Computing, Telecommunication and Control, 2023, Vol. 16, No. 4, Pp. 49–59.
Информатика, телекоммуникации и управление. 2023. Том 16, № 4. С. 49–59.

System Analysis and Control
Системный анализ и управление

Research article
DOI: https://doi.org/10.18721/JCSTCS.16405
UDC 681.51

PROJECTION OPERATOR FOR SOLVING GENERALIZED PROBLEMS OF PROGRAM MOTIONS STABILIZATION

A.A. Efremov ✉

Peter the Great St. Petersburg Polytechnic University,
St. Petersburg, Russian Federation

✉ Artem.Efremov@spbstu.ru

Abstract. Generalization and development of the projection operator method for solving problems of stabilization of given program motions seems to be an actual direction of research in the field of synthesis of optimal control systems for nonlinear dynamic stationary objects with limited phase coordinates and controls. In this paper, we formulate generalized stabilization problems for program motions given by a program-stabilizing vector C0 and a vector of admissible program motions C. We show the derivation of a projection operator for solving the specified class of problems. For a nonlinear locally controlled difference operator, admissible controls are synthesized that stabilize program motions under restrictions on phase coordinates and controls. An operator of a dynamical system is obtained for generalized problems of stabilization of program motions with restrictions on the vectors of phase coordinates and controls. Numerical simulation of the stabilization of the given program motions of a dynamic object is carried out. As an example of a dynamic object, a mathematical model of a synchronous generator is chosen, consisting of a system of bilinear differential equations with parameters corresponding to equations in the form of V. A. Venikov. A computational experiment confirmed the theoretical results obtained in the work.

Keywords: projection operators, stabilization of program motions, dynamical systems, optimization, nonlinear difference operator, locally admissible controls, restrictions on phase coordinates and controls

Citation: Efremov A.A. Projection operator for solving generalized problems of program motions stabilization. Computing, Telecommunications and Control, 2023, Vol. 16, No. 4, Pp. 49–59. DOI: 10.18721/JCSTCS.16405

Научная статья
DOI: https://doi.org/10.18721/JCSTCS.16405
УДК 681.51

ПРОЕКЦИОННЫЙ ОПЕРАТОР РЕШЕНИЯ ОБОБЩЕННЫХ ЗАДАЧ СТАБИЛИЗАЦИИ ПРОГРАММНЫХ ДВИЖЕНИЙ

А.А. Ефремов ✉

Санкт-Петербургский политехнический университет Петра Великого,
Санкт-Петербург, Российская Федерация

✉ Artem.Efremov@spbstu.ru

Аннотация. Обобщение и развитие проекционно-операторных методов для решения задач стабилизации заданных программных движений представляется актуальным направлением исследования в области синтеза систем оптимального управления нелинейными динамическими стационарными объектами с ограниченными фазовыми координатами и управлениями. В работе сформулированы обобщенные задачи стабилизации программных движений, заданных программным стабилизирующим вектором C0 и вектор допустимых программных движений C. Показан вывод проекционного оператора решения указанного класса задач. Для нелинейного локально управляемого разностного оператора объекта синтезированы допустимые управления, стабилизирующие программные движения при ограничениях на фазовые координаты и управления. Получен оператор динамической системы для обобщенных задач стабилизации программных движений с ограничениями на векторы фазовых координат и управлений. Проведено численное моделирование стабилизации заданных программных движений динамической объекта. В качестве примера динамического объекта выбрана математическая модель синхронного генератора, состоящая из системы билинейных дифференциальных уравнений с параметрами, соответствующими уравнениям в форме В.А. Веникова. Вычислительный эксперимент подтвердил теоретические результаты, полученные в работе.

Ключевые слова: проекционные операторы, стабилизация программных движений, динамические системы, оптимизация, нелинейный разностный оператор, локально допустимые управления, ограничения на фазовые координаты и управления

Для цитирования: Efremov A.A. Projection operator for solving generalized problems of program motions stabilization // Computing, Telecommunications and Control. 2023. Т. 16, № 4. С. 49–59. DOI: 10.18721/JCSTCS.16405

Introduction

Stabilization of program motions of dynamic objects is an urgent task of modern control theory [1–5]. Methods of inverse dynamics problems [6], Lyapunov barrier functions [7–9], mathematical programming [10, 11], etc. [12–15] are used to solve the problems of stabilization of program motions.

The projection operator method of mathematical programming is used in the paper to solve the specified class of problems [16]. The projection operator method is a universal technique of synthesis of locally admissible and quasi-optimal controlled nonlinear dynamic objects. The generalization and development of the projection operator method for solving tasks of stabilization of given program motions is a relevant research direction in the field of synthesis of optimal control systems for nonlinear dynamic objects with limited phase coordinates and controls.

1. Formulation of generalized problems of stabilization of program motions with inequality constraints on their vector

Generalized problems of stabilization of program motions, specified by the program-stabilizing vector $\mathbf{C}_0$ and the vector of permissible program motions $\mathbf{C}$ have the form: calculate vector

$$\mathbf{x}_* = \arg\min\left\{\varphi(\mathbf{x}) = \|\mathbf{x}-\mathbf{C}_0\|^2 \,\middle|\, \mathbf{A}\mathrm{x} = \mathbf{b},\ \mathbf{A}\in\mathbf{R}^{m\times n},\ \mathbf{C}_0^{\mathrm{T}}\mathbf{C}_0\in\mathbf{D}_x,\right.$$
$$\left.\mathbf{D}_x = \left\{\mathbf{x}\middle|(\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C})\le r^2\right\}\right\}\in\mathbf{R}^n, \tag{1}$$

where $\mathbf{C}_0\in\mathbf{R}^n$ is the stabilizing program vector of state coordinates, $\mathbf{C}\in\mathbf{R}^n$ is the vector of permissible program motions for inequality constraints, $\mathbf{C}_0 < \mathbf{C}$.

