1.00	1.35	4.29	4.26	4.00	1.00	1.07
		1.00	1.40	4.37	4.23	4.00	1.03	1.09
		1.00	1.45	4.35	4.19	4.00	1.04	1.09
		1.00	1.50	4.27	4.16	4.00	1.03	1.07
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		1.00	3.50	4.02	3.71	4.00	1.08	1.01
		1.00	3.75	3.98	3.69	4.00	1.08	1.00
		1.00	4.00	3.97	3.68	4.00	1.08	0.99

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
		1.00	5.00	3.97	3.63	4.00	1.09	0.99
		1.10	0.90	4.07	4.89	4.35	0.83	0.94
		1.10	0.95	4.06	4.79	4.35	0.85	0.93
		1.10	1.00	4.08	4.71	4.35	0.87	0.94
		1.10	1.05	4.11	4.63	4.35	0.89	0.95
		1.10	1.10	4.16	4.57	4.35	0.91	0.96
		1.10	1.15	4.23	4.51	4.35	0.94	0.97
		1.10	1.20	4.31	4.45	4.35	0.97	0.99
		1.10	1.25	4.39	4.40	4.35	1.00	1.01
		1.10	1.30	4.48	4.36	4.66	1.03	0.96
		1.10	1.35	4.52	4.32	4.66	1.05	0.97
		1.10	1.40	4.48	4.28	4.66	1.05	0.96
		1.10	1.45	4.41	4.25	4.66	1.04	0.95
		1.10	1.50	4.34	4.22	4.66	1.03	0.93
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		------	------	------	------	------	------	------
		1.10	3.50	4.11	3.78	4.66	1.09	0.88
		1.10	3.75	4.09	3.76	4.66	1.09	0.88
		1.10	4.00	4.09	3.74	4.66	1.09	0.88
		1.10	5.00	4.07	3.70	4.66	1.10	0.87
		1.20	0.80	4.26	5.17	4.71	0.82	0.90
		1.20	0.85	4.22	5.04	4.71	0.84	0.90
		1.20	0.90	4.21	4.93	4.71	0.85	0.89
		1.20	0.95	4.22	4.84	4.71	0.87	0.90
		1.20	1.00	4.26	4.75	4.71	0.90	0.91
		1.20	1.05	4.31	4.68	4.71	0.92	0.92
		1.20	1.10	4.39	4.61	4.71	0.95	0.93
		1.20	1.15	4.47	4.55	5.32	0.98	0.84
		1.20	1.20	4.55	4.50	5.32	1.01	0.86
		1.20	1.25	4.63	4.45	5.32	1.04	0.87
		1.20	1.30	4.64	4.41	5.32	1.05	0.87
		1.20	1.35	4.59	4.37	5.32	1.05	0.86
		1.20	1.40	4.52	4.33	5.32	1.04	0.85
		1.20	1.45	4.46	4.30	5.32	1.04	0.84
		1.20	1.50	4.41	4.27	5.32	1.03	0.83
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		1.20	3.50	4.17	3.84	5.32	1.09	0.78
		1.20	3.75	4.16	3.82	5.32	1.09	0.78
		1.20	4.00	4.15	3.80	5.32	1.09	0.78
		1.20	5.00	4.12	3.76	5.32	1.10	0.77
		1.37	0.70	4.56	5.54	5.07	0.82	0.90
		1.37	0.75	4.48	5.37	5.07	0.83	0.88
		1.37	0.80	4.44	5.23	5.07	0.85	0.88

