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Natural vibrations of buried pipeline section

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Abstract. To ensure reliable operation of the pipeline, the design takes into account the natural vibration of the structure caused by the uniform flow of the product along the pipeline. A pipeline section in the form of a steel-concrete cylindrical shell placed in the soil condition is considered. The inner part of the pipeline is made of steel, and the exterior is covered with a concrete layer 30–50 mm thick. Determination of natural vibration frequencies for a two-layer cylindrical shell in the soil is clarified using two methods. The first method is analytical, according to which the dependence for the frequency is obtained using the half momentless theory of cylindrical shells. The second is numerical and is based on the finite element method (FEM) with the construction of the computational model in the Lira Sapr environment. Modeling of steel and concrete layers of the composite shell in the software package was carried out by 4-node plates, which are combined into a common structure by means of absolutely rigid bodies (ARB). Two cases of taking into account the soil condition surrounding the shell were considered. In the first case, a soil mass (with dimensions of 5.3×5.3 m) is created by volumetric bodies, while in the second case, the pastel coefficient for the concrete layer is specified. It was found that the second method of setting the soil conditions allowed to reduce the time of data input by 5–6 times with the same results obtained. The discrepancy between the frequencies of natural vibration for the object of study determined by the analytical method and FEM does not exceed 10 %, and for the first 3 frequencies of the spectrum is not more than 6 %, therefore, both methods are applicable. The use of an analytical expression made it possible to obtain results an order of magnitude faster with the pastel coefficient than with the help of numerical soil modeling using volumetric elements.

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1. *Introduction*

Trunk pipelines are laid in different soil conditions, including areas with predicted watering and in waterlogged soils. For stability calculations, the pipeline section is represented as a cylindrical shell placed in the different soil condition. To prevent floating of the pipeline section, balancing measures are carried out for such sections. Steel pipelines are protected by concrete weights, which in the course of works can damage the original geometry of the pipe cross-section and negatively affect the reliable operation of the structure. One of the options to exclude such situations is the use of pipe-concrete products, the inner part of which is made of large-diameter steel pipes (d < 1000 mm) with the parameter $0.015 \le h/R \le 0.05$ and the outer part is formed by a concrete layer 40–100 mm thick. The reliability of such structures must be ensured by vibration resistance design calculations. Natural vibration can arise due to the movement of the pumped product. The sources offer calculation methods using analytical expressions and numerical solutions obtained by the finite element method (FEM) in various software packages. Below is an analysis of these works and the vibration resistance calculation models used in them.

In publications, there is the rod models of pipeline without influence of soil. In [1], the influence of critical values of fluid flow velocities in a pipe on the natural vibration of the pipe section is considered. The pipe in static equilibrium loses stability. Based on the Timoshenko beam theory, equations describing nonlinear vibration near a nontrivial static equilibrium configuration are formulated and the influence of system parameters on the equilibrium configuration, critical velocity, and frequency of natural vibration is shown.

In [2], the effect of fluid velocity on the dynamic characteristics of the pipe was studied using two methods: the derived analytical expressions for the critical flow velocity and natural frequencies of the pipe and the developed FEM. However, the developed FEM cannot reflect the effect of fluid flow on the pipe vibration mode.

In the study [3], the flow characteristics in the pipe are studied using computational fluid dynamics by numerical solution of low velocity compressible flow problems. In all these works, to determine the frequencies of natural vibrations of single-layer pipelines, taking into account the velocity of the flowing fluid, it is proposed to use analytical expressions or numerical solutions obtained for the calculation scheme of the pipeline in the form of a rod without taking into account the influence of the external environment on the pipe. The papers show the influence of static [4] and dynamic [5] ground modes on the vibrations of the underground pipeline. It is revealed that the oscillatory process of the pipeline can be realized at frequencies close to resonance frequencies. At frequencies close to resonance, the values of moments can be large in the pipeline sections, which is the cause of the pipeline stability loss. Pipelines partially resting on the ground based on the rod theory were studied in [6]. This approach does not allow to take into account the section deformation, and it was proposed to use it for the calculation of thick-walled cylindrical shells with parameters $0.07 < h/R < 0.125$.