Problems (1) generalize the requirements for the problems of stabilization of given program motions [16] by introducing a class of inequality class restrictions $(\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C})\le r^2$, which makes it possible to set more “flexible” requirements for these problems.

2. Synthesis of a projection operator for solving generalized problems of stabilization of program motions

Lemma 1 provides the derivation of the projection operator for solving generalized problems of program motions stabilization (1).

Lemma 1. The projection operator for solving the generalized problem (1) has the form:

$$\mathbf{x} = \mathbf{P}^{+}\mathbf{b} + (1+\lambda)^{-1}\mathbf{P}^{0}\mathbf{C}_0 + \lambda(1+\lambda)^{-1}\mathbf{P}^{0}\mathbf{C}, \tag{2}$$

where $\mathbf{P}^0 = \mathbf{E}_n - \mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{A}$ is a projector onto a linear manifold $\mathbf{Ax} = \mathbf{b}$, $\mathbf{P}^{+} = \mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}$ is a projector onto the orthogonal complement of a linear manifold. The scalar parameter λ is a Lagrange multiplier to restrict the type of inequality in (1).

Proof. The Lagrange function for problem (1) has the form:

$$L = \|\mathbf{x}-\mathbf{C}_0\|_2^2 + \lambda_0^{\mathrm{T}}(\mathbf{Ax}-\mathbf{b}) + \lambda\left((\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C}) - r^2\right). \tag{3}$$

The necessary conditions for Lagrange optimality are given in the form:

$$L'_x = 2(\mathbf{x}-\mathbf{C}_0) + \mathbf{A}^{\mathrm{T}}\lambda_0 + 2\lambda(\mathbf{x}-\mathbf{C}) = 0_n, \tag{4}$$

$$L'_{\lambda_0} = \mathbf{Ax}-\mathbf{b} = 0_m, \tag{5}$$

$$L'_{\lambda} = (\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C}) - r^2 = 0_1. \tag{6}$$

The necessary optimality condition (4) multiplied by matrix $\mathbf{A}$, considering equality (5) of the form $\mathbf{Ax} = \mathbf{b}$, determines the equation

$$2\mathbf{b} + 2\lambda\mathbf{b} - 2\mathbf{AC}_0 + \mathbf{AA}^{T}\lambda_0 - 2\lambda\mathbf{AC} = 0_n. \tag{7}$$

Then from (7) follows a set of transformations that determines the scalar parameter λ_0, which is the Lagrange multiplier for restricting the type of equality in problem (1).

$$\lambda_0 = 2\lambda\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{AC} + 2\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{AC}_0 - 2(\lambda+1)\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{b}. \tag{8}$$

Substituting the Lagrange multiplier λ_0 into (4) and equivalent transformations determine an equation of the form:

$$\mathbf{x} + \lambda\mathbf{x} - \mathbf{C}_0 + \lambda\mathbf{A}^{\mathrm{T}}\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{AC} + \mathbf{A}^{\mathrm{T}}\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{AC}_0 - (\lambda+1)\mathbf{A}^{\mathrm{T}}\left(\mathbf{AA}^{T}\right)^{-1}\mathbf{b} - \lambda\mathbf{C} = 0_n.$$

Replacing the projectors $\mathbf{P}^{+}$ and $\mathbf{P}^{0}$ with subsequent transformations makes it possible to obtain an expression that defines the projection operator for solving problem (1) as a function of the Lagrange multiplier λ for conditions of the type inequality:

$$\mathbf{x} = \mathbf{P}^{+}\mathbf{b} + (1+\lambda)^{-1}\mathbf{P}^{0}\mathbf{C}_0 + \lambda(1+\lambda)^{-1}\mathbf{P}^{0}\mathbf{C}.$$

Lemma is proven.

Next, we consider the solution of generalized problems of stabilization of program motions (1) in the case of equality of the stabilizing program vector of state coordinates and the vector of permissible program motions for inequality constraints $\mathbf{C}_0 = \mathbf{C}$.

Consequence. In the case of equality of the stabilizing program vector of state coordinates and the vector of permissible program motions for inequality constraints $\mathbf{C}_0 = \mathbf{C}$, the solution of generalized problems of program motions stabilization (1) does not depend on the Lagrange multiplier for conditions of the inequality type λ and has the form

$$\mathbf{x} = \mathbf{P}^{+}\mathbf{b} + \mathbf{P}^{0}\mathbf{C}. \tag{9}$$

Proof. Let us repeat the reasoning of Lemma 1 with the condition of equality of the stabilizing program vector of state coordinates and the admissible vector of program coordinates-controls $\mathbf{C}_0 = \mathbf{C}$. Problem (1) will take the form:

$$\begin{gathered} \mathbf{x}_* = \arg\min\left\{\varphi(\mathbf{x}) = \|\mathbf{x}-\mathbf{C}\|^2 \,\middle|\, \mathbf{A}\mathbf{x} = \mathbf{b},\ \mathbf{A} \in \mathbf{R}^{m\times n},\ \mathbf{C}^{\mathrm{T}}\mathbf{C} \in \mathbf{D}_x,\right. \\ \left.\mathbf{D}_x = \left\{\mathbf{x} \middle| (\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C}) \le r^2\right\}\right\} \in \mathbf{R}^n, \end{gathered} \tag{10}$$

