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ADAPTIVE WEIGHT ADJUSTMENT IN MULTICRITERIA CONTROL OF NONLINEAR SYSTEMS BASED ON A LYAPUNOV FUNCTION

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Abstract. The paper develops an approach to the control of nonlinear dynamic systems within a multicriteria optimization framework in which criterion weights are adjusted during the control process. The method relies on a Lyapunov function that is included both in the performance functional and in trajectory constraints. A numerical algorithm is proposed for dynamic tuning of the criterion weights. The criteria are stability and control energy consumption, while their relative importance changes depending on the current system state and trajectory. Adaptation is performed based on the current value of the Lyapunov function and the system state, which enables automatic prioritization of objectives without operator intervention. A numerical analysis of the system behavior under different weight-adaptation schemes is carried out. For the considered models, the simulation results show that adaptive weight adjustment can provide faster decay of the Lyapunov function and lower control energy consumption compared with fixed scalarization. The method can be applied to control problems for nonlinear and multi-agent systems, including cyber-physical complexes, autonomous agents, and energy nodes.

Keywords: multicriteria optimization, Lyapunov function, adaptive weights, nonlinear systems, stability

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АДАПТИВНАЯ НАСТРОЙКА ВЕСОВ В ЗАДАЧЕ МНОГОКРИТЕРИАЛЬНОГО УПРАВЛЕНИЯ НЕЛИНЕЙНЫМИ СИСТЕМАМИ С ИСПОЛЬЗОВАНИЕМ ФУНКЦИИ ЛЯПУНОВА

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Аннотация. В статье рассматривается подход к управлению нелинейными динамическими системами с использованием метода многокритериальной оптимизации, в котором веса критериев адаптируются в процессе управления. В основе метода лежит функция Ляпунова, которая входит как в функционал качества, так и выступает в качестве ограничения на траекторию системы. Предлагается численный алгоритм, позволяющий динамически настраивать веса критериев. Критериями являются устойчивость и энергозатраты, а их относительная значимость изменяется в зависимости от текущего состояния системы и траектории. Адаптация осуществляется на основе текущего значения функции Ляпунова и состояния системы, что обеспечивает автоматическую приоритизацию целей без вмешательства оператора. Проведен численный анализ поведения системы при различных схемах адаптации весов. Для рассмотренных моделей результаты численного моделирования показывают, что адаптивная настройка весов может обеспечивать более быстрое затухание функции Ляпунова и снижение энергозатрат по сравнению с фиксированной скаляризацией. Метод применим в задачах управления мультиагентными и нелинейными системами, включая киберфизические комплексы, автономные агенты и энергетические узлы.

Ключевые слова: многокритериальное управление, функция Ляпунова, адаптивные веса, нелинейные системы, устойчивость

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Introduction

In modern control problems of complex systems, it is often necessary to account for several criteria simultaneously: stability, efficiency, coordination among subsystems, and energy consumption. Such problems belong to the class of multi-criteria optimization problems, whose solution requires balancing mutually competing objectives. This is especially relevant for nonlinear and multi-agent systems, where the dynamics may be unstable, while the control inputs may be constrained both by admissible magnitudes and by available information.

In practice, one of the most common approaches is to scalarize the vector objective function, for example, as a weighted sum of criteria. However, this leads to a central question: whether the criterion weights should be prescribed in advance or updated during the control process.

The theoretical foundations of optimization and decision-making in the problem of multi-criteria control have been described in classical works on mathematical programming, numerical optimization, system analysis, and control theory [1–4, 21, 24–26]. In particular, the development of linear and nonlinear programming, numerical optimization algorithms, and multi-criteria optimization methods

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is associated with the works of L. V. Kantorovich, G. B. Dantzig, L. G. Khachiyan, R. Steuer, D. G. Luenberger, S. Boyd, L. Vandenberghe, V. G. Zhadan, I. G. Chernorutskiy, N. Yu. Prokopenko, V. N. Kozlov, N. V. Rostov, and other authors [1–4]. In these works, multi-criteria problems are usually considered from the point of view of Pareto optimality, scalarization, criterion convolution, minimax schemes, penalty functions, or numerical methods for finding compromise solutions [1–4, 21, 24–26].