The application of cylindrical shell theory to the calculation of pipelines made it possible to take into account vibrations in the transverse direction of the pipeline. In [7], the vibrations of an underwater pipeline for the pipe-liquid-soil system were investigated, but the issue of internal pressure acting on the pipeline walls was not considered. In [8, 9], the influence of internal unsteady pressure on the bending vibrations of the pipe was studied for closed cylindrical shells, but the influence of the external environment was not studied. The authors of [10–12] used different shell theories: Sodel, Flügge, Morley–Koiter and Donnell to determine the frequencies of natural vibrations of the pipeline. The result of the solution within the framework of these theories is the determinant for determining the natural frequency of vibrations, which was found in [13] for aluminum shells. In [14, 15], radial vibrations of the shell without taking into account the ground conditions are studied, the solution is obtained using the Vlasov–Novozhilov half momentless theory of cylindrical shells. In [16], analytical dependences for determining the frequencies of natural vibrations of a large-diameter pipeline partially buried in the ground were obtained. In [17], a similar approach is realized for a 2-parameter soil foundation. In [18], the natural frequencies are obtained for a metal-ceramic cylindrical shell placed in an elastic Pasternak base, but the internal pressure from the pumped product is not taken into account. Works [19, 20] are devoted to 3-layer shells, but the functional of the obtained solutions is extremely narrow, as it does not take into account the internal pressure on the wall of the shell, as well as the repulsion of the different soil conditions, preventing the deformation of the walls.

On the basis of the three-dimensional theory of elasticity, a parametric study was carried out, in which the influence of the parameters: shell thickness, mean radius, length and numbers of vibrational modes on the critical velocity of the shell without the influence of the ground was determined [21].

In [22], numerical modeling in ANSIS and, in [23], numerical modeling in ABAQUS were carried out, and the results of numerical modeling were compared with the results obtained from analytical formulas. Not always researchers compare the results of numerical modeling with analytical solutions. For example, in [24], the frequencies of natural vibrations for a subway tunnel are obtained in the MSC Patran Nastran software package, without comparison with analytical solutions, because they are not obtained for the problem under consideration. The issue of the influence of different soil conditions for buried pipelines is poorly studied.

The authors of [25, 26] investigated the free vibrations of cylindrical shells in an elastic inertia-free medium based on the Winkler–Pasternak hypothesis. The frequencies of free oscillations were obtained taking into account the deformation of the cross-section of the closed shell, but without taking into account the ground resistance forces. The influence of damping properties of the ground on the frequency characteristics of the pipeline is not taken into account.

In the problem [27] of static loss of pipeline stability solved by the FEM, the influence of rheological properties of the ground on the pipeline blowout is taken into account. The soil-pipeline interaction is considered using two approaches in [28]: laboratory experiment and numerical modeling using the discrete element method (DEM). The review article [29] analyzes experimental and numerical studies on pipeline deformations under the influence of dynamic processes in soils (seismic activity). Tests on a vibration test bench to estimate the soil resistance force required to move the buried pipeline in the transverse direction are presented. It is found that the drag force decreases at higher vibration amplitude and becomes more appreciable in case of complete liquefaction of sand, indicating its viscous behavior. The application of elasto-viscos-plastic soil models for structural design is shown in [30, 31]. However, these works do not take into account the natural vibrations of the pipeline in the ground caused by product transportation.

The authors of this article did not find any works, in which composite shells are considered in soil. This paper is devoted to partial elimination of the mentioned gaps, i.e., it is proposed to model the free vibrations of a steel-concrete cylindrical shell in an elastic soil medium analytically and numerically. A plane problem of the momentless shell theory is considered, in which the influence of the ground is taken into account through the radial ground repulsion. The ground repulsion varies along the circumference around the shell. The influence of the pumped product is taken into account through the longitudinal force.

The purpose of this paper is to study the influence of the soil on the configuration of the pipeline in its natural vibrations and to compare the natural frequencies of vibration for a closed two-layer cylindrical shell in the ground, obtained using the analytical formula and FEM.

2. *Materials and Methods*

The first method for determining the frequency of natural vibrations is analytical. The formula for the natural frequencies of vibrations for a two-layer cylindrical shell is obtained by the authors on the basis of the momentless theory of cylindrical shells and for the articulated support of the ends of the pipeline section under consideration. Here is the system of equations.

