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# On the use of constrained modulus for soil settlement analysis

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**Abstract.** Soil settlement is a key parameter in engineering design of geotechnical structures. Two approaches have been used for the characterization of soil behaviour under one dimensional compression: the linear - nonlinear, the traditional approach, and the modulus-based approach, the constrained modulus. The constrained modulus approach requires the knowledge of effective stress ( $\sigma'$ ), stress exponent ( $d$ ), and soil modulus number ( *m* ). In the current study, the constrained modulus approach was adopted in the settlement analysis. Compressibility data of 130 fine and mixed soils with a variety of composition and mineralogy was used in the analysis. In addition, three common clays, bentonite, sepiolite and attapulgite, was experimentally tested using one-dimensional oedometer compression test and the results were included in the analysis. The analysis shows that the approach of constrained modulus can be effectively used to calculate the settlement of fine and intermediate mixed soils, and the stress exponent ( *d* ) varies from 0 for clayey soils to 0.3–0.6 for intermediate silty and clayey sand soils depending on the soil plasticity and particle size distribution. Also, there is a simple relationship with  $R^2 = 0.83$  between the soil modulus number, *m* , and the liquid limit, *IL*. in that the higher the liquid limit, *IL*. the lower the soil modulus number, *m* .

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# *1. Introduction*

Loads of engineering earth structures are transferred down by foundations to underlaying soils and distributed as a stress over the affected layers. Generally, stresses compress soil particles quickly at a small rate without any moisture loose at immediate stage. With a stress increase, a reduction in soil volume occurs due to the expulsion of water from interconnected voids, quickly and insignificant in coarse soils, but slowly and significant in fine-grained soils depending on the soil thickness and the permeability. At the end of the process, that is, when dissipation of excess pore water pressure is all completed, some compression (so-called secondary compression) takes place due to the plastic status of soil mass fabrics. In design, it is necessary to assess the suitability of a soil against both excessive settlement failure and allowable bearing capacity. While stability issues of structures in soil mechanics can often be defined satisfactorily, it is still difficult to estimate settlements and deformations accurately in advance [1]. In spite of the growth of knowledge in the past years, the variation of displacements measured is still to be significantly larger, and sometimes much smaller, than those calculated. This is because of the inappropriate selection of stiffness parameters in characterization of soil settlement [1]. The linear elastic

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theory was the base of solutions in characterization of settlement  $(\varepsilon)$  of coarse-grained soils, which depends on the elastic modulus of a soil  $(E)$  and the flexibility of the resting foundation as:

$$
\varepsilon = \frac{\Delta \sigma}{E}.\tag{1}
$$

Most soils behave non-linearly under compression upon subjected to a stress change. As such, the linear elastic theory cannot be suitable, especially when dealing with compressible soils such as finegrained soils (more specifically clays). In this case, the settlement can be found by:

$$
\varepsilon = \frac{C_c}{1 + e_o} \log \frac{\sigma'_f}{\sigma'_o},\tag{2}
$$

where  $\sigma'_o$  and  $\sigma'_f$  represent the initial and final effective stress, respectively.

The equation (2) shows that the settlement of a fine soil due to primary consolidation is a function of two key parameters; the compression index  $C_c$ , and the initial void ratio  $e_o$ , which can be determined from the traditional one-dimensional compression laboratory test. A range of theoretical, empirical, and semiempirical relationships for  $C_c$  have been developed (e.g. [2]–[8]) depending on one (e.g. liquid limit, water content, field void ratio) or more soil properties. However, most civil engineers and soil researchers often report the values of  $C_c$  only, ignoring the magnitude of  $e_o$ . Sometimes, the reported  $e_o$  is determined from a soil sample different to that used in determining the  $C_c$ , which is not acceptable. The challenge of using two compression parameters is avoided by using soil modulus, the constrained modulus, the slope of stress-strain relationship which was first introduced by N. Janbu [9], using two non-dimensional parameters; a stress exponent, *d* , and a modulus number, *m* , as follows:

$$
\frac{d\sigma}{d\varepsilon} = m\sigma_o \left(\frac{\sigma'}{\sigma_r}\right)^{1-d} = M,\tag{3}
$$

where  $M$  is,  $\sigma'$  is an effective stress (kPa); and  $\sigma_r$  is a reference stress equal to (100 kPa).