Weighted sum scalarization is one of the most common methods for converting a vector criterion into a scalar objective function. In this approach, individual criteria are multiplied by weight coefficients, and then standard numerical optimization algorithms can be applied to the resulting scalar problem [1–4]. However, the result of such scalarization depends significantly on the chosen weighting factors [21, 26]. If the weighting factors are fixed before the control process begins, they only reflect the developer's a priori preferences and do not account for changes in the relative importance of stability and control energy throughout the system's trajectory.

In the theory of control, multi-criteria optimization of parameters is used to configure regulators of nonlinear automatic control systems [2, 5, 6, 9, 10, 27]. For example, I. G. Chernorutskiy discusses optimization methods in the theory of control, including parametric optimization, multi-criteria problems, and Pareto compromise [2]. N. V. Rostov proposes methods for multi-criteria optimization of digital regulator parameters for stabilization and tracking systems with electromechanical installations, taking into account nonlinearities, constraints, and external disturbances [5, 6, 27–30]. In these studies, the main goal is to determine technically sound compromise or Pareto optimal regulator parameters. In this case, the weight coefficients of the individual criteria are usually selected as adjustable parameters and are not recalculated during the control process depending on the current state of the nonlinear system [5, 6, 25, 26].

Similar results have also been obtained in the field of distributed predictive control and optimal consensus for nonlinear multi-agent systems [7, 8, 14, 15, 20, 22, 23]. A. N. Venkat has developed a distributed predictive control system aimed at improving the performance and stability of closed-loop systems for large-scale interacting systems [7]. Y. Tang and X. Wang have considered optimal output consensus for heterogeneous nonlinear multi-agent systems with uncertainty, combining distributed optimization, consensus, and adaptive control mechanisms [8]. These approaches aim to ensure stability, constraint satisfaction, coordination, and distributed optimization, but they are based on predictive optimization, consensus protocols, and adaptive uncertainty compensation [7, 8, 14, 15, 20, 22, 23]. These approaches do not use the Lyapunov function as a state-dependent variable that directly redistributes the weights of the stability and energy criteria during the transient state [14–18, 20].

Therefore, the approach proposed in this article differs from existing results in the following ways. The goal is not to construct the entire Pareto set, and it is not just about a posteriori stability checking. The Lyapunov function is used both as a stability criterion and as a feedback variable that adaptively changes the weights of the criteria during control [9, 10, 14, 16–18, 20]. This allows for higher priority of stabilization when the system is far from equilibrium, and increased relative importance of energy saving near the equilibrium state.

Fixed weights do not allow the system to adapt to changing conditions. At the initial stage of the control process, it may be critical to ensure rapid decay of the system state (stability), while at later stages, reducing control costs becomes more important. Without proper adaptation, such changes in priority are not reflected in the control law, leading to suboptimal behavior.

This article proposes an approach in which the weights in the multi-objective control problem are determined adaptively, as functions of the current system state and the observed trajectory behavior. A distinctive feature of the approach is the use of the Lyapunov function both as a stability-related criterion and as a variable that governs the redistribution of priorities. This enables the stability criterion to receive greater weight automatically when approaching the boundaries of the phase space and its weakening when the system is already close to equilibrium, while maintaining control over costs.

Thus, the article solves two problems: it proposes an adaptive scalarization mechanism for the objective function and evaluates its effectiveness for constrained nonlinear control and for a multi-agent configuration.

Problem statement for multicriteria control with adaptive weights

Consider a nonlinear controlled system in the form of ordinary differential equations:

$$\dot{x}(t) = f\left(x(t), u(t)\right);$$
$$x(0) = x_0;$$
$$x(t) \in X;$$
$$u(t) \in U,$$

where $x(t) \in \mathbb{R}^n$ – the state vector; $u(t) \in \mathbb{R}^n$ – control input; $f(\ ,\ )$ – a smooth vector function that defines the dynamics of the system; X, U – the admissible sets of states and control inputs, respectively.