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
		1.37	0.85	4.44	5.10	5.07	0.87	0.88
		1.37	0.90	4.47	4.99	5.07	0.89	0.88
		1.37	0.95	4.52	4.90	5.07	0.92	0.89
		1.37	1.00	4.58	4.82	5.07	0.95	0.90
		1.37	1.05	4.66	4.75	6.01	0.98	0.78
		1.37	1.10	4.74	4.68	6.01	1.01	0.79
		1.37	1.15	4.81	4.62	6.01	1.04	0.80
		1.37	1.20	4.83	4.57	6.01	1.06	0.80
		1.37	1.25	4.79	4.52	6.01	1.06	0.80
		1.37	1.30	4.73	4.48	6.01	1.05	0.79
		1.37	1.35	4.66	4.44	6.01	1.05	0.78
		1.37	1.40	4.61	4.41	6.01	1.05	0.77
		1.37	1.45	4.56	4.38	6.01	1.04	0.76
		1.37	1.50	4.52	4.35	6.01	1.04	0.75
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		1.37	3.50	4.24	3.93	6.01	1.08	0.70
		1.37	3.75	4.22	3.91	6.01	1.08	0.70
		1.37	4.00	4.20	3.89	6.01	1.08	0.70
		1.37	5.00	4.16	3.85	6.01	1.08	0.69
		1.57	0.65	4.83	5.80	5.46	0.83	0.89
		1.57	0.70	4.75	5.60	5.46	0.85	0.87
		1.57	0.75	4.71	5.43	5.46	0.87	0.86
		1.57	0.80	4.71	5.29	5.46	0.89	0.86
		1.57	0.85	4.75	5.17	5.46	0.92	0.87
		1.57	0.90	4.80	5.06	5.46	0.95	0.88
		1.57	0.95	4.87	4.97	6.73	0.98	0.72
		1.57	1.00	4.95	4.89	6.73	1.01	0.73
		1.57	1.05	5.00	4.82	6.73	1.04	0.74
		1.57	1.10	5.02	4.75	6.73	1.06	0.75
		1.57	1.15	4.99	4.70	6.73	1.06	0.74
		1.57	1.20	4.93	4.65	6.73	1.06	0.73
		1.57	1.25	4.87	4.60	6.73	1.06	0.72
		1.57	1.30	4.80	4.56	6.73	1.05	0.71
		1.57	1.35	4.75	4.52	6.73	1.05	0.71
		1.57	1.40	4.71	4.49	6.73	1.05	0.70
		1.57	1.45	4.67	4.45	6.73	1.05	0.69
		1.57	1.50	4.64	4.42	6.73	1.05	0.69
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		1.57	3.50	4.27	4.01	6.73	1.06	0.63
		1.57	3.75	4.25	4.00	6.73	1.06	0.63
		1.57	4.00	4.23	3.98	6.73	1.06	0.63
		1.57	5.00	4.17	3.94	6.73	1.06	0.62

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
		1.87	0.60	5.24	6.11	5.87	0.86	0.89
		1.87	0.65	5.14	5.87	5.87	0.87	0.88
		1.87	0.70	5.11	5.67	5.87	0.90	0.87
		1.87	0.75	5.12	5.51	5.87	0.93	0.87
		1.87	0.80	5.16	5.37	5.87	0.96	0.88
		1.87	0.85	5.22	5.25	7.50	0.99	0.70
		1.87	0.90	5.27	5.15	7.50	1.02	0.70
		1.87	0.95	5.30	5.06	7.50	1.05	0.71
		1.87	1.00	5.29	4.98	7.50	1.06	0.71
		1.87	1.05	5.24	4.91	7.50	1.07	0.70
		1.87	1.10	5.17	4.84	7.50	1.07	0.69
		1.87	1.15	5.10	4.79	7.50	1.06	0.68
		1.87	1.20	5.03	4.74	7.50	1.06	0.67
		1.87	1.25	4.97	4.69	7.50	1.06	0.66
		1.87	1.30	4.91	4.65	7.50	1.06	0.66
		1.87	1.35	4.87	4.62	7.50	1.05	0.65
		1.87	1.40	4.83	4.58	7.50	1.05	0.64
		1.87	1.45	4.80	4.55	7.50	1.05	0.64
		1.87	1.50	4.77	4.52	7.50	1.05	0.64
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		1.87	3.50	4.28	4.12	7.50	1.04	0.57
		1.87	3.75	4.25	4.11	7.50	1.04	0.57
		1.87	4.00	4.23	4.09	7.50	1.03	0.56
		1.87	5.00	4.16	4.06	7.50	1.02	0.55
		2.37	0.55	5.87	6.50	6.31	0.90	0.93
		2.37	0.60	5.79	6.21	6.31	0.93	0.92
		2.37	0.65	5.76	5.97	6.31	0.96	0.91
		2.37	0.70	5.77	5.78	6.31	1.00	0.91
		2.37	0.75	5.78	5.62	6.31	1.03	0.92
		2.37	0.80	5.78	5.48	8.33	1.06	0.69
		2.37	0.85	5.75	5.36	8.33	1.07	0.69
		2.37	0.90	5.67	5.26	8.33	1.08	0.68
		2.37	0.95	5.58	5.17	8.33	1.08	0.67
		2.37	1.00	5.48	5.10	8.33	1.08	0.66
		2.37	1.05	5.38	5.03	8.33	1.07	0.65
		2.37	1.10	5.30	4.97	8.33	1.07	0.64
		2.37	1.15	5.22	4.91	8.33	1.06	0.63
		2.37	1.20	5.15	4.86	8.33	1.06	0.62
		2.37	1.25	5.10	4.82	8.33	1.06	0.61
		2.37	1.30	5.04	4.78	8.33	1.06	0.61
		2.37	1.35	5.00	4.74	8.33	1.05	0.60
		2.37	1.40	4.96	4.71	8.33	1.05	0.59
		2.37	1.45	4.92	4.68	8.33	1.05	0.59