The equations of equilibrium for the cylindrical shell are written in the form [32]:

$$\frac{\partial T_1}{\partial \xi}+\frac{\partial S}{\partial \theta}+RQ_2\tau=-RX_1, \qquad \frac{\partial T_2}{\partial \theta}+\frac{\partial S}{\partial \xi}+\frac{R}{R_2^*}Q_2=-RX_2,$$
$$\frac{\partial Q_2}{\partial \theta}-\frac{R}{R_2^*}T_2-\frac{R}{R_1^*}T_1=-RX_3, \quad \frac{\partial M_1}{\partial \xi}+\frac{\partial H}{\partial \theta}-RQ_1=0, \quad \frac{\partial M_2}{\partial \theta}-\frac{\partial H}{\partial \xi}-RQ_2=0. \tag{1}$$

From the formulas of the momentless theory:

$$\left(\frac{\partial \nu}{\partial \theta}+w=0;\ \ \frac{\partial \nu}{\partial \xi}+\frac{\partial u}{\partial \theta}=0;\ \ \vartheta_2=\frac{\partial w}{\partial \theta}-\nu\right),$$

the equations (1) take the form:

$$\frac{\partial^2 T_1}{\partial \xi^2}+\frac{\partial}{\partial \xi}\left(\tau\frac{\partial M_2}{\partial \theta}\right)-\frac{1}{R^2}\cdot\frac{\partial^3}{\partial \theta^3}\left(R_2^*\frac{\partial M_2}{\partial \theta}\right)-\frac{\partial}{\partial \theta}\left(\frac{1}{R^2}\cdot\frac{\partial M_2}{\partial \theta}\right)+$$
$$+\frac{\partial^2}{\partial \theta^2}\left(\frac{R_2^*}{R_1^*}T_1\right)+R\frac{\partial X_1}{\partial \xi}-R\frac{\partial X_2}{\partial \theta}-\frac{\partial^2}{\partial \theta^2}\left(R_2^*X_3\right)=0. \tag{2}$$

Forces of inertia: in longitudinal direction $X_1=-\dfrac{Rh\rho_0\partial^2 u}{\partial t^2}$, circumferentially $X_2=-Rh\rho_0\partial^2 v/\partial t^2$, radially $X_3=-\dfrac{Rh\rho_0\partial^2 w}{\partial t^2}+p_0-C_{1z}Rw\left(0.5-\alpha_1\cos\theta-\alpha_2\cos 3\theta\right)$. The components of the inertia force were substituted into the expression (2), taking into account the linear relationship between forces and deformations, displacements and deformations, we obtained the linearized differential equation of motion of the shell in displacements:

$$\frac{\partial^3 u}{\partial \xi^3}+\eta h_v^2\frac{\partial^3}{\partial \theta^3}\left(\frac{\partial^2 \vartheta_2}{\partial \theta^2}+\vartheta_2\right)+2\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 w}{\partial \xi^2}\varepsilon_0\right)-\frac{R}{E_0 h}p_0\frac{\partial^3 \vartheta_2}{\partial \theta^3}+\frac{1}{2}\frac{R^2 C_{1z}}{E_0 h}\frac{\partial^2 w}{\partial \theta^2}-$$
$$-\frac{R^2\rho_0}{Eh}\left(\frac{\partial^3 u}{\partial \xi \partial t^2}-\frac{\partial^3 v}{\partial \xi \partial t^2}-\frac{\partial^3 w}{\partial \theta^2 \partial t}\right)-\frac{R^2\alpha_1 C_{1z}}{E_0 h}\left(\frac{\partial^2 w}{\partial \theta^2}\cos\theta-2\frac{\partial w}{\partial \theta}\sin\theta-w\cos\theta\right)- \quad (3)$$
$$-\frac{R^2\alpha_2 C_{1z}}{E_0 h}\left(\frac{\partial^2 w}{\partial \theta^2}\cos 3\theta-\frac{\partial w}{\partial \theta}6\sin 3\theta-9w\cos 3\theta\right)=0.$$

The terms with the multiplier C_{1z} in the equation (3) describe the influence of the soil on the pipeline section. In problems on vibration stability in the framework of shell theory, these additional summands are considered for the first time. The backpressure of the ground medium in the radial direction (see Fig. 1) is described by the expression:

$$q_{gr}=C_{1z}\times R_w\times\left(0.5-\alpha_1\times\cos\theta-\alpha_2\times\cos 3\theta\right).$$

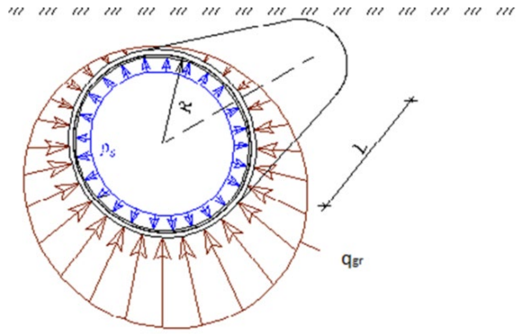


Figure 1. The effect of soil pressure on a pipeline section.