For application to real objects, the proposed method assumes that the state vector x is available for feedback control. This means that all components of x are either directly measured by sensors or reconstructed with sufficient accuracy using a state observer. This article focuses on the case of full state feedback. The synthesis of observers for partially measured systems is beyond the scope of this work.

The proposed approach is applicable to nonlinear controlled objects for which: a state-space model is known in the admissible operating domain; a continuously differentiable positive-definite Lyapunov function $V(x)$ can be constructed; the admissible set of controls is non-empty; and the available control actions allow for the reduction of the function $V(x)$. Such objects include electromechanical systems, robotic modules, active stabilization systems, power electronic converters, and homogeneous groups of mobile agents, provided that the necessary state variables are measurable [10, 14, 18–20, 22, 27].

In the following, by the Lyapunov function $V(x)$, we mean a continuously differentiable positive-definite function such that

$$V(0) = 0, \quad V(x) > 0, \quad x \neq 0.$$

The value of $V(x)$ is used as a quantitative measure of the system's departure from equilibrium, and a decrease in $V(x)$ along the trajectory indicates the system's stabilizing behavior. In simulations, a quadratic function is used for a two-dimensional model:

$$V(x) = x_1^2 + x_2^2.$$

For other nonlinear systems, the form of the Lyapunov function may differ and should be determined based on the structure of the specific model. Thus, the proposed approach is intended for nonlinear controlled systems for which the Lyapunov function is known or can be constructed in the admissible state domain, and for which there exists an admissible control input that can reduce this function, at least in the sense of the penalty used in the numerical algorithm. In the case of multiple agents, discussed below, it is assumed that the agents are homogeneous, the interaction graph is connected, and the agents share a common goal of achieving equilibrium. Systems with heterogeneous agent dynamics or incompatible local goals require additional assumptions and are not considered in this work.

The goal of control is to minimize a vector criterion that includes at least two individual criteria:

– the criterion of stability $J_1(u) = \int_0^T V\left(x(t)\right) dt$;

– control cost criteria $J_2(u) = \int_0^T \|u(t)\|^2 dt$.

Scalarization form with fixed weights:

$$J(u) = \int_0^T \left(\alpha V(x(t)) + \beta \|u(t)\|^2\right) dt,$$

where $\alpha > 0$ and $\beta > 0$; α and β are predefined parameters that specify the relative priority of the objectives.

Fixed values of α and β do not adapt to the system dynamics and do not reflect the current importance of the criteria at different points along the trajectory.

Adaptive weight structure

It is proposed to define weights as functions of the current state:

$$\alpha(t) = \phi_1(x(t));$$
$$\beta(t) = \phi_2(x(t)),$$

where ϕ_1 and ϕ_2 are smooth functions with the following properties:

– ϕ_1 increases with $V(x)$: the stability-related component receives greater weight as the system moves farther from equilibrium;

– ϕ_2 increases as we approach equilibrium: the control-cost component becomes more influential as the state approaches equilibrium.

The specified properties do not set the only possible form of weight functions, but define a class of admissible adaptive scalarizations [21, 25, 26]. In general, any smooth positive functions $\phi_1(V)$, $\phi_2(V)$ can be used, for which the normalized weight of the stability criterion:

$$\omega_1(V) = \frac{\varphi_2(V)}{\varphi_1(V) + \varphi_2(V)},$$

does not decrease as $V(x)$ increases, and the normalized weight of the energy consumption criterion:

$$\omega_2(V) = \frac{\varphi_2(V)}{\varphi_1(V) + \varphi_2(V)},$$

does not increase with $V(x)$. Therefore, the proposed below form of weights is considered not as the only possible, but as a simple numerically implementable option, allowing to investigate the effect of dynamic redistribution of priorities between stability and energy saving.

Taking into account the adaptive weights, a generalized scalarized functional is introduced:

$$J(u) = \int_0^T \left[\phi_1(x(t)) \cdot V(x(t)) + \phi_2(x(t)) \cdot \|u(t)\|^2\right] dt.$$

An example of admissible weight functions is

$$\phi_1(x) = 1 + \kappa_1 \cdot V(x);$$
$$\phi_2(x) + 1 + \kappa_2 \cdot \frac{1}{1 + V(x)},$$

where $\kappa_1, \kappa_2 \geq 0$ are the adaptation coefficients.