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
		2.37	1.50	4.88	4.65	8.33	1.05	0.59
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		2.37	3.50	4.26	4.27	8.33	1.00	0.51
		2.37	3.75	4.23	4.25	8.33	1.00	0.51
		2.37	4.00	4.20	4.24	8.33	0.99	0.50
		2.37	5.00	4.12	4.20	8.33	0.98	0.49
		3.33	0.35	7.98	8.89	6.80	0.90	1.17
		3.33	0.40	7.49	8.06	6.80	0.93	1.10
		3.33	0.45	7.20	7.45	6.80	0.97	1.06
		3.33	0.50	7.03	6.99	6.80	1.01	1.03
		3.33	0.55	6.91	6.63	6.80	1.04	1.02
		3.33	0.60	6.82	6.35	6.80	1.07	1.00
		3.33	0.65	6.70	6.12	6.80	1.10	0.99
		3.33	0.70	6.55	5.93	9.25	1.10	0.71
		3.33	0.75	6.38	5.77	9.25	1.11	0.69
		3.33	0.80	6.20	5.64	9.25	1.10	0.67
		3.33	0.85	6.04	5.52	9.25	1.09	0.65
		3.33	0.90	5.89	5.42	9.25	1.09	0.64
		3.33	0.95	5.76	5.34	9.25	1.08	0.62
		3.33	1.00	5.64	5.26	9.25	1.07	0.61
		3.33	1.05	5.53	5.19	9.25	1.07	0.60
		3.33	1.10	5.44	5.13	9.25	1.06	0.59
		3.33	1.15	5.36	5.08	9.25	1.05	0.58
		3.33	1.20	5.28	5.03	9.25	1.05	0.57
		3.33	1.25	5.21	4.99	9.25	1.04	0.56
		3.33	1.30	5.15	4.95	9.25	1.04	0.56
		3.33	1.35	5.09	4.92	9.25	1.04	0.55
		3.33	1.40	5.04	4.89	9.25	1.03	0.54
		3.33	1.45	4.99	4.86	9.25	1.03	0.54
		3.33	1.50	4.94	4.83	9.25	1.02	0.53
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		3.33	3.50	4.21	4.46	9.25	0.95	0.46
		3.33	3.75	4.18	4.44	9.25	0.94	0.45
		3.33	4.00	4.15	4.43	9.25	0.94	0.45
		3.33	5.00	4.05	4.40	9.25	0.92	0.44
		6.20	0.45	9.09	7.68	7.36	1.18	1.23
		6.20	0.50	8.53	7.23	7.36	1.18	1.16
		6.20	0.55	8.04	6.88	7.36	1.17	1.09
		6.20	0.60	7.61	6.60	7.36	1.15	1.03
		6.20	0.65	7.25	6.38	7.36	1.14	0.99
		6.20	0.70	6.94	6.19	7.36	1.12	0.94
		6.20	0.75	6.67	6.04	7.36	1.10	0.91