The boundary conditions described the articulated resting of the shell ends:

$$v\left\{\xi=0,\ \xi=\frac{L}{R}=0\right\},\quad \theta_2\left\{\xi=0,\ \xi=\frac{L}{R}=0\right\},$$
$$w\left\{\xi=0,\ \xi=\frac{L}{R}=0\right\},\quad \frac{\partial^2 w}{\partial \xi^2}\left\{\xi=0,\ \xi=\frac{L}{R}=0\right\}. \quad (4)$$

Solving the equations (3) with the conditions (4), obtain the displacements in the form of Fourier series.

Free oscillations of the cylindrical shell are represented by the harmonic law $\varphi(t)$ in the form of:

$$\varphi(t)=\sin\omega_{mn}\times t;\quad \varphi''=-\omega_{mn}^2\times\sin\omega_{mn}\times t, \quad (5)$$

where ω_{mn} is natural frequency.

Expression for determination of natural frequencies for the pipeline:

$$\omega_{mn}=\frac{1}{2\pi}\cdot\sqrt{\frac{\lambda_n^4+\eta\cdot m^4\left(m^2-1\right)\left(m^2-1+\frac{p^*}{\eta}\right)+C_{1z}^*\cdot m^4}{\rho_{sh}^*\cdot R_0\cdot h\left(\lambda_n^4 h_v+m^4+m^2\right)}}, \quad (6)$$

here: n is number of half-waves in the longitudinal direction; m is number of half-waves in the circumferential direction; $\lambda_n=\frac{n\times\pi\times R_0}{L\times\sqrt{h_v}}$ is length parameter of a two-layer cylindrical shell; L is section length (m); $R_0=R-Z_0$ is shell radius (m); R is radius of the steel layer of the shell (m);

$Z_0 = \frac{E_1 h_1^2 - E_2 h_2^2}{2\left(E_1 h_1 + E_2 h_2\right)}$ is distance from the jointing layer to the original surface (m) (Fig. 2); h_1, h_2 is thickness of concrete and steel layer of the shell, respectively (m); $h = h_1 + h_2$ is wall thickness of the two-layer shell (m); E_1 is modulus of elasticity of concrete layer (N/m²); E_2 is modulus of elasticity of steel layer (N/m²); $h_v = h/R_0 \sqrt{12\left(1-v^2\right)}$ is relative shell thickness parameter; ν is Poisson's ratio; $\eta = E_v/E_0$ is heterogeneity factor; $E_v = \left(1-v^2\right)\cdot 12D/h^3$ is reduced modulus of elasticity (bending) (N/m²); $D = \frac{1}{3\left(1-v^2\right)}\left[E_1\left\{\left(h_1 - Z_0\right)^3 + Z_0^3\right\} + E_2\left\{\left(h_2 - Z_0\right)^3 - Z_0^3\right\}\right]$ is flexural stiffness (Nm); $E_0\left[E_1 h_1 + E_2 h_2\right]/h$ is reduced modulus of elasticity (tensile/compression) (N/m²); $p^* = p_0\left(R_0/E_0 h \cdot h_v^2\right)$ is internal working pressure parameter; p_0 – internal pressure in the two-layer shell (N/m²); $\rho_{sh}^* = \rho_0\left(R_0/E_0 \cdot h \cdot h_v^2\right)$ is shell material density parameter (c²/m²); $\rho_0 = \frac{1}{g}\left[\left(\gamma_1 h_1 + \gamma_2 h_2\right)/h\right]$ is reduced density of the shell material (N·c²/m⁴); γ_1 is concrete density N/m³; γ_2 is steel density N/m³; $C_{1z}^* = R_0^2 C_{1z}/E_{gr} h \cdot h_v^2$ is reduced coefficient of soil rigidity (posteli coefficient); $C_{1z} = E_{gr}/R_0\left(1 + v_{gr}\right)$ is soil stiffness coefficient (N/m³); E_{gr} is ground elastic modulus (N/m²).

