Magazine of Civil Engineering. 2024. 17(6). Article No. 13001

Magazine of Civil Engineering

ISSN
2712-8172

journal homepage: http://engstroy.spbstu.ru/

Research article

UDC 624.04:531.391.3

DOI: 10.34910/MCE.130.1

Simplified method for estimating the first natural frequency of a symmetric arch truss

M.N. Kirsanov ✉ **, *C.L. Luong***

National Research University "Moscow Power Engineering Institute", Moscow, Russian Federation

✉ *c216@ya.ru*

Keywords: Dunkerley method, Maple, analytical solution, lowest frequency, regular trusses, spectra of natural frequencies

Abstract. A planar model of a symmetric statically determinate arch truss is considered. The mass of the truss is evenly distributed over its nodes. The Maxwell–Mohr formula, assuming that the truss rods have the same cross-section, allows one to determine the stiffness matrix of a given structure. The Dunkerley method and a variant of the Rayleigh method are proposed to be used to estimate the first natural frequency of the truss. The mass of the structure is conditionally concentrated in its nodes. Only small vertical oscillations are considered. The generalization of a number of solutions for trusses with different number of panels to the general case is carried out by induction. A simplified method for calculating the first frequency based on the Rayleigh method is proposed. To simplify the sums of partial frequencies and squared frequencies included in the Rayleigh solution, the area of the curve limiting the frequency values is replaced by its approximate value, which is calculated by the triangle area formula. The solution includes the value of the maximum deflection of the truss from the action of a distributed load. The results obtained by different analytical methods are compared with the results obtained by the numerical method. All transformations were carried out analytically using the Maple computer mathematical system. The results showed that with an increase in the number of panels, the accuracy of the Dunkerley analytical estimate increases, and the proposed method changes insignificantly. Spectral constants and frequency safety regions were found in the spectra of a family of trusses of different orders.

Citation: Kirsanov, M.N., Luong, C.L. Simplified method for estimating the first natural frequency of a symmetric arch truss. Magazine of Civil Engineering. 2024. 17(6). Article no. 13001. DOI: 10.34910/MCE.130.1

1. *Introduction*

The characteristic properties of truss racks are low material consumption, ease of installation, ease of maintenance and convenient transportation. Therefore, this type of structure is widely applied in construction, mechanical engineering, aviation technology, shipbuilding, etc. One of the main tasks, when considering a structure in dynamics, is the calculation of the natural frequency of oscillation. The first, the lowest frequency is of particular importance to practice. To calculate the natural frequency of the truss oscillation in the general case, numerical methods are used based on the finite element method [1–3]. Usually only the upper or lower estimates of the first frequency are of interest, the Dunkerley (for the lower limit) or Rayleigh (for the upper limit) methods are used [4, 5]. A two-sided analytical estimate of the fundamental natural frequency of the spatial contour coverage was obtained in [6]. The formulas obtained based on the Dunkerley method are relatively simple, but in some cases, the accuracy of this method is not satisfactory. Usually the accuracy of the Dunkerley method ranges from 10 % to 45 %. General problems with conventional statically determinate trusses have been presented in [7, 8]. The formula for the lower estimate of natural oscillations of a planar regular externally statically indeterminate beam truss with a rectilinear upper chord was obtained in [9]. The existence of conventional statically determinate truss systems was first announced in [10, 11]. Algorithms for calculating normal truss deflection based on inductive method using the ability of Maple notation have been used in [12, 13]. The lower limit of the first frequency by the Dunkerley method for a regular planar beam truss was obtained in [14]. In [15], a formula was derived for the dependence of the first frequency of natural vibrations of two span trusses on the number of panels. The dependence of the truss spatial deflection on the number of slabs in the Maple system is found in [16, 17]. Analytical calculations of elements of building structures using the method of superposition of analysis and expansions into series in the Maple computer mathematics system were used in [18, 19]. The spectrum of natural frequencies of the spatial model of a hexagonal rod prism and an analytical estimate of the first frequency were obtained in [20]. The Galerkin method for the analysis of non-linear parametric oscillations of plates was used in [21]. Vibrations and stability of a reinforced rectangular plate were studied in [22].

The Rayleigh method usually gives results with high accuracy. However, this method introduces overly complex formulas and in many cases may not provide a solution. In this paper, a variant of the Dunkerley method with relatively simple coefficients with good accuracy is proposed. The essence of this method is that if the potential energy is calculated from the sum of the potential energies of all the masses, then the sum of the kinetic energies of the masses is replaced by an approximate expression calculated from the maximum kinetic energy of one of the nodes. Refined versions of the Dunkerley method are considered in [23, 24].