This construction automatically increases the priority of stabilization in the early stages of control (when $V(x)$ is large) and increases the relative importance of reducing control energy when $V(x) \to 0$.

The specific form of the functions ϕ_1, ϕ_2 and the values of the parameters κ_1, κ_2 affect the quantitative characteristics of the transient process. Increasing κ_1 enhances the stabilizing priority for large deviations, which can reduce the convergence time but also increase the initial control input and lead to saturation of the u constraint. Increasing κ_2 increases the importance of energy saving near the equilibrium, which reduces the integral energy consumption but may slow down the final convergence phase. Therefore, the numerical results should be interpreted as a result for the chosen parametric family of weight functions, rather than as a universal statement for all possible adaptive dependencies [21, 26].

Limitations of stability

In addition to including $V(x)$ in the objective function, the following condition may be imposed as a constraint:

$$\dot{V}\left(x(t)\right) = \nabla V\left(x(t)\right)^T f\left(x(t), u(t)\right) \le -\delta \left\| x(t) \right\|^2, \quad \delta > 0.$$

If this condition is imposed as a hard constraint, monotone decrease of the Lyapunov function is enforced. If the condition is introduced through a penalty term, this decrease is promoted but is not guaranteed by construction.

Discretization of the Problem

For numerical implementation, the problem is discretized with respect to time:

$$x_{k+1} = f_d\left(x_k, u_k\right), \quad k = 0, \ldots, N-1,$$

where f_d is a discrete model of the system obtained, for example, by the Euler method:

$$x_{k+1} = x_k + \Delta t \cdot \left(x_k, u_k\right),$$

or by a Runge–Kutta method.

The scalarized functional is then written in discrete form:

$$J(u) = \sum_{k=0}^{N-1} \left[\phi_1\left(x_k\right) \cdot V\left(x_k\right) + \phi_2\left(x_k\right) \cdot \left\| u_k \right\|^2 \right] \cdot \Delta t.$$

Numerical Implementation of the Algorithm with Adaptive Weights

Consider the numerical implementation of the proposed approach to multi-objective control with adaptive tuning of the functional weights. The problem is formulated as a multi-objective problem (stability and energy consumption), and a feedback law is then introduced, in which the adaptive gain $G(x_k)$ is determined based on the weights $\phi_1(x_k)$, $\phi_2(x_k)$. The functional J is used as a quality criterion and for selecting the controller structure and weight adaptation mechanism. In this case, the minimization of J does not have to be repeated at every control step.

Solution algorithm

1. **Initialization of parameters**:

Initial state $x_0 \in \mathbb{R}^n$;

Simulation horizon T and sampling step $\Delta t = \frac{T}{N}$;

Admissible bounds on the control input $u_k \in \left[-u_{\max}, u_{\max}\right]$;

The Lyapunov function $V\left(x\right) = \left\|x\right\|^2$;

Adaptive weights $\phi_1(x_k)$, $\phi_2(x_k)$.

2. **Construction of the objective function**:

$$J\left(u\right) = \sum_{k=0}^{N-1}\left[\left(1+\kappa_1 V\left(x_k\right)\right)\cdot V\left(x_k\right) + \left(1+\kappa_2\cdot\frac{1}{1+V\left(x_k\right)}\right)\cdot\left\|u_k\right\|^2\right]\cdot\Delta t.$$

3. **The system dynamics may be integrated numerically by several scheme**s:

The simplest model: Euler method

$$x_{k+1} = x_k + \Delta t\cdot f\left(x_k, u_k\right).$$

For improved accuracy of trajectory and stability estimates, a fourth-order Runge–Kutta method may be used:

$$x_{k+1} = x_k + \frac{\Delta t}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$

where $k_1 = f\left(x_k, u_k\right)$, $k_2 = f\left(x_k + \frac{\Delta t}{2}k_1, u_k\right)$, $k_3 = f\left(x_k + \frac{\Delta t}{2}k_2, u_k\right)$, $k_4 = f\left(x_k + \Delta t\cdot k_3, u_k\right)$.