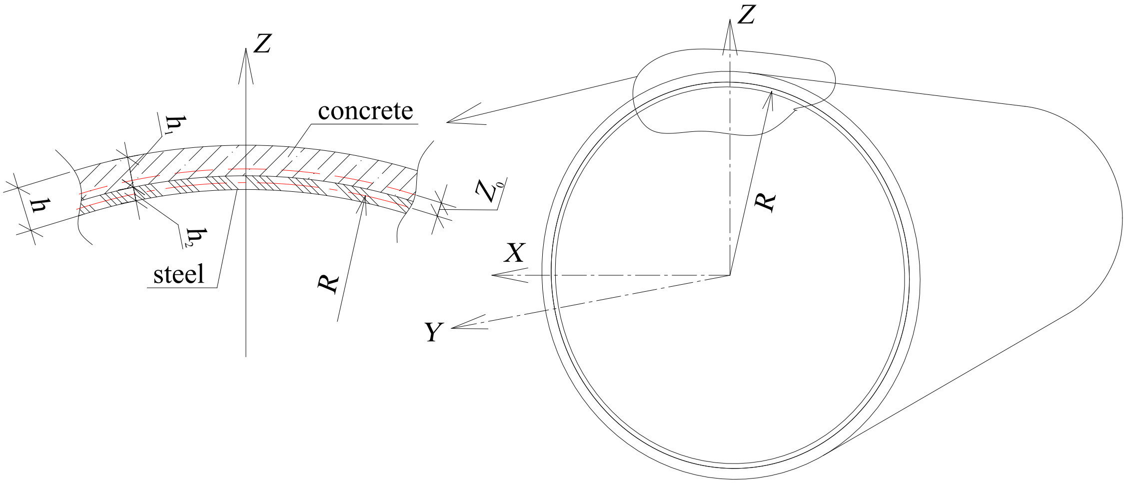


Figure 2. Geometric dimensions of the two-layer shell.

Lira Sapr software was used to calculate the natural vibration frequencies using the second method. Modeling of each layer of the two-layer shell (Fig. 3) was carried out by plates, and to ensure the joint operation of the layers were used to combine the displacements for each corresponding node by staging absolutely rigid bodies (ARB). The linear displacement constraint of the boundary nodes along the Z (X3) and Y (X2) axis was introduced to create a hinge-unstiffened shell end fixation.

Consideration of the soil, in which the shell is placed, was performed in two ways. In the first method, a ground mass of volumetric bodies with a cross-section size of 5.3×5.3 m was created. In the second method, the mass was not created, and the elastic rebound of the soil was taken into account by assigning the pastel coefficient C_{1z} = 473620 N/m³. As external loads, only the own weight of the shell layers was taken into account (without taking into account the weight of the soil conditions). Determination of natural vibration frequencies was carried out using “Modal Analysis”, which was used to form a matrix of the masses of the structure and the number of vibration forms.

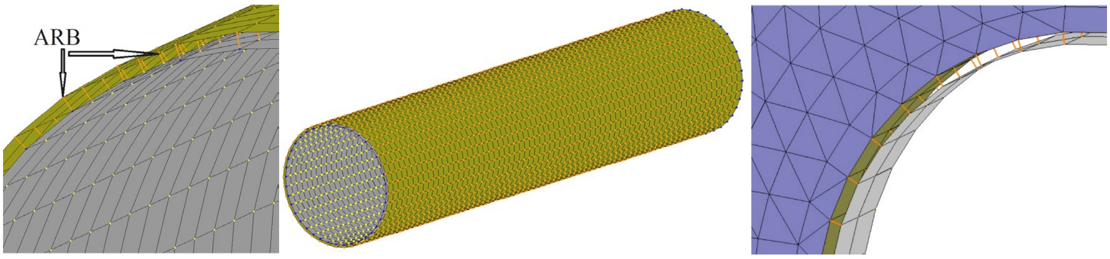


Figure 3. Modeling of a cylindrical shell.