Lagrange function for problem (10):

$$L = \|\mathbf{x}-\mathbf{C}\|_2^2 + \lambda_0^{\mathrm{T}}(\mathbf{A}\mathbf{x}-\mathbf{b}) + \lambda\left((\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C}) - r^2\right). \tag{11}$$

The necessary conditions for function (11) have the form:

$$L'_x = 2(\mathbf{x}-\mathbf{C}) + \mathbf{A}^{\mathrm{T}}\lambda_0 + 2\lambda(\mathbf{x}-\mathbf{C}) = 0_n, \tag{12}$$

$$L'_{\lambda_0} = \mathbf{A}\mathbf{x} - \mathbf{b} = 0_m, \tag{13}$$

$$L'_{\lambda} = (\mathbf{x}-\mathbf{C})^{\mathrm{T}}(\mathbf{x}-\mathbf{C}) - r^2 = 0_1. \tag{14}$$

The first equation (12) multiplied by matrix $\mathbf{A}$, considering equality (13) of the form $\mathbf{A}\mathbf{x} = \mathbf{b}$, determines the equation

$$2\mathbf{b} - 2\mathbf{A}\mathbf{C} + \mathbf{A}\mathbf{A}^{T}\lambda_0 + 2\lambda\mathbf{b} - 2\lambda\mathbf{A}\mathbf{C} = 0_n. \tag{15}$$

Then the scalar parameter is expressed from the resulting equation λ_0,

$$\lambda_0 = 2\lambda\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{A}\mathbf{C} + 2\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{A}\mathbf{C} - 2\lambda\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{b} - 2\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{b}. \tag{17}$$

Substituting λ_0 into (12) followed by opening the parentheses determines the equation:

$$\mathbf{x} + \lambda\mathbf{x} = \mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{A}^{T}\right)^{-1}\mathbf{b} + \lambda\mathbf{A}^{T}\left(\mathbf{A}\mathbf{A}^{T}\right)^{-1}\mathbf{b} - \mathbf{A}^{\mathrm{T}}\left(\mathbf{A}\mathbf{A}^{T}\right)^{-1}\mathbf{A}\mathbf{C} - \lambda\mathbf{A}^{T}\left(\mathbf{A}\mathbf{A}^{T}\right)^{-1}\mathbf{A}\mathbf{C} + \lambda\mathbf{C} + \mathbf{C}.$$

Reducing similar ones and substituting the projection operators $\mathbf{P}^{+}$ and $\mathbf{P}^{0}$, allows us to obtain an expression that determines the projection operator for solving problem (10), which does not depend on the Lagrange multiplier λ:

$$\mathbf{x} = \mathbf{P}^{+}\mathbf{b} + \mathbf{P}^{0}\mathbf{C}.$$

Consequence is proven.

3. Synthesis of a dynamic system with restrictions on phase coordinates and controls

This is a method for synthesizing locally admissible controls for the generalized problem of stabilizing a single stationary equilibrium position or program motions specified by the stabilizing program vector $\mathbf{C}_0 = \mathbf{C}_{0k}$ and limited by the vector of admissible program motions $\mathbf{C} = \mathbf{C}_k$, $k \in N$, for an object in the form of a difference operator with restrictions on phase coordinates and controls. In this case, it is assumed that the nonlinear control object is locally controllable according to N.N. Petrov [17–20].

Let a nonlinear locally controlled object be defined by a difference operator:

$$\mathbf{x}_{k+1} = \mathbf{H}(\mathbf{x}_k) + \mathbf{F}\mathbf{u}_k,\quad \mathbf{y}_k = \mathbf{c}_y \mathbf{x}_k,\quad \mathbf{x}_{k_0} = \mathbf{x}_0 \in \mathbf{D}, \tag{15}$$

where $\mathbf{D} \subset \mathbf{R}^n$ is the neighborhood of attraction as a set of initial states from which the system returns to the equilibrium position. Vectors and matrices of operator (15) have the form $\mathbf{x}_{k+1} \in \mathbf{R}^n$, $\mathbf{x}_k \in \mathbf{D} \subset \mathbf{R}^n$, $\mathbf{y}_k \in \mathbf{R}^l$, $\mathbf{u}_k \in \mathbf{R}^m$, $\mathbf{F} \in \mathbf{R}^{n\times m}$, $\mathbf{c}_y \in \mathbf{R}^{l\times n}$; $\mathbf{H}(\mathbf{x}_k) \in \mathbf{R}^n$ is the function vector.