2. *Methods*

2.1. *The Truss Scheme*

The truss under consideration is a symmetrical planar truss consisting of interconnected trusses with different number of slabs and different slopes (Fig. 1). The truss has a movable articulated support and a fixed articulation post. All truss posts have the same height h. The slope of the side (lower) parts of the truss with m panels is determined by the ratio b/a, the slope of the middle parts (n panels in each part) is c/a. Truss length $L = a(m+n)$. In a truss of $\eta = 8(m+n)+4$ rods, of which $4(m+n)$ are in the upper and lower chords, $4(m+n+1)$ rods form a lattice, and three rods are support rods. Using two balanced equations for each node, a closed system of equations for the forces in the rods and the reactions of the supports can be obtained. It is necessary to solve the problem of oscillation frequency in an analytical form.

It is assumed that the mass of the truss is concentrated in its nodes, oscillations occur along the vertical axis y.

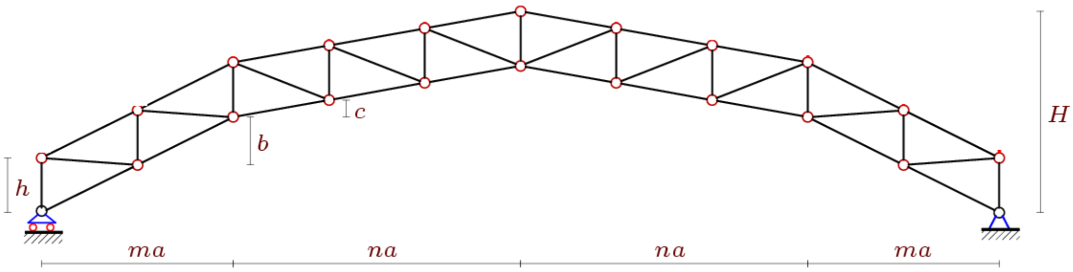

Figure 1. Truss, *n* = 3, *m* = 2.

The coordinates of the nodes and the connection order of the bars in the structure are entered into the program like problems in discrete mathematics, the edges and vertices of the graph are specified. Fig. 2 shows the numbering order of the bars and the nods, nods and special feature bars are marked with formulas. The program to input coordinates into the Maple system has the following form:

```
> for i to 2*n+2*m+1 do
> x[i]:=a*(i-1);
> x[i+2*n+2*m+1]:=x[i];
> end:
> for i to m+1 do
> y[i]:=b*(i-1);
```

```
> y[i+m+2*n]:=m*b-b*i+b:
> end:
> for i to n do y[i+m+1]:=m*b+c*i; end:
> for i to n-1 do y[i+m+1+n]:=m*b+n*c-c*i; end:
> x[m3-2]:=0: y[m3-2]:=-4:
> x[m3-1]:=x[2*n+2*m+1]: y[m3-1]:=-4:
> x[m3]:= x[2*n+2*m+1]+3: y[m3]:=0:
> for i to 2*n+2*m+1 do
> y[i+2*n+2*m+1]:=y[i]+h:
> end:
```

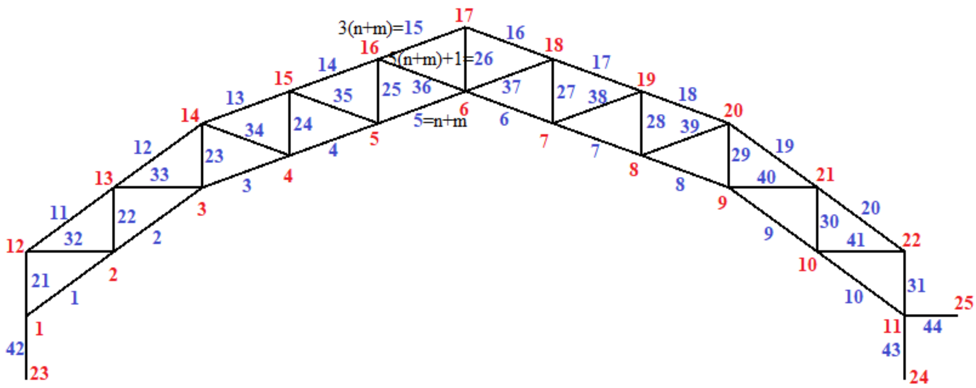

Figure 2. Numbering of bars and nodes, *m* = 3, *n* = 2. Supports are modeled by bars.

2.2. *Dunkerley Method*

The lowest estimate of the frequency of the first oscillations is obtained by the Dunkerley formula:

$$\omega_D^{-2} = \sum_{i=1}^{K} \omega_i^{-2}, \tag{1}$$

where ω_p are partial frequencies calculated for each mass separately, $K = 4n + 4m + 2$ is the number of degrees of freedom of the cargo system in the truss nodes.