4. **Constraint Handling**:

Control: $u_k \in \left[-u_{\max}, u_{\max}\right]$;

States: $x_k \in \left[-x_{\max}, x_{\max}\right]$;

The Lyapunov derivative:

$$\dot V\left(x_k\right) = \nabla V\left(x_k\right)^T f\left(x_k, u_k\right) = 2x_k^T f\left(x_k, u_k\right) \le -\delta\left\|x_k\right\|^2.$$

The constraints may be incorporated in two forms: either as soft penalties in the objective function by adding a term $\lambda\max\left\{0, \nabla V\left(x_k\right)^T f\left(x_k, u_k\right) + \delta\left\|x_k\right\|^2\right\}^2$, $\lambda > 0$, or as inequality constraints included directly in the optimization formulation.

Optimization

To implement the proposed approach, a feedback control scheme with an adaptively adjusted gain is used. At each step, the control u_k is calculated as follows:

1. The current adaptive weights $\phi_1(x_k)$ and $\phi_2(x_k)$ are computed from the system state;
2. The control input is defined as a function of the weight ratio:

$$u_{i,k} = -G\left(\phi_1\left(x_{i,k}\right), \phi_2\left(x_{i,k}\right)\right)\cdot\left(Kx_{i,k} + \rho\sum_{j\in N_i}\left(x_{i,k} - x_{j,k}\right)\right),$$

where G is an adaptive gain that balances the convergence rate (ϕ_1) and control-resource expenditure (ϕ_2).

Stabilization and regularization

To prevent divergence of the numerical procedure, the following methods are used:

1. Projection of variables onto the admissible domain;
2. Penalization of Lyapunov-function increase in the case of model inaccuracies (penalty for $\dot{V}(x_k) + \delta\|x_k\|^2 > 0$
3. Preventing oscillations at the equilibrium point (threshold-based setting of u_k to zero);
4. Filtering of numerical noise and low-amplitude oscillations (additional filtering).

Simulation of the Adaptive-Weight Method

To evaluate the effectiveness of the proposed approach with adaptive weight adjustment, numerical experiments were conducted for models with nonlinear dynamics to assess how adaptive weights affect system behavior, control energy, trajectory stability, and Lyapunov-function decay.

Experiment 1: A Single Nonlinear System

Model:

$$\begin{cases} \dot{x}_1 = -x_1 + x_2^2 + u \\ \dot{x}_2 = -x_2 + x_1 x_2 \end{cases}; \quad V(x) = x_1^2 + x_2^2.$$

Initial state: $x_0 = [1.5, -1.0]$;

Simulation horizon: $T = 15$, $N = 1500$, $\Delta t = 0.01$;

Constraints: $u_k \in [-1.5, 1.5]$, $x_k \in [-10, 10]$;

In experiment 1, the values $\kappa_1 = 5.0$, $\kappa_2 = 2.0$, $\delta = 0.05$ are used, which were selected based on the results of preliminary numerical experiments to ensure a pronounced priority of stabilization when the state is far from equilibrium and consistent redistribution of weights in the vicinity of the equilibrium state.

To evaluate the effectiveness, we compare the baseline method with fixed scalarization and the proposed method with adaptive weights. In both cases, we use the same initial conditions, control and state constraints, and the same numerical integration scheme.

In the fixed-weight baseline, the coefficients were set to constant values $\phi_1 = 2.0$, $\phi_2 = 3.0$. Therefore, the corresponding constant gain in the feedback law was

$$G = \frac{\varphi_1}{\varphi_1 + \varphi_2} = 0.4.$$

These values remained unchanged throughout the trajectory. In the adaptive case, the same feedback structure was used, but the coefficients $\phi_1(x_k)$ and $\phi_2(x_k)$ were recalculated at each time step as functions of the current value of the Lyapunov function. Both simulations used the same initial conditions, integration scheme, control bounds, state bounds, and Lyapunov-based correction procedure. Therefore, the comparison highlights the effect of replacing a constant priority ratio with state-dependent priority redistribution [1–4, 21, 26].