3. Results and Discussion

For numerical implementation, a section of a cylindrical two-layer shell with the radius of the first layer R = 0.71 m and thickness h_1 = 18 mm is considered. The thickness of the second layer is h_2 = 40 mm. The length of the section under consideration is taken as 7, 8 and 9 m. The modulus of elasticity of concrete, modulus of elasticity of steel and density of layers are respectively equal to E_1 = 3.24711·10^{10} N/m^2, E_2 = 2.06·10^{11} N/m^2, γ_1 = 24516.6 N/m^3, γ_2 = 76982.2 N/m^3. Poisson's ratio for steel and concrete of class B30 is taken equal to ν = 0.3. The structure is placed in a ground medium with different ground deformation modulus and parameters: ground density is γ_{gr} = 11770 N/m^3; ground Poisson's ratio is ν_{gr} = 0.49. Internal pressure is assumed to be p_0 = 0 MPa.

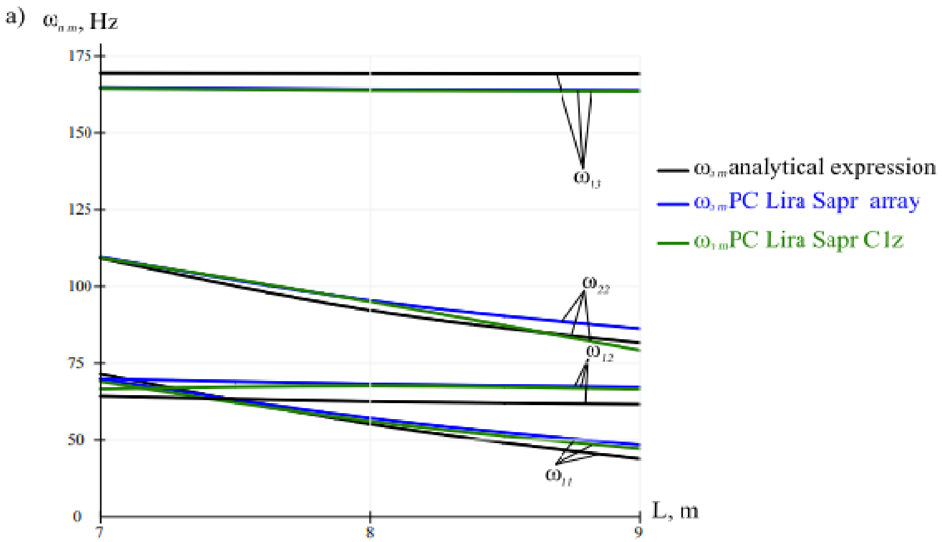
The values of natural vibration frequencies for different lengths of pipe sections are summarized in Table 1. The first column shows the values of vibration frequencies using the analytical formula (6), the second column shows numerical solutions of vibration frequencies found using the FEM method when the soil medium array is created, the third column shows the frequencies found using the FEM method when the soil stiffness coefficient is set.

Analysis of the data in Table 1 shows that the difference in the frequency values determined by the FEM with the soil array assignment (column 2) and by assigning the bedding coefficient C_{1z} = 473620 (N/m^3) (column 3) does not exceed 2 %, therefore, in order to reduce the labor input during model creation, it is recommended to use the second method of taking into account different soil conditions using the C_{1z} coefficient. The second method allowed to reduce the time of data processing by the processor by 5–6 times.

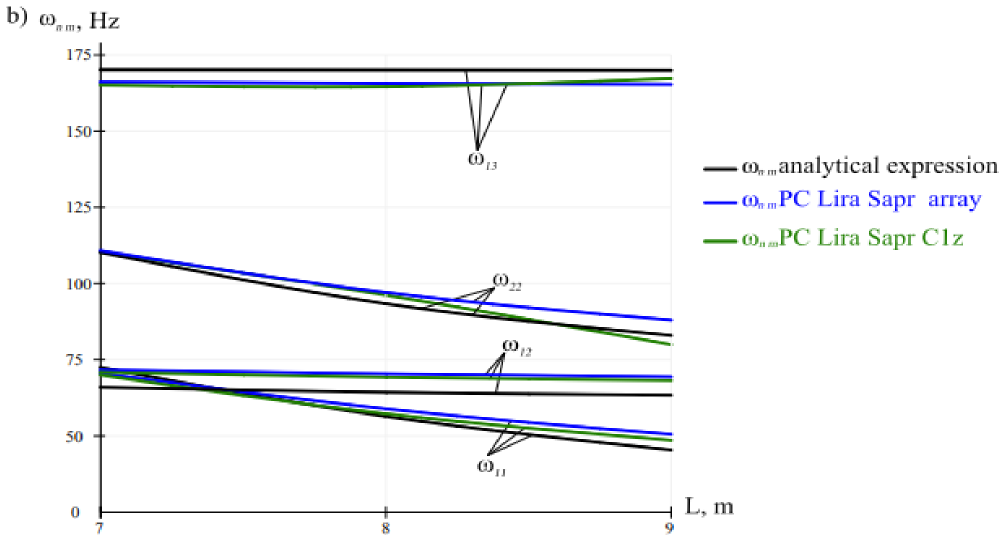
Table 1. The results of determining the frequencies of natural oscillations in various ways.