To calculate the partial frequencies, we compose the mass μ motion equation:

$$\mu \ddot{y}_p + D_p y_p = 0, \; p = 1, 2, \; \ldots, \; K. \tag{2}$$

The stiffness coefficient D_p, inverse to the compliance coefficient, is calculated using the Maxwell–Mohr formula:

$$\delta_p = 1/D_p = \sum_{\alpha=1}^{\eta-3} \left(S_\alpha^{(p)}\right)^2 l_\alpha / (EF). \tag{3}$$

From the Dunkerley formula for $y_p = A_p \sin(\omega t + \varphi)$, $\omega_p = \sqrt{D_p/\mu}$ follows. Hence, we have the expression for the Dunkerley frequency:

$$\omega_D^{-2} = \mu \sum_{p=1}^{K} \delta_p = \mu \Delta_n. \tag{4}$$

To obtain the dependence of the solution on the number of panels n in the crossbar and the number of panels m in the supporting parts, double induction is required. To do this, first, for $m = 1$, according to the solutions for a sequence of trusses with $n = 1, 2, 3 \ldots$, a general formula is obtained, then the same procedure is repeated for $m = 2,3,4\ldots$. A number of solutions obtained for various m are generalized to an arbitrary case. For $m = 1$, we have the following solution for the amount corresponding to vertical oscillations:

$$n = 1: \Delta_{(1,1)} = \left(344a^3 + 842h^3 + 984d^3 + 163g^3\right) / \left(36EFh^2\right),$$
$$n = 2: \Delta_{(1,2)} = \left(116a^3 + 346h^3 + 340d^3 + 153g^3\right) / \left(8EFh^2\right),$$
$$n = 3: \Delta_{(1,3)} = \left(392a^3 + 1470h^3 + 1160d^3 + 1027g^3\right) / \left(20EFh^2\right),$$
$$n = 4: \Delta_{(1,4)} = \left(446a^3 + 2093h^3 + 1326d^3 + 1995g^3\right) / \left(18EFh^2\right),$$
$$n = 5: \Delta_{(1,5)} = \left(840a^3 + 4862h^3 + 2504d^3 + 5861g^3\right) / \left(28EFh^2\right),$$
$$\ldots$$

Using the Maple system operators, the common terms of the resulting sequence of coefficients are calculated for the degrees of truss sizes: a^3, h^3, d^3, g^3, where $b = h$, $c = h/2$, $d = \sqrt{a^2 + h^2}$, $g = \sqrt{4a^2 + h^2}$. It follows from the formula that the dependence of the deflection on the number of panels and the dimensions of the structure has the form:

$$\Delta_n = \left(\mathrm{C}_1 a^3 + \mathrm{C}_2 h^3 + \mathrm{C}_3 d^3 + \mathrm{C}_4 g^3\right) / \left(h^2 \mathrm{EF}\right). \quad (5)$$

The coefficients have the following formula:

$$C_{1,1} = \left(8n^2 + 10n + 3\right) / 3,$$
$$C_{1,2} = \left(4n^4 + 32n^3 + 50n^2 + 82n + 45\right) / 12,$$
$$C_{1,3} = \left(8n^2 + 10n + 3\right) / 3,$$
$$C_{1,4} = \left(16n^5 + 80n^4 + 200n^3 + 160n^2 + 39n\right) / 180.$$

Similarly, when $m = 2$, the coefficients have the following formula:

$$C_{2,1} = \left(16n^2 + 40n + 30\right) / 3,$$
$$C_{2,2} = \left(2n^4 + 24n^3 + 81n^2 + 176n + 138\right) / 6,$$
$$C_{2,3} = 16n^2 + 40n + 26,$$
$$C_{2,4} = \left(16n^5 + 160n^4 + 600n^3 + 10400n^2 + 549n\right) / 180.$$

Carry out the same process, when $m = 3,4, \ldots$, until $m = 12$, the general formula for the coefficients according to (5) is found and has the following form:

$$C_1 = \left(4m^3 + 10m^2 n + \left(8n^2 - 1\right)m\right) \Big/ 3,$$

$$C_2 = \left(6m^4 + 8(3n+2)m^3 + \left(32n^2 + 32n + 3\right)m^2 + \right.$$

$$\left. +2\left(8n^3 + 8n^2 + 3n + 10\right)m + 4n^4 + 16n^3 + 2n^2 + 20n\right) \Big/ 12,$$

$$C_3 = m\left(32m^4 + 100m^3 n + 20\left(4n^2 + 1\right)m^2 + 50mn + 40n^2 - 7\right) \Big/ 45,$$

$$C_4 = \left(30m^4 n + 120m^3 n^2 + 20\left(8n^2 + 1\right)nm^2 + 40\left(2n^2 + 1\right)mn^2 + 16n^5 + 40n^3 - 11n\right) \Big/ 180.$$

The final expression for the dependence of the lowest limit of the first frequency on the geometrical parameters of the structure, including the number of plates in the crossbar and the load-bearing side truss, has the following form:

$$\omega_D = h\sqrt{\frac{EF}{\mu\left(C_1 a^3 + C_2 h^3 + C_3 d^3 + C_4 g^3\right)}}. \quad (6)$$

The result can be checked by doing it in reverse order, that is first carry out induction on m, then on n.