Then the linear manifold for the operator optimization problem, considering the difference operator (15), will be written in the form:

$$\mathbf{A}\mathbf{z}_k = \left[\mathbf{E} \middle| -\mathbf{F}\right] \times \begin{bmatrix} \mathbf{x}_{k+1} \\ \mathbf{u}_k \end{bmatrix} = \mathbf{H}(\mathbf{x}_k) = \mathbf{b}_k, \tag{16}$$

where the object operator, the vector "state-control" and the vector of the right side of the linear manifold have the form:

$$\mathbf{A} = \left[\mathbf{E} \middle| -\mathbf{F}\right] \in \mathbf{R}^{n\times(n+m)},\ \mathbf{z}_k = \left[\mathbf{x}_{k+1} \middle| \mathbf{u}_k\right]^{\mathrm{T}} \in \mathbf{R}^{(n+m)},\ \mathbf{b}_k = \mathbf{H}(\mathbf{x}_k) \in \mathbf{R}^n.$$

Representing the difference operator of an object in the form of a linear manifold (16) allows one to synthesize locally admissible controls by reducing the problem of calculating controls to a countable number of projection operator optimization problems. In this case, the problem of finite-dimensional mathematical programming (1), considering the linear manifold (16), is transformed into a problem of the form: calculate the "generalized" state-control vector (17)

$$\begin{gathered} \mathbf{z}_k^* \triangleq \begin{bmatrix} \mathbf{x}_{k+1}^* \\ \mathbf{u}_k^* \end{bmatrix} = \arg\min \left\{ \varphi(\mathbf{z}_k) = \left\|\mathbf{z}_k - \mathbf{C}_{0k}\right\|^2 \middle| A\mathbf{z}_k = \left[\mathbf{E}_1 \middle| -\mathbf{F}\right] \begin{bmatrix} \mathbf{x}_{k+1} \\ \mathbf{u}_k \end{bmatrix} = \right. \\ \left. = \mathbf{H}(\mathbf{x}_k) = \mathbf{b}_k,\ \mathbf{C}_{0k} \in \mathbf{D}_z,\ \mathbf{D}_z = \left\{ \mathbf{z}_k \middle| (\mathbf{z}_k - \mathbf{C}_k)^T (\mathbf{z}_k - \mathbf{C}_k) \le r^2 \right\}^2 \right\} \in \mathbf{R}^{m+n}, \end{gathered} \tag{17}$$

where $\mathbf{C}_{0k} \in \mathbf{R}^n$ is the stabilizing program vector of state coordinates, $\mathbf{C}_k \in \mathbf{R}^n$ is the vector of permissible program motions for inequality constraints, $\mathbf{C}_k < \mathbf{C}_{0k}$.

The countable set of solutions to mathematical programming problems (17) determines the "state-control" vectors. The structure of the admissible control operator is determined by the generalized projection operator of finite-dimensional optimization (2) and has the form:

$$\mathbf{z}_k^*(\varsigma_i, \sigma_i) = \mathbf{P}^{+}\mathbf{b}_k + \varsigma_i \mathbf{P}^0 \mathbf{C}_{0k} + \sigma_i \mathbf{P}^0 \mathbf{C}_k,\ i = 1,\ 2,$$

where $\varsigma_1 = (1+\lambda_1)^{-1}$, $\varsigma_2 = (1+\lambda_2)^{-1}$, $\sigma_1 = \lambda_1(1+\lambda_1)^{-1}$, $\sigma_2 = \lambda_1(1+\lambda_2)^{-1}$, λ_1, λ_2 are a pair of Lagrange multipliers for a condition of the inequality type of the problem (1).

The solution vector of the optimization problem under consideration is defined as the image of a convex linear combination of two “boundary generalized operators” $\mathbf{z}_k^*(\varsigma_1, \sigma_1)$ and $\mathbf{z}_k^*(\varsigma_2, \sigma_2)$,

$$\hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta) = \theta \mathbf{z}_k^*(\varsigma_1, \sigma_1) + (1-\theta)\mathbf{z}_k^*(\varsigma_2, \sigma_2), \quad \theta \in [0;1],$$
$$\mathbf{z}_k^*(\varsigma_i, \sigma_i) = \mathbf{P}^+\mathbf{b}_k + \varsigma_i \mathbf{P}^0 \mathbf{C}_{0k} + \sigma_i \mathbf{P}^0 \mathbf{C}_k, \quad i = 1,\ 2. \tag{18}$$

Vector $\hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta)$ in (18) includes vectors of locally admissible controls $\mathbf{u}_k = \mathbf{T}_u \hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta)$ and vectors of phase coordinate predictions $\mathbf{x}_{k+1} = \mathbf{T}_x \hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta)$ “filtered” using matrices $\mathbf{T}_u = \left[0_{m\times n} \middle| \mathbf{E}_{m\times m}\right]$ and $\mathbf{T}_x = \left[\mathbf{E}_{n\times n} \middle| 0_{n\times m}\right]$ respectively.