It should be noted that a specially tuned pair of constant weights may reduce the quantitative advantage of the adaptive approach [21, 26]. Therefore, the obtained results should not be interpreted as proof of superiority over all possible fixed-weight scalarizations.

Results

For the selected nonlinear model and the specified parameters, the Lyapunov function $V(x_k)$ demonstrates a higher convergence rate to zero when using adaptive weights compared with the selected

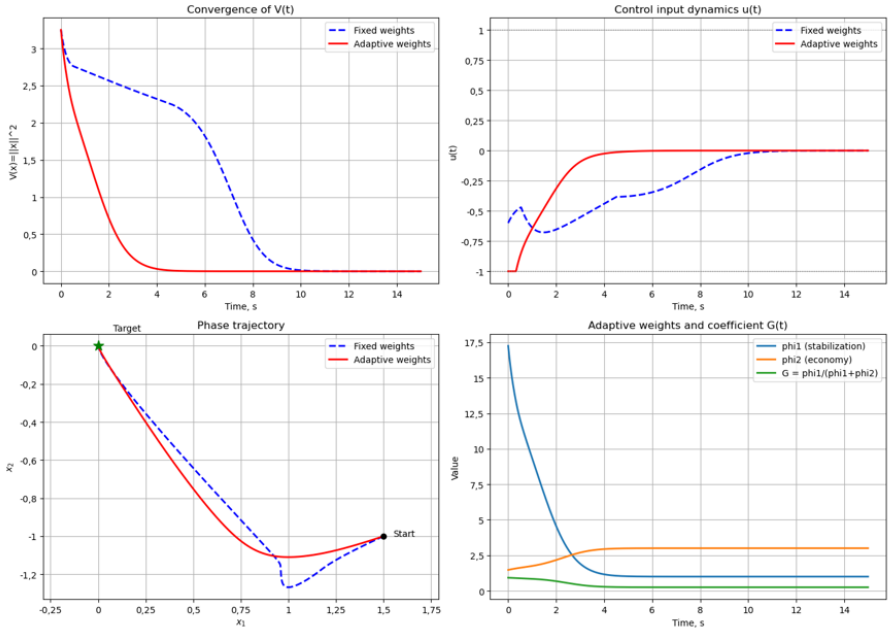


Fig. 1. Comparison of fixed-weight and adaptive-weight control for a single system: (a) – dependence $V(t)$; (b) – dependence $u(t)$; (c) – phase trajectory; (d) – change in weights ϕ_1, ϕ_2 and coefficient $G(t)$

fixed-weight scalarization baseline. This indicates a reduction in the transition time in this numerical experiment, but it does not necessarily generalize to all nonlinear systems;

In the simulation, the adaptive redistribution of priorities leads to a larger control input at the initial stage and to a decrease in it near the equilibrium. As a result, the integral control energy $E = \int_0^T u^2(t)\,dt$ is lower than that of the considered fixed-weight baseline. The magnitude of this reduction depends on the selected functions ϕ_1, ϕ_2, parameters κ_1, κ_2, control constraints, and the selected fixed-weight baseline level.

The phase trajectory also illustrates the effect of adaptive priority reallocation for the selected test model. This result should be interpreted as a numerical illustration of the proposed mechanism, rather than as a general proof of the superiority of arbitrary nonlinear systems. The adaptation mechanism is manifested in the fact that at the beginning of the process, the stabilizing component has a greater relative influence, while near equilibrium, the relative influence of the control cost component increases.

Experiment 2: A Multi-Agent Consensus System

Consider a distributed system of $M = 3$ agents. In this simulation, the agents are homogeneous: each agent has the same local dynamics, but different initial conditions. Let the state of the i-th agent be denoted by

$$x_i = \left(x_{1,i}, x_{2,i}\right)^T, \quad i = 1, \ldots, M.$$

The local dynamics of each agent are defined by the system

$$\dot{x}_{1,i} = -x_{1,i} + x_{2,i}^2 + u;$$
$$\dot{x}_{2,i} = -x_{2,i} + x_{1,i}x_{2,i}.$$

Thus, the control input ui enters the first equation of the agent's local dynamics. The connection between agents is introduced not as an additional physical term in the right-hand side of the system, but through a matching component in the control law.