Soil moduli have been used by several researchers as a measure of soil stiffness (e.g. [10]–[16]). Measurements of soil modulus by means of laboratory tests are quite difficult and often inaccurate because of the significant disturbance [17]. Therefore, correlations were always required to estimate the specific soil modulus. The constrained modulus approach for settlement, equation 3, is used where is no strain in the perpendicular direction which is the case of the soil behavior under one-dimensional compression. It combines the principles of linear and non-linear behavior [9]. It is also applicable for all types of soils even intermediate soils (e.g. silty sand, clayey sand) that exhibit transitional behavior which are not addressed properly by the traditional approaches (equation 1 and 2) [9] and [18]. According to Byington [19], the traditional approaches are limited and cannot be used with silt containing 15 % or less clay content. The interaction and arrangement of the fine and coarse particles in these soils affects the compressibility behavior such that they can be expected to exhibit behaviors and modes associated with clay and sands soils.

Despite the applicability of the constrained modulus approach, the approach is not commonly used in soil mechanics for soil compressibility and settlement. Very few studies were undertaken and provided preliminary information. N. Janbu [9] was the first who used the modulus approach to describe the compressibility behavior of rock, sand, silt and clay soils tested by oedometer and triaxial. He showed that the approach can describe the compressibility behavior provided that the accurate values of soil modulus number and stress exponent are selected. Schanz and Vermeer [1] tested the stiffness of three types of sands – Toyoura, Karlsruhe and Hostun sands – using the oedometer and triaxial modulus and showed that both normalized moduli have nearly same values.

The modulus number and stress exponent are of high importance in modulus-based settlement calculation. According to N. Janbu [9], the stress component ( *d* ) and the modulus number ( *m* ) both varies with soil initial porosity ( *n* ), the coarser the soil, the larger the stress exponent and modulus numbers assumed that,  $d = 1$  for non-plastic soils, and  $d = 0$  for plastic soils. The variation range of the modulus number, *m* , is really quite wide, from 6 orders of magnitude between coarse- and fine-grained soils.

According to the Canadian Foundation Engineering Manual 1992, the value of *m* for coarse soils, gravels and sands, ranges between 100 and 400 and for fine soils, silts, and clays, it is between 5 and 200. These values of *m* are approximate, and for intermediate soils such as silty clays or sandy clays, the values will be different as the compressibility behavior is different to that in pure soils. Therefore; the objective of this study is to determine both the modulus number, *m* , and the stress component, *d* , of 133 fine and mixed soils using the constrained modulus-based approach, and also to establish a new empirical relationship for the modulus number, *m* .

# *1.1 Soil settlement determination*

With the knowledge of soil modulus number, *m* , stress exponent, *d* , and effective stress, σ′, soil settlement can be determined. If, by the assumptions of N. Janbu [9], the stress exponent is fixed to unity for very coarse soils, rocks and gravels, and integrating the equation (3) gives:

$$
\varepsilon = \frac{\sigma_f' - \sigma_o'}{100m}.\tag{4}
$$

For fine soils, it equals to zero and equation (3) be:

$$
\varepsilon = \frac{1}{m} \ln \left( \frac{\sigma'_f}{\sigma'_o} \right). \tag{5}
$$

However, for intermediate, composite, and transitional soils such as sandy clays, sandy silts and silty sands that are not covered properly by the traditional approaches, an intermitted value corresponds to 0.5 can be used which lies in an agreement with the fact that for each stress increment, the strain of a soil gets gradually smaller

$$
\varepsilon = \frac{1}{5m} \left( \sqrt{\sigma'_f} - \sqrt{\sigma'_o} \right). \tag{6}
$$

To estimate the modulus number, *m*, the normal compression curves  $(e - log \sigma')$  of soils were first converted to constrained modulus  $(M - \sigma')$  using the definition of volume compressibility  $(m_v)$  as:

$$
M = \frac{1}{m_v} = \frac{\delta \sigma'}{\delta e} \frac{(1+e)}{1000}.
$$
 (7)

Then, applying the best fit to the equation (3).

## *2. Materials and Methods*

It is known that soil type, stress history and magnitude of effective stress play an important role in compressibility behavior of soils so a range of data from different sources varying in geological origin, soil type, sample preparation and index properties was selected in analysis. Published data of [5–7, 20–31] was included in the analysis. The soil samples in these sources are natural, remolded, and reconstituted samples prepared with an initial water content of 1–2 times of liquid limit. The standard one-dimensional oedometer test was used in these papers to assess the compression characteristics with a stress varying between (2.5–40000 kPa). Table 1 summarizes the soil data used in the analysis.