As a result, it follows from relations (15)–(18) that the operator of a nonlinear dynamic system with feedback for generalized problems of stabilization of program motions with restrictions on the vectors of phase coordinates and controls specified by the program stabilizing vector $\mathbf{C}_{0k}$ and the vector of permissible program motions $\mathbf{C}_k$ is written as:

$$\mathbf{x}_{k+1} = \mathbf{H}(\mathbf{x}_k) + \gamma \mathbf{F}\mathbf{T}_u \hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta),$$
$$\hat{\mathbf{z}}_k(\varsigma_1, \varsigma_2, \sigma_1, \sigma_2, \theta) = \left[\theta\left(\mathbf{P}^+\mathbf{b}_k + \varsigma_1 \mathbf{P}^0 \mathbf{C}_{0k} + \sigma_1 \mathbf{P}^0 \mathbf{C}_k\right)\right] + (1-\theta)\left(\mathbf{P}^+\mathbf{b}_k + \varsigma_2 \mathbf{P}^0 \mathbf{C}_{0k} + \sigma_2 \mathbf{P}^0 \mathbf{C}_k\right), \tag{19}$$

where $\varsigma_1 = (1+\lambda_1)^{-1}$, $\varsigma_2 = (1+\lambda_2)^{-1}$, $\sigma_1 = \lambda_1(1+\lambda_1)^{-1}$, $\sigma_2 = \lambda_2(1+\lambda_2)^{-1}$, $\gamma \in R$ – feedback parameter, θ – "acceptability" parameter, $\theta \in [0;1]$.

4. Computational experiment

The section presents the results of a computational experiment of the dynamic system under study with restrictions on phase coordinates and controls (19).

As an example of a dynamic object, we used a vector-matrix bilinear differential model of a synchronous generator [21] with parameters of the Gorev-Park system of equations in the form of V.A. Venikov [22]. To calculate the values of the vector-matrix model, the technical parameters of the TBB-320-2 synchronous turbogenerator were used [23]:

$$\begin{bmatrix} i_d' \\ i_q' \\ i_f' \\ i_{rd}' \\ i_{rq}' \\ \omega' \\ \varphi' \end{bmatrix} = \begin{bmatrix} -3.05 i_d + 9.65\omega i_q - 0.22 i_f - 2.8 i_{rd} - 5.78 i_{rq} \\ -6.18\omega i_d - 1.95 i_q + 3.7\omega i_f + 3.7\omega i_{rd} - 5.99 i_{rq} \\ -0.98 i_d + 3.1\omega i_q - 1.92 i_f + 5.32 i_{rd} - 1.85\omega i_{rq} \\ -3.59 i_d + 11.35\omega i_q + 1.55 i_f - 10 i_{rd} - 6.79\omega i_{rq} \\ -8.64\omega i_d - 2.73 i_q + 5.18\omega i_f + 5.18\omega i_{rd} - 10 i_{rq} \\ 67.34 i_q i_d - 67.34 i_d i_q - 40.32 i_q i_f - 40.32 i_q i_{rd} + 40.32 i_d i_{rq} - 4.03\omega \\ \omega \end{bmatrix} +$$
$$+ \begin{bmatrix} -5.78 & 0 & 0.22 & 0 & 0 & 0 & 0 \\ 0 & -3.7 & 0 & 0 & 0 & 0 & 0 \\ -1.85 & 0 & 1.92 & 0 & 0 & 0 & 0 \\ -6.8 & 0 & -1.55 & 0 & 0 & 0 & 0 \\ 0 & -5.18 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 40.32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_f \\ 0 \\ 0 \\ M_{mx} \\ 0 \end{bmatrix}. \tag{20}$$

Discretization of the mathematical model of a synchronous generator (20) is carried out by difference operators of the implicit Euler method [24], implemented in the environment for dynamic modeling of technical systems SimInTech[1].

When conducting a computational experiment, the limitation parameter in the condition of inequality r is taken equal to 1. Feedback parameter $\gamma = -0.001$. The Lagrange multipliers for limiting the type of inequality and the "admissibility" parameter were selected experimentally and are equal to $\lambda_1 = -0.998$, $\lambda_1 = -1.0017$ and $\theta = 0.509$. The generalized vector of stabilization of program motions and controls has the form:

$$\mathbf{C}_{0k} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{14}.$$

Vector $\mathbf{C}_{0k}$ sets the stabilizing value of the synchronous generator frequency ω. The generalized vector of permissible program motions and controls is defined by the equality:

$$\mathbf{C}_{k} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1.001 & 0 & | & 0 & 0 & 0 & 0 & 0 & 0.03 & 0 \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{14},$$

and in accordance with (20) sets the permissible restrictions on the frequency ω and mechanical torque M_{mx}. To calculate the stresses u_d and u_q, an approximate load model was used [25].

Considering the structure of the linear manifold (16) and the vector-matrix Park–Gorev model for the TBB-320-2 synchronous turbogenerator (20), the calculated projector onto the linear manifold will take the form:

$$\mathbf{P}^0 =$$

$$= \begin{bmatrix}
0.45 & 0 & 0.31 & 0.37 & 0 & 0 & 0 & -0.07 & 0 & 0.09 & 0 & 0 & 0 & 0 \\
0 & 0.33 & 0 & 0 & 0.46 & 0 & 0 & 0 & -0.09 & 0 & 0 & 0 & 0 & 0 \\
0.31 & 0 & 0.66 & -0.18 & 0 & 0 & 0 & -0.04 & 0 & 0.3 & 0 & 0 & 0 & 0 \\
0.37 & 0 & -0.18 & 0.73 & 0 & 0 & 0 & -0.07 & 0 & -0.16 & 0 & 0 & 0 & 0 \\
0 & 0.46 & 0 & 0 & 0.65 & 0 & 0 & 0 & -0.12 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.07 & 0 & -0.04 & -0.07 & 0 & 0 & 0 & 0.01 & 0 & -0.01 & 0 & 0 & 0 & 0 \\
0 & -0.09 & 0 & 0 & -0.12 & 0 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 \\
0.09 & 0 & 0.3 & -0.16 & 0 & 0 & 0 & -0.01 & 0 & 0.15 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0006 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.$$