Analytical formula (Hz)	PC Lira Sapr mass (Hz)	PC Lira Sapr ratio C_{1z} (Hz)	Analytical formula (Hz)	PC Lira Sapr mass (Hz)	PC Lira Sapr ratio C_{1z} (Hz)	Analytical formula (Hz)	PC Lira Sapr mass (Hz)	PC Lira Sapr ratio C_{1z} (Hz)
1	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
L=7 m (R/L=1/10)			L=8 m (R/L=1/11)			L=9 m (R/L=1/13)		
ω_{11}=71.52	ω_{11}=69.80	ω_{11}=68.99	ω_{11}=55.16	ω_{11}=57.08	ω_{11}=56.04	ω_{11}=43.86	ω_{11}=48.4	ω_{11}=47.14
ω_{12}=64.23	ω_{12}=69.90	ω_{12}=63.13	ω_{12}=62.53	ω_{12}=68.21	ω_{12}=67.58	ω_{12}=61.61	ω_{12}=67.2	ω_{12}=66.54
ω_{13}=169.4	ω_{13}=164.7	ω_{13}=164.4	ω_{13}=169.3	ω_{13}=164.1 0	ω_{13}=163.8	ω_{13}=169.2	ω_{13}=163.8	ω_{13}=163.5
ω_{22}=109.2	ω_{22}=109.6	ω_{22}=109.3	ω_{22}=92.18	ω_{22}=95.40	ω_{22}=95.01	ω_{22}=81.62	ω_{22}=86.2	ω_{22}=79.20
ω_{23}=174.3	ω_{23}=174.6	ω_{23}=174.3	ω_{23}=172.1	ω_{23}=170.9	ω_{23}=170.6	ω_{23}=170.9	ω_{23}=168.7	ω_{23}= 168.4
ω_{32}=211.9	ω_{32}=186.5	ω_{32}=186.3	ω_{32}=167.4	ω_{32}=154.8	ω_{32}=154.6	ω_{32}=137.6	ω_{32}=132.3	ω_{32}=132.0
ω_{33}=194.6	ω_{33}=199.7	ω_{33}=199.5	ω_{33}=184.4	ω_{33}=188.2	ω_{33}= 1879	ω_{33}=178.7	ω_{33}= 181.3	ω_{33}= 180.7

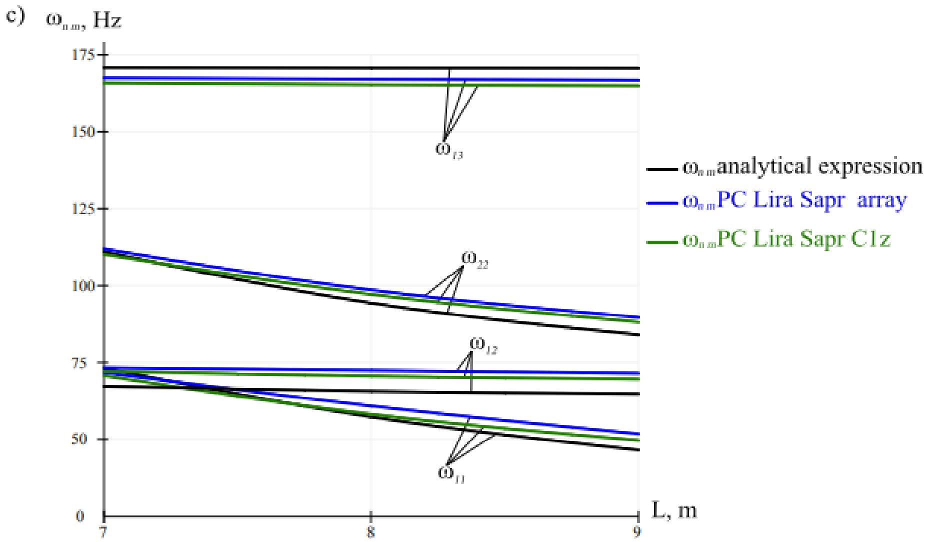
Comparison of the results of determining the frequencies of natural vibrations using the expression (6) and FEM shows that the difference for the first 3 frequencies does not exceed 6 %, and for the rest of the results 10 % (Fig. 4).