2.3. A Simplified Version of the Calculation of the First Frequency

When using the Rayleigh method to find the upper estimate of the fundamental frequency, the biggest difficulty in deducing the dependence of the oscillator frequency on the number of plates is the denominator. The essence of the Rayleigh method is based on the law of conservation of energy. During each period of the harmonic oscillation, there is a conversion of potential energy to kinetic energy and vice versa. The transformation can be expressed as the following equation:

$$\omega_R^2 = \sum_{i=1}^{K} \tilde{u}_i \Big/ \sum_{i=1}^{K} \mu \tilde{u}_i^2, \quad (7)$$

where $\tilde{u}_i = u_i / P = \sum_{\alpha=1}^{\eta-3} \tilde{S}_\alpha^{(P)} \tilde{S}_\alpha^{(i)} l_\alpha / (EF)$ – displacement amplitude of mass μ at a node i under the effect of load distributed on the nodes of the truss, is called the value of vertical nodal force P. Notations are used: $S_\alpha^{(P)}$ is the force in the rod $\alpha = 1, \ldots, \eta - 3$ from the action of the load P, uniformly distributed over the nodes: $\tilde{S}_\alpha^{(P)} = S_\alpha^{(P)} / P$. Three support rods in this arrangement are assumed to be rigid. The proposed method for the simplified calculation of the Rayleigh frequency consists in replacing the sums in (7) by half the value of the product of the truss-averaged value of the partial frequency and the number of degrees of freedom. This is illustrated in Figs. 3 and 4. The figure for a specific truss shows the distribution of partial frequencies over the nodes of the truss. In Fig. 4, on the horizontal axis, not the number of marked nodes but the number of the list of u^* values ranked in ascending order. The sum $\sum_{i=1}^{N} \tilde{u}_i$ can be interpreted as the area bounded by the distribution curve. This area can be calculated more simply by sorting the frequencies in ascending order and calculating the area using the triangle area formula $\sum_{i=1}^{N} \tilde{u}_i = K u_* / 2$ (Fig. 4). Here u_* is the maximum deflection of the truss from the action of a single force distributed over all nodes. From (7) a formula is obtained for an approximate expression of the frequency based on the simplified Rayleigh method:

$$\omega_*^2 = \sum_{i=1}^{K} \tilde{u}_i \Big/ \sum_{i=1}^{K} \mu \tilde{u}_i^2 = 2\Big/\left(\mu K u_*\right). \tag{8}$$

The value of the maximum deflection is obtained by the formula $u_* = \sum_{\alpha=1}^{\eta-3} \tilde{S}_\alpha^{(P)} \tilde{S}_\alpha^{(*)} l_\alpha / (EF),$ where $\tilde{S}_\alpha^{(*)}$ is the force in the rod with number α from the action of a unit vertical force on the node, the deflection of which is tentatively estimated as maximum. This value for an arbitrary number of panels has the form:

$$u_* = \left(C_1 a^3 + C_2 h^3 + C_3 d^3 + C_4 g^3\right)\Big/\left(h^2 EF\right). \tag{9}$$

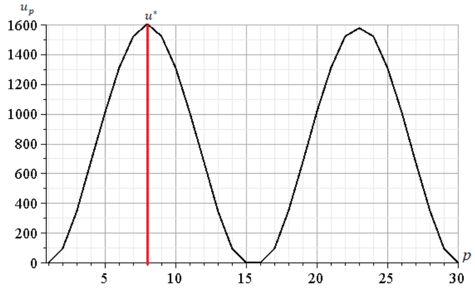

Figure 3. The dependence of the coefficient u_p on the number of the truss node for m = 4, n = 3.

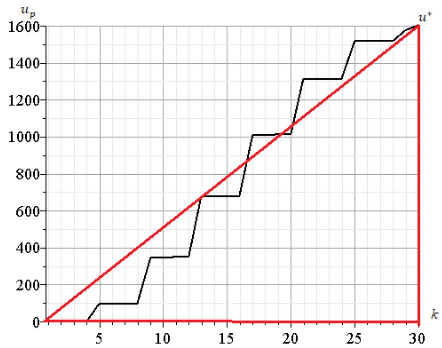

Figure 4. Coefficients u_p in ascending order for m = 4, n = 3.

Calculations testing the value of u_p for different values of m and n show that the maximum value of u_* falls on a node of order $m+n+1$.