The projector onto the orthogonal complement of a linear manifold is defined by the equality:

[1] Dynamic Simulation Environment. Available at: https://simintech.ru

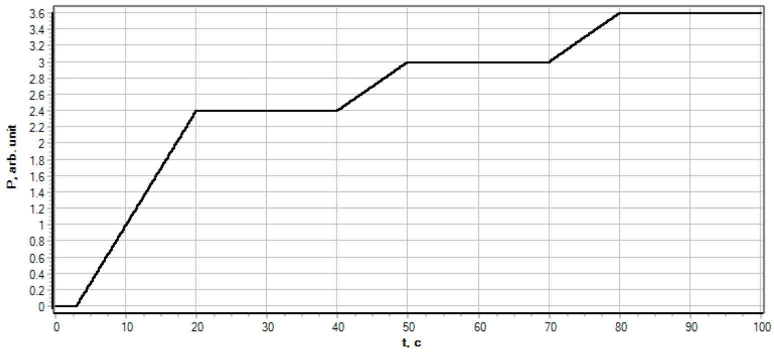

Fig. 1. Power change graph

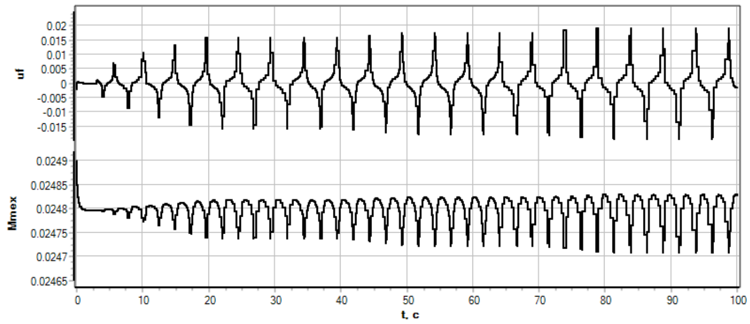

Fig. 2. Synthesized control actions of a synchronous generator
with restrictions specified by the vector of permissible program motions $\mathbf{C}_k$

$$\mathbf{P}^{+} = \begin{bmatrix} 0.55 & 0 & -0.31 & -0.37 & 0 & 0 & 0 \\ 0 & 0.67 & 0 & 0 & -0.46 & 0 & 0 \\ -0.31 & 0 & 0.34 & 0.18 & 0 & 0 & 0 \\ -0.37 & 0 & 0.18 & 0.27 & 0 & 0 & 0 \\ 0 & -0.46 & 0 & 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.006 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.07 & 0 & 0.04 & 0.07 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 & 0.12 & 0 & 0 \\ -0.09 & 0 & -0.3 & 0.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The graph of changes in the power of a synchronous generator, specified by a piecewise linear function at constant intervals, is shown in Fig. 1.

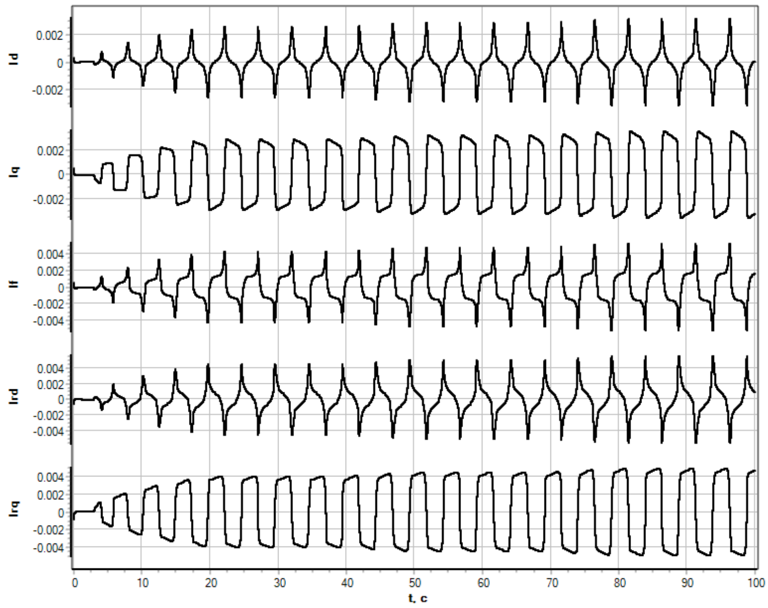

Fig. 3. Dynamics of synchronous generator currents

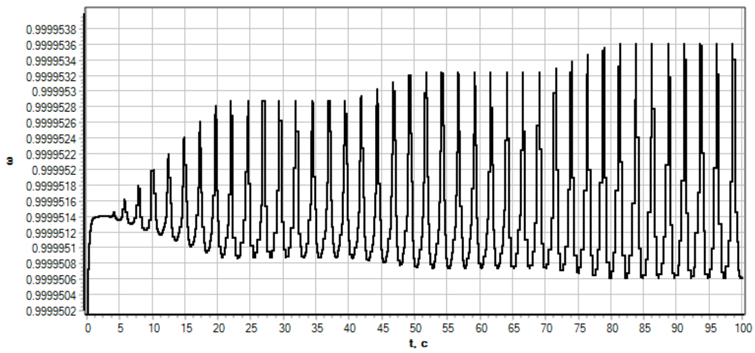

Fig. 4. Dynamics of the permissible change in the "frequency" ω of the synchronous generator, specified by the stabilizing program vector $\mathbf{C}_{k0}$ and the vector of permissible program motions $\mathbf{C}_k$

Limited by the vector of admissible program motions $\mathbf{C}_k$, the locally admissible controls u_f and M_{mx}, defined by operator (19), considering the change in power, have the form shown in Fig. 2.