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{Diez}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{Diez}
		6.20	0.80	6.44	5.91	7.36	1.09	0.87
		6.20	0.85	6.24	5.79	7.36	1.08	0.85
		6.20	0.90	6.06	5.70	7.36	1.06	0.82
		6.20	0.95	5.91	5.61	7.36	1.05	0.80
		6.20	1.00	5.77	5.54	7.36	1.04	0.78
		6.20	1.05	5.65	5.48	7.36	1.03	0.77
		6.20	1.10	5.54	5.42	7.36	1.02	0.75
		6.20	1.15	5.44	5.37	7.36	1.01	0.74
		6.20	1.20	5.35	5.32	7.36	1.01	0.73
		6.20	1.25	5.27	5.28	7.36	1.00	0.72
		6.20	1.30	5.20	5.24	7.36	0.99	0.71
		6.20	1.35	5.13	5.21	7.36	0.98	0.70
		6.20	1.40	5.06	5.18	7.36	0.98	0.69
		6.20	1.45	5.01	5.15	7.36	0.97	0.68
		6.20	1.50	4.95	5.12	7.36	0.97	0.67
		------	------	------	------	------	------	------
		------	------	------	------	------	------	------
		6.20	3.50	4.12	4.77	7.36	0.87	0.56
		6.20	3.75	4.09	4.75	7.36	0.86	0.56
		6.20	4.00	4.05	4.74	7.36	0.85	0.55
		6.20	5.00	3.95	4.71	7.36	0.84	0.54
Mean							1.02	0.75
STD							0.07	0.17

3.4. Comparison with design rules

The elastic local buckling coefficient obtained from the proposed formulas was compared with the buckling coefficient predicted by the Eurocode [3]. The EC3 provides the following buckling coefficient formulas, as shown in Table 5 to calculate the ultimate strength of a slender plate of internal compression elements. Eurocode 3 only provides a single provision for tapered plates: it suggests that general design rules may be applied by assuming that the panel is rectangular with the maximum width.

Table 5. Buckling factor formulas for internal compression elements according to Eurocode 3.

Stress gradients ($\psi = \sigma_2 / \sigma_1$)	Buckling factor K_σ
ψ = 1.00	4.00
1.00 > ψ > 0.00	$8.2/(1.05+\psi)$
ψ = 0.00	7.81
0.00 > ψ > –1.00	$7.81-6.29\psi+9.78\psi^2$
ψ = –1.00	23.90

Table 6 displays the comparison between the buckling coefficient obtained from the proposed formulas (Eqs.2 to 4) and the buckling factor formulas from EC3 for internal compression elements. For all analyses, plate width "h" is equal 1000 mm, plate thickness is equal 10 mm , six values of stress gradients ψ (1;2/3;1/3;0;-2/3;-1/3), five values of plate tapering ratio R (1;2;3;5;8), and eight values of normalized plate length α (1;1.5;2;2.5;3;4;6;8). It can be seen that the Eurocode are conservative in calculating the buckling coefficient of trapezoidal plates under stress gradients to calculate the ultimate strength of slender plates. The reason is that the EC3 does not take into consideration the normalized plate length and tapering ratio, which have a very significant effect on the buckling coefficient. The mean value of the $K_{FE} / K_{Prop.}$ ratio is 1.04 with a corresponding coefficient of variation (COV) of 0.13 and the mean value of the K_{FE} / K_{EC3} ratio is 1.14 with a corresponding coefficient of variation (COV) of 0.17. The buckling coefficient predicted by EC3 is generally conservative; except for the pure bending case, it was close to that of the FE model prediction.

Table 6. Comparison of the buckling coefficient obtained from the proposed formulas and the EC3 formulas for internal compression elements.