The interaction graph is assumed to be fully connected. A normalized adjacency matrix is used for three agents [7, 8, 18, 20, 22]

$$A = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

The following adaptive control law is used:

$$u_{i,k} = -G\left(\varphi_{1,i}, \varphi_{2,i}\right)\left(Kx_{1,i,k} + \rho \sum_{j \in N_i} a_{ij}\left(x_{1,i,k} - x_{1,j,k}\right)\right),$$

where K determines the intensity of local agent stabilization, and ρ specifies the strength of the consensus interaction with neighboring agents; in the simulation under consideration, K = 1.0 and ρ = = 1.0 were used, a_{ij} are the elements of adjacency matrix, N_i is the set of neighbors of the i-th agent.

A local Lyapunov function is used for each agent:

$$V_i\left(x_i\right) = x_{1,i}^2 + x_{2,i}^2.$$

Weight functions:

$$\phi_1\left(x_i\right) = 1 + \kappa_1 \left\|x_i\right\|^2, \quad \phi_2\left(x_i\right) = 1 + \frac{\kappa_2}{1 + \left\|x_i\right\|^2}.$$

Thus, each agent adapts the local weights based on its own state, but the consensus coupling term in the control law takes into account the deviation from neighboring agents. The local decrease in $V_i(x_i)$ alone is not sufficient to prove the stability and coordination properties of the entire multi-agent system, so the consensus error and the average value of the Lyapunov function over the group are also analyzed [8, 18, 20, 22].

Constraints: $u_k \in [-1.5, 1.5], \quad x_k \in [-10, 10]$;

The parameters: κ_1 = 2.0, κ_2 = 10.0, δ = 0.05.

Heterogeneous agents with different local dynamics $f_i(x_i, u_i)$ are not considered in this article and require a separate analysis of the conditions for consensus and stability.

For the considered homogeneous system of three agents with a fully connected interaction graph, the simulation demonstrates a decrease in the disagreement between the agents and convergence of their states to a neighborhood of a common equilibrium.

In this experiment, the total control energy is reduced by more than 40% compared to the considered fixed-weight baseline. This effect is associated with an earlier reduction of the control effort near equilibrium. The quantitative value of the gain depends on the selected adaptive weight functions, the parameters κ_1, κ_2, the control constraints, and the selected fixed-weight baseline used for comparison.

The control inputs reach their highest values at the beginning of the transition process, and then decrease as the agent trajectories approach each other. In the simulation, the average Lyapunov function for the group tends to zero. This indicates a stabilization of the behavior of the chosen multi-