For $m=1$, the calculations give the following sequence:

$$n=1: u_* = \left(240a^3 + 2044h^3 + 19380d^3 + 633g^3\right) \Big/ \left(2EFh^2\right),$$
$$n=2: u_* = \left(560a^3 + 5096h^3 + 44120d^3 + 3139g^3\right) \Big/ \left(4EFh^2\right),$$
$$n=3: u_* = \left(320a^3 + 3128h^3 + 24740d^3 + 2867g^3\right) \Big/ \left(2EFh^2\right),$$
$$n=4: u_* = \left(180a^3 + 1895h^3 + 13710d^3 + 2294g^3\right) \Big/ \left(EFh^2\right),$$
$$n=5: u_* = \left(200a^3 + 2270h^3 + 15050d^3 + 3400g^3\right) \Big/ \left(EFh^2\right),$$
$$\ldots$$

Formulas for u_* are obtained similarly for $m = 2, 3, \ldots$. The generalization of the series of these formulas to an arbitrary number of panels m gives the following final formulas:

$$\begin{aligned} C_1 &= m^2 + 2mn, \\ C_2 &= \left(3m^3 + 2n^3 + 2\left(4n^2+1\right)m^2 + 4n^2 + 2\left(3n^2+1\right)m + 2n\right) \Big/ 4, \\ C_3 &= m\left(5m^3 + 8m^2n + m + 4n\right) \Big/ 6, \\ C_4 &= \left(12m^3n + 30m^2n^2 + 2n\left(10n^2-1\right)m + 4n^4 + 7n^2\right) \Big/ 48. \end{aligned} \tag{10}$$

The sequence of coefficients with this approach were relatively complex, for problems like this, there are many advantages to using the *rsolve* and *rgf_findrecur* operators or the Maple operator to generalize to sequences.

Thus, the approximate value of the first frequency (8) is:

$$\omega_* = h\sqrt{\frac{EF}{\mu(2n+2m+1)\left(C_1a^3 + C_2h^3 + C_3d^3 + C_4g^3\right)}} \tag{11}$$

with coefficients (10).

This solution is obtained under the assumption that the three rods modeling the left movable and right fixed supports are rigid. This is why the summation in the Maxwell–Mohr formulas is carried out up to $\eta - 3$. However, this solution can be refined by conditionally accepting that all support rods have a length h and a modulus of elasticity E. Only the coefficient C_2 in (10) will have a small difference:

$$C_2 = \left(3m^3 + 2n^3 + 2\left(4n^2+1\right)m^2 + 4n^2 + 2\left(3n^2+5\right)m + 10n + 4\right) \Big/ 4.$$

Considering that the new value of the coefficient C_2 is greater than the initial one, and this coefficient is in the denominator of formula (11), the solution for the natural frequency, taking into account the elasticity of the supports, will be slightly smaller. This change has almost no effect on the magnitude of the natural frequency.

When $m = n$, the proposed simplified solution (11) has the form:

$$\omega_* = \\ = 4h\sqrt{\frac{3EF}{\mu n\left(\left(67g^3 + 104d^3\right)n^3 + 228h^3n^2 + \left(72h^3 + 144a^3 + 40d^3 + 5g^3\right)n + 48h^3\right)}}. \tag{12}$$

3. *Results and Discussion*

3.1. *Numerical Example*

To illustrate the dependence of the natural oscillator frequency on the number of panels found by the Dunkerley method, the new method has been proposed, and to evaluate the accuracy of the analytical solution, the first frequency from the frequency of the device natural motion of the structure will be found. Using the special operator Eigenvalues from the LinearAlgebra package, the Maple system finds eigenvalues and matrix vectors. The truss under consideration corresponds to the dimensions a = 5 m, h = 4 m. The cross-sectional area of the grids and supports is assumed to be the same: F = 4 cm^2. Elastic modulus of steel E = 2·10^5 MPa, mass at nodes μ = 150 kg. Fig. 5 shows the dependence on the number of panels of the frequency ω_D according to the standard Dunkerley formula (6), the frequency ω_* according to the simplified formula (11), and the numerically found value of the first frequency ω_1 of the spectrum of a system with K degrees of freedom.

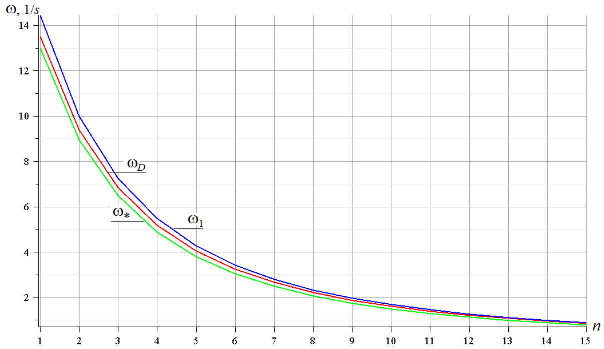

Figure 5. Comparison of analytical solutions with numerical, *m* = 3. The first oscillation frequency of the truss ω_1 and its lower estimate ω_D according to Dunkerley, ω_* is the lower estimate according to the simplified formula.