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
G1	1	1	1	3.91	4.66	4.00	0.84	0.98
			1.5	4.27	4.16	4.00	1.03	1.07
			2	3.95	3.95	4.00	1.00	0.99
			2.5	4.08	3.83	4.00	1.07	1.02
			3	3.96	3.76	4.00	1.05	0.99
			4	3.97	3.68	4.00	1.08	0.99
			6	3.97	3.60	4.00	1.10	0.99
			8	3.98	3.57	4.00	1.11	0.99
		2	1	5.36	5.01	4.00	1.07	1.34
			1.5	4.81	4.56	4.00	1.06	1.20
			2	4.59	4.37	4.00	1.05	1.15
			2.5	4.44	4.27	4.00	1.04	1.11
			3	4.35	4.21	4.00	1.03	1.09
			4	4.22	4.14	4.00	1.02	1.06
			6	4.10	4.08	4.00	1.01	1.02
			8	4.03	4.05	4.00	1.00	1.01
		3	1	5.60	5.21	4.00	1.07	1.40
			1.5	4.93	4.78	4.00	1.03	1.23
			2	4.62	4.59	4.00	1.00	1.15
			2.5	4.43	4.50	4.00	0.99	1.11
			3	4.31	4.44	4.00	0.97	1.08
			4	4.16	4.37	4.00	0.95	1.04
			6	4.01	4.32	4.00	0.93	1.00
			8	3.93	4.29	4.00	0.92	0.98
		5	1	5.75	5.45	4.00	1.05	1.44
			1.5	4.96	5.03	4.00	0.99	1.24
			2	4.59	4.85	4.00	0.95	1.15
			2.5	4.38	4.76	4.00	0.92	1.10
			3	4.25	4.70	4.00	0.90	1.06
			4	4.08	4.64	4.00	0.88	1.02
			6	3.92	4.59	4.00	0.85	0.98
			8	3.83	4.56	4.00	0.84	0.96
		8	1	5.79	5.65	4.00	1.03	1.45
			1.5	4.95	5.23	4.00	0.95	1.24
			2	4.56	5.06	4.00	0.90	1.14
			2.5	4.34	4.97	4.00	0.87	1.08

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
			3	4.20	4.91	4.00	0.85	1.05
			4	4.02	4.85	4.00	0.83	1.01
			6	3.85	4.80	4.00	0.80	0.96
			8	3.77	4.78	4.00	0.79	0.94
G2	0.67	1	1	4.75	4.26	4.77	1.12	1.00
			1.5	5.17	4.72	4.77	1.10	1.09
			2	4.77	4.30	4.77	1.11	1.00
			2.5	4.91	4.47	4.77	1.10	1.03
			3	4.77	4.32	4.77	1.11	1.00
			4	4.78	4.33	4.77	1.10	1.00
			6	4.78	4.34	4.77	1.10	1.00
			8	4.78	4.34	4.77	1.10	1.00
		2	1	6.52	6.10	4.77	1.07	1.37
			1.5	5.96	5.40	4.77	1.10	1.25
			2	5.74	5.12	4.77	1.12	1.20
			2.5	5.58	4.93	4.77	1.13	1.17
			3	5.48	4.81	4.77	1.14	1.15
			4	5.33	4.66	4.77	1.15	1.12
			6	5.18	4.49	4.77	1.15	1.09
			8	5.10	4.41	4.77	1.16	1.07
		3	1	6.91	6.40	4.77	1.08	1.45
			1.5	6.30	5.56	4.77	1.13	1.32
			2	5.99	5.16	4.77	1.16	1.26
			2.5	5.79	4.92	4.77	1.18	1.22
			3	5.65	4.77	4.77	1.19	1.19
			4	5.48	4.58	4.77	1.20	1.15
			6	5.29	4.39	4.77	1.20	1.11
			8	5.18	4.29	4.77	1.21	1.09
		5	1	7.23	6.59	4.77	1.10	1.52
			1.5	6.55	5.59	4.77	1.17	1.37
			2	6.18	5.12	4.77	1.21	1.30
			2.5	5.95	4.86	4.77	1.23	1.25
			3	5.79	4.69	4.77	1.24	1.21
			4	5.58	4.47	4.77	1.25	1.17
			6	5.36	4.26	4.77	1.26	1.12
			8	5.25	4.16	4.77	1.26	1.10
		8	1	7.41	6.65	4.77	1.11	1.55