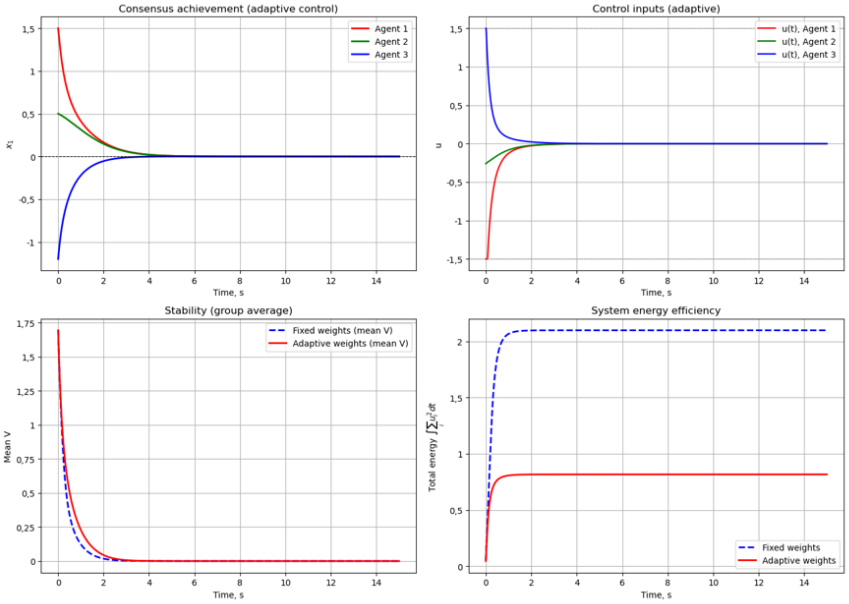


Fig. 2. Results of a numerical experiment for a multi-agent system:
(a) – consensus formation process; (b) – control inputs of the agents;
(c) – group dynamics of the Lyapunov function; (d) – total energy consumption

agent model, but this should be interpreted together with the consensus-error curve, since local Lyapunov reduction and network-level consensus are different properties of a distributed system.

The Python code to generate the plots (Figs. 1, 2) is available for download and can be run locally[1].

Conclusions

The conducted research shows that adaptive adjustment of the weight coefficients in a multi-criteria functional can be used as a control tool for nonlinear systems. The obtained results are applicable to models for which a Lyapunov function can be constructed, the state variables are measurable, control constraints are specified, and a stabilizing control law can be implemented. The main conclusions are as follows:

1. A mechanism is proposed in which the weight coefficients of the objective function are dynamically changed depending on the current state of the system through the Lyapunov function. Unlike classical weighted scalarization, in which the weight coefficients are fixed before the optimization or control process begins, the proposed approach allows the weight coefficients of the criteria to be recalculated in real-time. This allows for a shift in priority towards stabilization in the case of large deviations from equilibrium and towards energy conservation as the system approaches equilibrium. The chosen form of the weight coefficients is not the only possible choice and is considered as a representative of the class of smooth positive adaptive functions.

2. The proposed approach is a development of existing approaches to multi-criteria control by using the Lyapunov function not only as a stability criterion, but also as a variable that regulates the redistribution of priorities between individual criteria. This distinguishes this method from known approaches to multi-criteria parameter optimization, where the Pareto-optimal regulator parameters are typically selected with fixed-weight coefficients for the criteria [2, 5, 6, 21, 25, 26].

[1] GitHub – fershtadt/Adaptive-scale-adjustment · GitHub, Available: https://github.com/fershtadt/Adaptive-scale-adjustment (Accessed 21.07.2026)

3. A numerically implementable condition for reducing the Lyapunov function is introduced. This condition can be used either as an explicit constraint or as a penalty term in the quality functional. If there exists a valid control that satisfies this condition, it helps to bring the system to an equilibrium state. The fulfillment of this condition depends on the nonlinear model, the initial state, the set of valid controls, and the chosen Lyapunov function.

4. For the considered multi-agent system, where the communication between the subsystems is provided by a connected graph, it is shown by numerical methods that the local adaptation of weights based on the Lyapunov functions is compatible with the coordination term in the control law and leads to a reduction in the discrepancy between the agents. Unlike distributed model predictive control approaches, the proposed scheme does not require complex numerical optimization at each time step [7, 14, 15, 20, 23]. Instead, the redistribution of priorities is performed directly using the adaptive weights based on the Lyapunov functions.

5. The algorithm can be implemented in a centralized form and, under additional assumptions, in a distributed multi-agent form, provided that each agent has access to its own state and the states of neighboring agents in the graph. In this work, the multi-agent implementation was confirmed by numerical simulations for a system of three agents. The use of simple numerical integrators, such as the Euler method or the Runge-Kutta method, makes the scheme convenient for computational implementation.

6. The method was simulated in Python. Under the simulation parameters, the results demonstrate a faster decrease in the Lyapunov function compared to the selected baseline with fixed weights and a decrease in energy consumption near the system's equilibrium. The proposed method can be used to achieve a trade-off between fast stabilization and reduced control inputs [9, 10, 14, 16–18, 20]. These findings are applicable only to the class of systems under consideration. Future work will address the scaling of this approach to more complex real-world systems.

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