The dynamics of synchronous generator currents considering the "load graph" (Fig. 1) is shown in Fig. 3.

The dynamics of the permissible change in the "frequency" ω of the synchronous generator, specified by the program stabilizing vector $\mathbf{C}_0$ and limited by the vector of permissible program motions $\mathbf{C}$, considering the "load schedule" (Fig. 1), is shown in Fig. 4.

From Fig. 4 the value of frequency ω, considering the change in power, does not exceed the limitations specified by the vector of permissible program motions $\mathbf{C}_k$.

Conclusions

The paper presents the formulation of generalized problems of stabilization of given program motions and provides the derivation of the projection operator for solving this class of problems.

A dynamic system operator was synthesized for generalized problems of stabilization of program motions with restrictions on the vectors of phase coordinates and controls.

Using the example of a synchronous generator model consisting of a system of bilinear differential equations with parameters corresponding to equations in the form of V.A. Venikov, the use of a synthesized projection operator for calculating controls and stabilizing the phase coordinates of a dynamic system, considering restrictions on coordinates and controls, is demonstrated.

Computational experiments were performed to confirm the correctness of the results obtained.

REFERENCES

1. **Vorob'ev E.I., Mikheev A.V., Morgunenko K.O.** Construction of program motions of relative manipulation mechanisms with three degrees of freedom. Journal of Machinery Manufacture and Reliability, 2019, no. 6, pp. 510–516. DOI: 10.3103/S1052618819060104

2. **Meshchanov A.S., Bikmurzin A.M.** Stabilizatsiya programmnoy posadki letatelnykh apparatov bez shassi na podvizhnuyu platformu v atmosfere pri neopredelennykh vozmeshcheniya. Ch. 1 [Stabilization of program landing of aircraft without landing gear on a moving platform in the atmosphere with uncertain compensation. Part 1.]. Vestnik tekhnologicheskogo universiteta, 2020, vol. 23, № 12, pp. 109–117. (rus)

3. **Alekseev A., Stukonog S.** An algorithm for constructing an optimal ship's path with elements of the ship's program motion systems on the course. Transp. Bus. Russ., 2022, № 2, pp. 217–221. DOI: 10.52375/20728689_2022_2_217/issn2072-8689

4. **Li T., Ren H., Li C.** Intelligent electric vehicle trajectory tracking control algorithm based on weight coefficient adaptive optimal control, Trans. Inst. Meas. Control, 2023, pp. 1–17. DOI: 10.1177/01423312221141591

5. **Yu H., Liang X., H J., F Y.** Adaptive Trajectory Tracking Control for the Quadrotor Aerial Transportation System Landing a Payload Onto the Mobile Platform, IEEE Trans. Ind. Informatics, 2023, pp. 1–12.

6. **Golubev A.Ye.** Stabilizatsiya programmnykh dvizheniy mekhanicheskikh sistem s uchetom ogranicheniy [Stabilization of program motions of mechanical systems considering constraints]. Izvestiya Rossiyskoy akademii nauk. Teoriya i sistemy upravleniya, 2023, № 4, pp. 153–167. (rus) DOI: 10.31857/S0002338823040054

7. **Sachan K., Padhi R.** Lyapunov function based output-constrained control of nonlinear Euler–Lagrange systems. 15th Intern. Conf. on Control, Automation, Robotics and Vision (ICARCV), IEEE, Singapore, November 18–21, 2018, pp. 686–691. DOI: 10.1109/ICARCV.2018.8581068

8. **Wang X., Xu J., Lv M., Zhang L., Zhao Z.** Barrier Lyapunov function-based fixed-time FTC for high-order nonlinear systems with predefined tracking accuracy. Nonlinear Dyn., 2022, vol. 110, no. 1, pp. 381–394. DOI: 10.1007/s11071-022-07627-9/issn0924-090X

9. **Golubev A.Ye.** Postroyeniye programmnykh dvizheniy mekhanicheskikh sistem s uchetom ogranicheniy pri pomoshchi mnogochlenov tretyego poryadka [Construction of program motions of mechanical systems considering restrictions using third-order polynomials]. Izvestiya Rossiyskoy akademii nauk. Teoriya i sistemy upravleniya, 2021, № 2, pp. 126–137. (rus) DOI: 10.31857/S0002338820060049/issn0002-3388

10. **Kozlov V.N., Efremov A.A.** Operators of bounded locally optimal controls for dynamic systems. Lect. Notes Networks Syst. Springer, 2020, vol. 95, pp. 140–145. DOI: 10.1007/978-3-030-34983-7_14

11. **Kozlov V.N., Efremov A.A.** Projection-operator optimization of controls of dynamic objects. J. Phys. Conf., Ser. IOP Publishing Ltd, 2021, vol. 1864, no. 1, pp. 140–145. DOI: 10.1088/1742-6596/1864/1/012076/issn17426596