The Dunkerley method, its simplified version, and the numerical method for values do not differ too much. At the same time, over the entire range of values of the number of panels, as expected, the Dunkerley estimates are less than the first frequency of the spectrum obtained numerically, taking into account all degrees of freedom. From Fig. 5 it can be concluded that the more cells in the crossbar, the smaller the error obtained. The lower the truss height, the smaller the error.

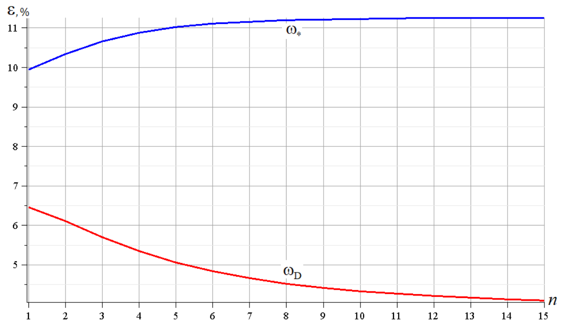

Figure 6. Comparative error of methods, *m* = 3.

Fig. 6 shows how the error of approximate solutions' changes as the number of tables changes. Values of relative error are introduced: $\varepsilon = \left(\omega_1 - \omega_D\right)/\omega_1$. With increasing truss order, the error of the Dunkerley method decreases and reaches a value of about 1 % with $k > 12.$ In contrast, with the proposed method, with $k < 8,$ the accuracy is gradually reduced, when $k > 8,$ this error does not change at 12 %. In summary, for the selected design features, this value should not exceed 12 % for the proposed method and 6.5 % at most for the Dunkerley method.

3.2. *The Level Lines of the Value of the First Frequency*

The level lines of the value of the first frequency in the *m-n* axes, with increasing frequency, thicken (Fig. 7). The figure is constructed for the a = 5 m, h = 10 m and previous values of the mass, modulus of elasticity and stiffness of the rods. As the height of the truss decreases, the curves in this figure straighten.

Figure 7. First frequency level lines.
1: ω = 4*s*$^{-1}$; 2: ω = 6*s*$^{-1}$; 3: ω = 8*s*$^{-1}$; 4: ω = 10*s*$^{-1}$; 5: ω = 12*s*$^{-1}$.

The curves thicken with increasing frequency of natural oscillations. Outwardly, this resembles the Doppler effect for a moving source of oscillations, although, of course, it has a completely different nature.

3.3. Natural Frequency Spectra of Regular Trusses

Usually, high vibration frequencies are not used in engineering calculations, except in resonance case studies. The natural frequency of vibration caused by the operation of different equipment may coincide with the natural frequency of the structure's vibration. The natural frequency spectrum is used, for example, to monitor the dynamics of railway bridges [25, 26]. The frequency spectrum is also required for multi-scale modeling and design of sandwich metastructures with a lattice core to suppress broadband low-frequency vibrations [27].

The analytical method cannot account for these oscillation frequencies, but the well-tuned mathematical engine in numerical mode gives an interesting picture of the spectral set of conventional systems. In Fig. 8, twelve curves connect the points corresponding to the oscillation frequency of the truss of order $n = 1, 2, \ldots 12$. Each curve corresponds to a given truss, and the coordinates of the points on it are the frequencies. The horizontal axis shows the number of natural frequencies in the ordered spectrum.

Some features of the frequency distribution are noted here. All spectra are characterized by significant frequency jumps, and all higher frequencies of the spectrum are multiples. For the accepted truss sizes a = 5 m, h = 4 m, F = 4 cm^2, there appear to be frequencies of $\omega_1 = 250s^{-1}$, $\omega_2 = 520s^{-1}$, $\omega_3 = 600s^{-1}$ (spectral constant) constant for rigs of different orders. The presence of this regularity allows one to predict several truss frequencies with large regularity as calculated by a truss with a small number of plates, which greatly simplifies the solution. The first frequency has a fairly accurate analytical estimate, which is found by the Dunkerley formula. A similar spectrum with multiple higher frequencies was obtained for a planar regular truss that allows kinematic changes for a certain number of panels [28].

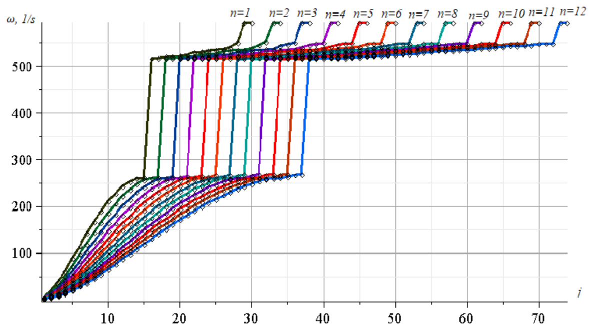

Figure 8. Spectra of regular trusses, *m* = 6.