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
			1.5	6.68	5.57	4.77	1.20	1.40
			2	6.29	5.08	4.77	1.24	1.32
			2.5	6.04	4.80	4.77	1.26	1.27
			3	5.87	4.62	4.77	1.27	1.23
			4	5.64	4.40	4.77	1.28	1.18
			6	5.41	4.18	4.77	1.29	1.13
			8	5.28	4.07	4.77	1.30	1.11
G3	0.33	1	1	5.88	5.66	5.94	1.04	0.99
			1.5	6.38	6.22	5.94	1.03	1.07
			2	5.90	5.72	5.94	1.03	0.99
			2.5	6.07	5.92	5.94	1.03	1.02
			3	5.91	5.74	5.94	1.03	0.99
			4	5.92	5.75	5.94	1.03	1.00
			6	5.92	5.76	5.94	1.03	1.00
			8	5.92	5.76	5.94	1.03	1.00
		2	1	7.87	7.91	5.94	1.00	1.32
			1.5	7.06	7.05	5.94	1.00	1.19
			2	6.73	6.71	5.94	1.00	1.13
			2.5	6.52	6.48	5.94	1.00	1.10
			3	6.38	6.34	5.94	1.01	1.07
			4	6.20	6.15	5.94	1.01	1.04
			6	6.02	5.95	5.94	1.01	1.01
			8	5.92	5.85	5.94	1.01	1.00
		3	1	8.23	8.27	5.94	1.00	1.39
			1.5	7.25	7.24	5.94	1.00	1.22
			2	6.79	6.75	5.94	1.01	1.14
			2.5	6.53	6.47	5.94	1.01	1.10
			3	6.35	6.28	5.94	1.01	1.07
			4	6.13	6.05	5.94	1.01	1.03
			6	5.92	5.82	5.94	1.02	1.00
			8	5.80	5.70	5.94	1.02	0.98
		5	1	8.52	8.50	5.94	1.00	1.43
			1.5	7.36	7.28	5.94	1.01	1.24
			2	6.82	6.71	5.94	1.02	1.15
			2.5	6.51	6.39	5.94	1.02	1.10
			3	6.31	6.18	5.94	1.02	1.06
			4	6.07	5.93	5.94	1.02	1.02

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
			6	5.82	5.67	5.94	1.03	0.98
			8	5.70	5.54	5.94	1.03	0.96
		8	1	8.63	8.57	5.94	1.01	1.45
			1.5	7.37	7.26	5.94	1.02	1.24
			2	6.80	6.66	5.94	1.02	1.14
			2.5	6.47	6.32	5.94	1.02	1.09
			3	6.26	6.11	5.94	1.03	1.05
			4	6.01	5.84	5.94	1.03	1.01
			6	5.75	5.57	5.94	1.03	0.97
			8	5.63	5.44	5.94	1.03	0.95
G4	0	1	1	7.68	8.12	7.81	0.95	0.98
			1.5	8.25	8.78	7.81	0.94	1.06
			2	7.70	8.18	7.81	0.94	0.99
			2.5	7.90	8.43	7.81	0.94	1.01
			3	7.72	8.21	7.81	0.94	0.99
			4	7.72	8.22	7.81	0.94	0.99
			6	7.73	8.23	7.81	0.94	0.99
			8	7.73	8.24	7.81	0.94	0.99
		2	1	10.21	10.76	7.81	0.95	1.31
			1.5	9.17	9.76	7.81	0.94	1.17
			2	8.73	9.35	7.81	0.93	1.12
			2.5	8.47	9.09	7.81	0.93	1.08
			3	8.29	8.91	7.81	0.93	1.06
			4	8.06	8.69	7.81	0.93	1.03
			6	7.83	8.46	7.81	0.93	1.00
			8	7.71	8.34	7.81	0.93	0.99
		3	1	10.73	11.18	7.81	0.96	1.37
			1.5	9.46	9.97	7.81	0.95	1.21
			2	8.87	9.40	7.81	0.94	1.14
			2.5	8.53	9.07	7.81	0.94	1.09
			3	8.30	8.85	7.81	0.94	1.06
			4	8.03	8.58	7.81	0.94	1.03
			6	7.75	8.30	7.81	0.93	0.99
			8	7.60	8.16	7.81	0.93	0.97
		5	1	11.19	11.46	7.81	0.98	1.43
			1.5	9.67	10.02	7.81	0.96	1.24
			2	8.97	9.35	7.81	0.96	1.15