# *Table 1. Data used in analysis.*





In addition, three common commercially clays; bentonite, sepiolite, and attapulgite were chosen in the present study which covers a range of plasticity (105 % ≤ *IL* ≤ 220 %). The index properties, determined by ASTM D4318–17e1 [32], D854–14 [33], and D2487–17 [34] of the materials are shown in Table 2. The plasticity chart of the data used is shown in Fig. 1. As per Fig. 1, the classification of clays is high plasticity silts (MH). Also, the used data covers a range of clayey, silty, and intermediate soils with high and low plasticity which is very important as to cover a range of soils for settlement analysis.

The clays were prepared in a blender as slurry by adding de-aired water using 1.75 times of liquid limit ensuring a fully saturated and homogeneous sample. The mixture was poured into a rigid stainless steel consolidometer tube with a 14 cm diameter and 25 cm height and then consolidated to the desired stress. The compressibility test was conducted using the standard oedometer equipment following ASTM D2435/D2435M-11 [35] procedures. The internal area of the consolidation ring was lubricated with silicon grease so that the side friction can be minimized, and the test was carried out at maintained room temperature of 20 °C. The ring was inserted into the consolidated sample to bring the target sample. The assembled cell was mounted and placed on the platform of the oedometer equipment with an adjusted dial gage of a 0.001 mm resolution. Before applying loads, the soil sample was kept in saturated condition by submerging it in de-aired water for the whole test period. Increments of load increments were applied up to maximum stress of 1280 kPa and each increment was doubled when the primary compressibility for the current load was completed.







**Figure 1. The plasticity chart of present clays and data used.**

## *3. Results and Discussion*

Fig. 2 shows the normal compression curves of the bentonite, sepiolite and attapulgite clays that were tested in the present study. It can be seen that the curves are slightly concave up for the stresses larger than 10 kPa which is the most common.



**Figure 2. Normal compression curves of clays (bentonite, sepiolite, and attapulgite).**

The normal compression curves in the preset study (Fig. 2) and in previous selected studies were replotted in terms of the constrained modulus,  $M$ , versus effective stress,  $\sigma'$  (as seen in Fig. 3). It can be seen that the constrained modulus is a function of effective stress, stress history, and soil type. The modulus increases when the stress increases.

The modulus number, *m,* and stress exponent, *d ,* were determined using the best-fit of the equation (3). The values are presented in Table 2 and correlated in Fig. 4 and 5 with liquid limit, *IL*. It can be observed that there is a good relationship for *m* with IL with  $R^2 = 0.83$  in the form:

$$
m = 264.11(IL)^{-0.841}.
$$
 (8)

This is acceptable because the liquid limit is a material property that is relied on soil composition, particle size and surface characteristics. This single relationship has an advantage of the fact that liquid limit test is easy to conduct in lab.

The stress exponent, d, shows a scatter as it depends on soil plasticity and particle size distribution and varies from 0 for clayey soils to 0.3–0.6 for silty and clayey sand soils. This is similar to the assumptions of N. Janbu [9] for normal consolidated clays and intermediate soils.



**(b)** 



**(e)** 







**Figure 4. The relation between: (a) modulus number,** *m***; (b) stress exponent,** *d***, with liquid limit,**  *IL***.** 

# *4. Conclusion*

This study is devoted to exploring the suitability of the constrained modulus approach for soil settlement determination. An analysis of a range of compressibility data taken from different high-quality papers along with the present experimental results on three commercial clays (bentonite, sepiolite and attapulgite) using the modulus-based approach of N. Janbu shows that:

- 1. The constrained modulus-based approach is a convenient approach to adequately describe the settlement phenomena of fine and intermediate transitional soils using the stress exponent ( *d* ) and the modulus number (*m*).
- 2. The results show that the stress exponent  $(d)$  in the approach depends on soil plasticity and particle size distribution and varies from 0 for clayey soils to 0.3–0.6 for silty and clayey sand soils.
- 3. There is a relationship between the soil modulus number (*m*) and the liquid limit (*IL)* in that an increase in the liquid limit leads to a decrease in the modulus number.
- 4. An empirical relationship for modulus number, *m*, based on the soil liquid limit, *IL,* was established with  $R^2 = 0.83$ .

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