It should also be noted the area of frequency safety, enclosed between frequencies $\omega_1 = 250 s^{-1}$ and $\omega_2 = 520 s^{-1}$ in the spectral picture. This means that placing on the truss some source of external excitation with a natural frequency from this gap guarantees the safety of the system from the occurrence of resonance. It is also characteristic that this area does not depend on the order of the truss (in this case, the number of panels n).

4. Conclusion

A new scheme for a statically determinate symmetric truss is proposed. The design under consideration could be used in the design of a bridge or other transport structure. The Dunkerley method and the proposed method allow one to obtain an analytical solution to estimate the first natural frequency of oscillation of a conventional truss. One of the objectives of the study was to obtain an analytical solution in the form of a relatively compact formula for the frequency of natural oscillations. Therefore, fairly strong assumptions were introduced into the model of the object: the oscillations of the nodes were assumed to be vertical, and neither physical nor geometric nonlinearities were taken into account. In favor of the latter, it should be noted that the oscillations were assumed to be small, where various nonlinearities are irrelevant.

Any other conventional mechanical system with a large number of degrees of freedom can also use this proposed method. The following main conclusions can be drawn from the analyzed results:

1. The coefficients obtained from the proposed method are much simpler than those obtained from the Dunkerley method, and the accuracy of these two methods is almost the same.
2. With an increase in the number of panels in the truss structure and a sufficiently large number of crossbars, the accuracy of the proposed method remains unchanged, while the accuracy of the Dunkerley method increases.
3. In the frequency spectrum of a family of regular trusses, spectral constants and a frequency safety region are found.

References

1. Macareno, L.M., Agirrebeitia, J., Angulo, C., Avilés, R. FEM subsystem replacement techniques for strength problems in variable geometry trusses. Finite Elements in Analysis and Design. 2008. 44(6–7). Pp. 346–357. DOI: 10.1016/j.finel.2007.12.003
2. Vatin, N., Havula, J., Martikainen, L., Sinelnikov, A.S., Orlova, A.V., Salamakhin, S.V. Thin-walled cross-sections and their joints: Tests and FEM-modelling. Advanced Materials Research. 2014. 945–949. Pp. 1211–1215. DOI: 10.4028/www.scientific.net/AMR.945-949.1211
3. Ignatyev, A.V., Ignatyev, V.A. On the Efficiency of the Finite Element Method in the Form of the Classical Mixed Method. Procedia Engineering. 2016. 150. Pp. 1760–1765. DOI: 10.1016/J.PROENG.2016.07.167
4. Kirsanov, M.N., Luong, C. Frequency spectrum of natural oscillations of the spatial structure of the rod pyramid. Construction of Unique Buildings and Structures. 2023. 107(2). Article no. 10702. DOI: 10.4123/CUBS.107.2
5. Komerzan, E.V., Maslov, A.N. Estimation of the L-shaped spatial truss fundamental frequency oscillations. Structural Mechanics and Structures. 2023. 37(2). Pp. 35–45. DOI: 10.36622/VSTU.2023.37.2.004
6. Kirsanov, M.N., Luong, C. Natural frequency spectra of spatial structure. Construction of Unique Buildings and Structures. 2023. 106(1). Article no. 10604. DOI: 10.4123/CUBS.106.04
7. Kaveh, A., Rahami, H., Shojaei, I. Swift Analysis of Civil Engineering Structures Using Graph Theory Methods. Springer, Cham, 2020. 302 p. DOI: 10.1007/978-3-030-45549-1
8. Ignatiev, V.A., Ignatiev, A.V. Metod konechnykh elementov v forme klassicheskogo smeshannogo metoda stroitel'noi mekhaniki (teoriia, matematicheskie modeli i algoritmy) [Finite element method in the form of a classical mixed method of structural mechanics (theory, mathematical models and algorithms)]. Moscow: ASV Publ., 2022. 306 p. DOI: 10.12731/978-5-4323-0431-5
9. Shchigol, E.D. The formula for the lower estimate of the natural oscillations of a flat regular girder truss with a rectilinear upper belt. Structural Mechanics and Structures. 2023. 37(2). Pp. 46–53. DOI: 10.36622/VSTU.2023.37.2.005
10. Hutchinson, R.G., Fleck, N.A. The structural performance of the periodic truss. Journal of the Mechanics and Physics of Solids. 2006. 54(4). Pp. 756–782. DOI: 10.1016/j.jmps.2005.10.008
11. Hutchinson, R.G., Fleck, N.A. Microarchitectured cellular solids – the hunt for statically determinate periodic trusses. Zeitschrift für Angewandte Mathematik und Mechanik. 2005. 85(9). Pp. 607–617. DOI: 10.1002/zamm.200410208
12. Ivanitskii, A.D. Formulas for calculating deformations of a planar frame. Structural Mechanics and Structures. 2022. 34(3). Pp. 90–98. DOI: 10.36622/VSTU.2022.34.3.007
13. Komerzan, E.V., Lushnov, N.A., Osipova, T.S. Analytical calculation of the deflection of a planar truss with an arbitrary number of panels. Structural Mechanics and Structures. 2022. 33(2). Pp. 17–25. DOI: 10.36622/VSTU.2022.33.2.002
14. Komerzan, E.V., Maslov, A.N. Analytical evaluation of a regular truss natural oscillations fundamental frequency. Structural Mechanics and Structures. 2023. 37(2). Pp. 17–26. DOI: 10.36622/VSTU.2023.37.2.002