12. **Kotina Ye.D., Ovsyannikov D.A.** Matematicheskaya model sovmestnoy optimizatsii programmnogo i vozmushchennykh dvizheniy v diskretnykh sistemakh [Mathematical model of joint optimization of program and perturbed motions in discrete systems]. Vestnik Sankt-Peterburgskogo universiteta. Prikladnaya matematika. Informatika. Protsessy upravleniya, 2021, vol. 17, no. 2, pp. 213–224. (rus) DOI: 10.21638/11701/spbu10.2021.210

13. **Andreev A.S., Kolegova L.V.** PID controllers with delay in a problem of stabilization of robotic manipulators' desired motions. Zhurnal Sredn. Mat. Obs., 2022, vol. 34, no. 3, pp. 267–279. DOI: 10.15507/2079-6900.24.202203.267-279/issn20796900

14. **Aleksandrov O.V., Kozik A.A.** Minimaksnaya optimizatsiya parametrov stabilizatsii programmnogo poleta [Minimax optimization of program flight stabilization parameters] // Vestnik Moskovskogo universiteta. Seriya 1. Matematika. Mekhanika, 2019, vol. 3, pp. 45–49. (rus)

15. **Polyanina A.S.** Synthesis of stabilizing control in the generation of multi-body systems program motions, Sovremennyye naukoyemkiye tekhnologii [Modern High Technology], 2020, no. 2, pp. 45–50.

16. **Kozlov V.N.** Proyektsionnyy metod sinteza ogranichennykh optimalnykh upravleniy dinamicheskikh sistem energetiki [Projection method for the synthesis of limited optimal controls of dynamic energy systems]. SPb.: Izd-vo Politekhn. un-ta., 2019. 161 pp. (rus)

17. **Petrov N.N.** Ob upravlyayemosti avtonomnykh sistem [On the controllability of autonomous systems]. Differentsialnyye uravneniya, 1968, vol. 4, no. 4, pp. 606–617. (rus)

18. **Kalman R.E.** Discussion: "On the Existence of Optimal Controls" (Markus, L., and Lee, E.B., 1962, ASME J. Basic Eng., 84, pp. 13–20). J. Fluids Eng., 1962, vol. 84. Available: https://asmedigitalcollection.asme.org/fluidsengineering/article/84/1/21/424598/Discussion-On-the-Existence-of-Optimal-Controls (accessed 30 May 2023).

19. **Avakov Ye.R., Magaril-Ilyayev G.G**. Lokalnaya upravlyayemost i optimalnost [Local controllability and optimality]. Matematicheskiy sbornik, 2021, vol. 212, no. 7, pp. 3–38. (rus) DOI: 0.4213/sm9434

20. **Avakov Ye.R., Magaril-Ilyayev G.G.** Upravlyayemost i neobkhodimyye usloviya optimalnosti vtorogo poryadka [Controllability and necessary conditions for second-order optimality]. Matematicheskiy sbornik, 2019, vol. 210, no. 1, pp. 3–26. (rus) DOI: 10.4213/sm9013

21. **Kozlov V.N., Ryabov G.A., Yefremov A.A., Trosko I.U.** Strukturno-invariantnyye uravneniya energoobyedineniy dlya sinteza sistem ogranicheniya peretokov i regulirovaniya napryazheniya [Structural-invariant equations of power interconnections for the synthesis of flow limitation and voltage regulation systems]. XIX International Scientific and Practical Conference. "System analysis in design and management", St. Petersburg, July 1–3, 2015, pp. 207–217. (rus)

22. **Veretennikov L.P.** Klassifikatsiya uravneniy Goreva–Parka [Classification of Gorev-Park equations]. Elektrichestvo, 1959, no. 11, pp. 13–20. (rus)

23. **Neklepayev B.N.** Elektricheskaya chast elektrostantsiy i podstantsiy. 2-ye izd [Electrical part of power plants and substations. 2nd ed]. M., Energoatomizdat, 1986. (rus)

24. **Skvortsov L.M.** Chislennoye resheniye obyknovennykh differentsialnykh i differentsialno-algebraicheskikh uravneniy [Numerical solution of ordinary differential and differential-algebraic equations]. M.: DMK Press, 2023. (rus)

25. **Leonov G.A., Kuznetsov N.V., Andriyevskiy B.R., Yuldashev M.V., Yuldashev R.V.** Matematicheskoye modelirovaniye perekhodnykh protsessov gidroagregata Sayano-Shushenskoy GES [Mathematical modeling of transient processes of the hydraulic unit of the Sayano-Shushenskaya HPP]. Differentsialnyye uravneniya i protsessy upravleniya, 2018, vol. 4, no. 4. Available: https://diffjournal.spbu.ru/RU/numbers/2018.4/article.1.5.html (accessed: 25.05.2023). (rus)

INFORMATION ABOUT AUTHOR / СВЕДЕНИЯ ОБ АВТОРЕ

Efremov Artem A.
Ефремов Артем Александрович
E-mail: Artem.Efremov@spbstu.ru
ORCID: https://orcid.org/0000-0002-0224-2412

Submitted: 05.09.2023; Approved: 14.11.2023; Accepted: 15.12.2023.
Поступила: 05.09.2023; Одобрена: 14.11.2023; Принята: 15.12.2023.