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
			2.5	8.57	8.98	7.81	0.95	1.10
			3	8.31	8.73	7.81	0.95	1.06
			4	7.99	8.43	7.81	0.95	1.02
			6	7.68	8.13	7.81	0.94	0.98
			8	7.52	7.97	7.81	0.94	0.96
		8	1	11.38	11.54	7.81	0.99	1.46
			1.5	9.74	10.00	7.81	0.97	1.25
			2	8.99	9.30	7.81	0.97	1.15
			2.5	8.56	8.90	7.81	0.96	1.10
			3	8.28	8.64	7.81	0.96	1.06
			4	7.95	8.32	7.81	0.95	1.02
			6	7.62	8.01	7.81	0.95	0.98
			8	7.45	7.86	7.81	0.95	0.95
		1	1	10.71	11.63	10.95	0.92	0.98
			1.5	11.18	12.39	10.95	0.90	1.02
			2	10.75	11.70	10.95	0.92	0.98
			2.5	10.85	11.98	10.95	0.91	0.99
			3	10.76	11.73	10.95	0.92	0.98
			4	10.76	11.75	10.95	0.92	0.98
			6	10.76	11.76	10.95	0.91	0.98
			8	10.75	11.76	10.95	0.91	0.98
G5	-0.33	2	1	14.30	14.66	10.95	0.98	1.31
			1.5	12.92	13.51	10.95	0.96	1.18
			2	12.33	13.04	10.95	0.95	1.13
			2.5	11.98	12.74	10.95	0.94	1.09
			3	11.74	12.54	10.95	0.94	1.07
			4	11.44	12.28	10.95	0.93	1.05
			6	11.14	12.02	10.95	0.93	1.02
			8	10.98	11.88	10.95	0.92	1.00
		3	1	15.29	15.15	10.95	1.01	1.40
			1.5	13.53	13.76	10.95	0.98	1.24
			2	12.72	13.10	10.95	0.97	1.16
			2.5	12.25	12.72	10.95	0.96	1.12
			3	11.95	12.47	10.95	0.96	1.09
			4	11.57	12.16	10.95	0.95	1.06
			6	11.19	11.84	10.95	0.94	1.02
			8	10.99	11.68	10.95	0.94	1.00