15. Petrenko, V.F. The natural frequency of a two-span truss. AlfaBuild. 2021. 20. Article no. 2001. DOI: 10.34910/ALF.20.1
16. Astakhov, S.V., Kirsanov, M.N., Vorobyev, O.V. Formulas for Calculating Deformations of Power Line Supports. IOP Conference Series: Earth and Environmental Science. 2022. 988(4). Article no. 052058. DOI: 10.1088/1755-1315/988/5/052008
17. Komerzan, E., Sviridenko, O. Static deformations of the truss of a composite spatial frame. Analytical solutions. Structural mechanics and structures. 2022. 35(4). Pp. 40–48. DOI: 10.36622/VSTU.2022.35.4.005
18. Matrosov, A.V. Computational Peculiarities of the Method of Initial Functions. Computational Science and Its Applications – ICCSA 2019. Springer, Cham, 2019. 11619. Pp. 37–51. DOI: 10.1007/978-3-030-24289-3_4
19. Goloskokov, D.P. Analyzing simply supported plates using Maple system. 2014 International Conference on Computer Technologies in Physical and Engineering Applications (ICCTPEA). IEEE, 2014. Pp. 55–56. DOI: 10.1109/ICCTPEA.2014.6893273
20. Kirsanov, M. Model of a hexagonal prismatic truss. Oscillation frequency spectrum. Construction of Unique Buildings and Structures. 2023. 106(1). Article no. 10601. DOI: 10.4123/CUBS.106.1
21. Khodzhaev, D.A., Abdikarimov, R.A., Amabili, M., Normuminov, B.A. Parametric oscillations of a viscous-elastic orthotropic shell of variable thickness. Magazine of Civil Engineering. 2023. 120(4). Article no. 12010. DOI: 10.34910/MCE.120.10
22. Eshmatov, B.Kh., Abdikarimov, R.A., Amabili, M., Vatin, N. Nonlinear vibrations and dynamic stability of viscoelastic anisotropic fiber reinforced plates. Magazine of Civil Engineering. 2023. 118(2). Article no. 11811. DOI: 10.34910/MCE.118.11
23. Low, K.H. Modified Dunkerley formula for eigenfrequencies of beams carrying concentrated masses. International Journal of Mechanical Sciences. 2000. 42(7). Pp. 1287–1305. DOI: 10.1016/S0020-7403(99)00049-1
24. Rutenberg, A. A lower bound for Dunkerley's formula in continuous elastic systems. Journal of Sound and Vibration. 1976. 45(2). Pp. 249–252. DOI: 10.1016/0022-460X(76)90599-X
25. Vanova, P., Sun, Z., Odinson, O.-E., Jiang, Z. Dynamic response analysis of a model truss bridge considering damage scenarios. Engineering Failure Analysis. 2023. 151. Article no. 107389. DOI: 10.1016/j.engfailanal.2023.107389
26. Torres, B., Poveda, P., Ivorra, S., Estevan, L. Long-term static and dynamic monitoring to failure scenarios assessment in steel truss railway bridges: A case study. Engineering Failure Analysis. 2023. 152. Article no. 107435. DOI: 10.1016/j.engfailanal.2023.107435
27. Guo, J., Li, Y., Xiao, Y., Fan, Y., Yu, D., Wen, J. Multiscale modeling and design of lattice truss core sandwich metastructures for broadband low-frequency vibration reduction. Composite Structures. 2022. 289. Article no. 115463. DOI: 10.1016/j.compstruct.2022.115463
28. Kirsanov, M.N., Luong, C. Deformations and natural frequency spectrum of a planar truss with an arbitrary number of panels. AlfaBuild. 2022. 25. Article no. 2507. DOI: 10.57728/ALF.25.7

Information about the authors:

Mikhail Kirsanov
ORCID: https://orcid.org/0000-0002-8588-3871
E-mail: c216@ya.ru

Cong Luan Luong
ORCID: https://orcid.org/0000-0002-6848-0974
E-mail: luongcongluan96@gmail.com

Received 20.08.2024. Approved after reviewing 02.09.2024. Accepted 02.09.2024.