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
		5	1	16.23	15.47	10.95	1.05	1.48
			1.5	14.08	13.81	10.95	1.02	1.29
			2	13.08	13.05	10.95	1.00	1.19
			2.5	12.52	12.61	10.95	0.99	1.14
			3	12.15	12.33	10.95	0.99	1.11
			4	11.70	11.98	10.95	0.98	1.07
			6	11.25	11.64	10.95	0.97	1.03
			8	11.03	11.46	10.95	0.96	1.01
		8	1	16.70	15.56	10.95	1.07	1.53
			1.5	14.33	13.79	10.95	1.04	1.31
			2	13.24	12.98	10.95	1.02	1.21
			2.5	12.62	12.52	10.95	1.01	1.15
			3	12.23	12.23	10.95	1.00	1.12
			4	11.74	11.87	10.95	0.99	1.07
			6	11.26	11.51	10.95	0.98	1.03
			8	11.01	11.33	10.95	0.97	1.01
G6	-0.67	1	1	16.00	16.20	16.41	0.99	0.97
			1.5	15.67	17.07	16.41	0.92	0.95
			2	15.88	16.29	16.41	0.97	0.97
			2.5	15.57	16.61	16.41	0.94	0.95
			3	15.64	16.32	16.41	0.96	0.95
			4	15.57	16.34	16.41	0.95	0.95
			6	15.58	16.35	16.41	0.95	0.95
			8	15.57	16.36	16.41	0.95	0.95
		2	1	22.52	19.64	16.41	1.15	1.37
			1.5	20.89	18.34	16.41	1.14	1.27
			2	20.15	17.80	16.41	1.13	1.23
			2.5	19.70	17.46	16.41	1.13	1.20
			3	19.41	17.24	16.41	1.13	1.18
			4	19.03	16.95	16.41	1.12	1.16
			6	18.64	16.65	16.41	1.12	1.14
			8	18.43	16.49	16.41	1.12	1.12
		3	1	25.67	20.19	16.41	1.27	1.56
			1.5	23.22	18.62	16.41	1.25	1.41
			2	22.05	17.88	16.41	1.23	1.34
			2.5	21.38	17.44	16.41	1.23	1.30
			3	20.93	17.16	16.41	1.22	1.28

Group	Specimen parameters			Buckling Coefficient (K)			Comparison	
	ψ	R	α	K_{FE}	$K_{Prop.}$	K_{EC3}	$K_{FE} / K_{Prop.}$	K_{FE} / K_{EC3}
			4	20.37	16.80	16.41	1.21	1.24
			6	19.81	16.45	16.41	1.20	1.21
			8	19.51	16.26	16.41	1.20	1.19
		5	1	29.11	20.55	16.41	1.42	1.77
			1.5	25.62	18.68	16.41	1.37	1.56
			2	23.98	17.81	16.41	1.35	1.46
			2.5	23.05	17.32	16.41	1.33	1.40
			3	22.44	17.00	16.41	1.32	1.37
			4	21.69	16.61	16.41	1.31	1.32
			6	20.94	16.22	16.41	1.29	1.28
			8	20.55	16.02	16.41	1.28	1.25
		8	1	31.16	20.65	16.41	1.51	1.90
			1.5	26.99	18.65	16.41	1.45	1.64
			2	25.07	17.74	16.41	1.41	1.53
			2.5	23.97	17.22	16.41	1.39	1.46
			3	23.26	16.88	16.41	1.38	1.42
			4	22.39	16.47	16.41	1.36	1.36
			6	21.53	16.07	16.41	1.34	1.31
			8	21.09	15.87	16.41	1.33	1.28
				Mean			1.04	1.14
				COV			0.13	0.17

4. Conclusions

With the aim of providing technical information that may be used to improve design codes in dealing with tapered structural elements, the buckling behaviour of trapezoidal plates subjected to stress gradients was studied by the finite element method. The critical buckling coefficients were estimated for different compression and bending cases with simply supported boundary conditions, including uniform compression load, trapezoidal compression load, triangle compression load, two unequal reverse triangular load, and pure bending load. More than 650 geometries were analysed for each case, covering tapering ratios from 0.25 to 8, and normalized plate lengths from 1 to 8. The regression analysis was employed to propose approximate closed-form expressions that can be used directly to compute the local buckling coefficient for trapezoidal plates. Based on the findings, the following conclusions and recommendations were reached:

1. Where the load is applied on the shorter edge, the buckling resistance of the tapered plate is directly proportional to the tapering of the web plate because the smaller edge provides a stiffer zone compared to the larger edge.
2. The proposed formulas represent a significant improvement over current Eurocode 3 and most design codes in predicting the critical buckling coefficient under compression and bending stresses and can be used to compute the effective width of a slender cross section.
3. For the pure bending moment case, it is recommended to ignore the decrease in k values for a tapering ratio of less than 2.00.
4. The buckling coefficient predicted by EC3 is generally conservative; except for the pure bending case, it was close to that of the FE model prediction.

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