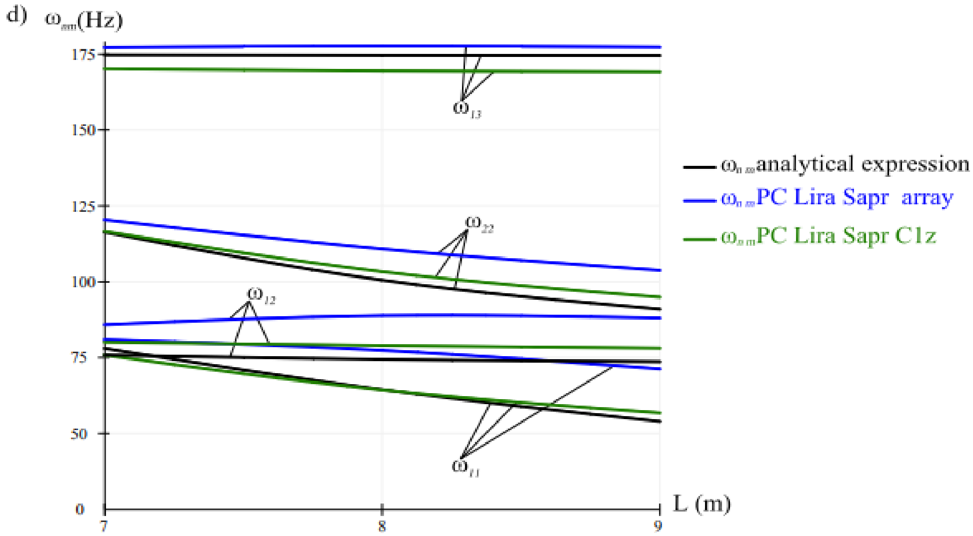
Peat C_{1z} = 473620 N/m³; γ_{gr} = 11770 N/m³; E_{gr} = 5·10⁵ N/m²; ν_{gr} = 0.49



Loose bulk soil C_{1z} = 3136416 N/m³; γ_{gr} = 16660 N/m³; E_{gr} = 3·10⁶ N/m²; ν_{gr} = 0.35



Bulk compacted soil C_{1z} = 5227360 N/m³; γ_{gr} = 17660 N/m³; E_{gr} = 5·10⁶ N/m²; ν_{gr} = 0.35



Clay C_{1z} = 19878694 N/m³; γ_{gr} = 19620 N/m³; E_{gr} = 2·10⁷ N/m³; ν_{gr} = 0.42

Figure 4. Variation of natural oscillation frequency from the length of the pipeline section.

For the pipe section with length of 7 m, the minimum frequency was ω_{12} (Fig. 4c), the change in cross-section corresponded to the shell form of oscillations. For the pipe sections of lengths 8 and 9 m, the minimum frequencies corresponded to the ω_{11} form (beam without cross-sectional deformation, Fig. 4a). All other vibration shapes were shell-shaped. Increasing the length of the pipe section by 1 m resulted in an average 1.5–3.1 % decrease in natural vibration frequencies (Figs. 4a–d). The increase in the stiffness of the soil led to an increase in the frequency of natural vibrations of the system steel-concrete shell+soil. The stiffness of the system in the cross-section of the pipe increased due to the soil (Figs. 5, 6). For the seven vibration forms, the changes in the pipe configuration taking into account the soil stiffness coefficient are shown in Fig. 5. Taking into account the influence of the soil massif, the first three vibration forms are shown (Fig. 6).

The analytical method has clear advantages over the FEM, since the calculation of frequencies took 10 times less time with almost the same results. Also, using the analytical method, it is possible to take into account the influence of internal working pressure by the parameter p*, while using the FEM, this factor cannot be taken into account, because when modeling this loading, it is taken into account not as a force preventing the deformation of the cross section but as an additional mass taken into account when calculating frequencies, so all the data in Table 1 are obtained with zero internal pressure.

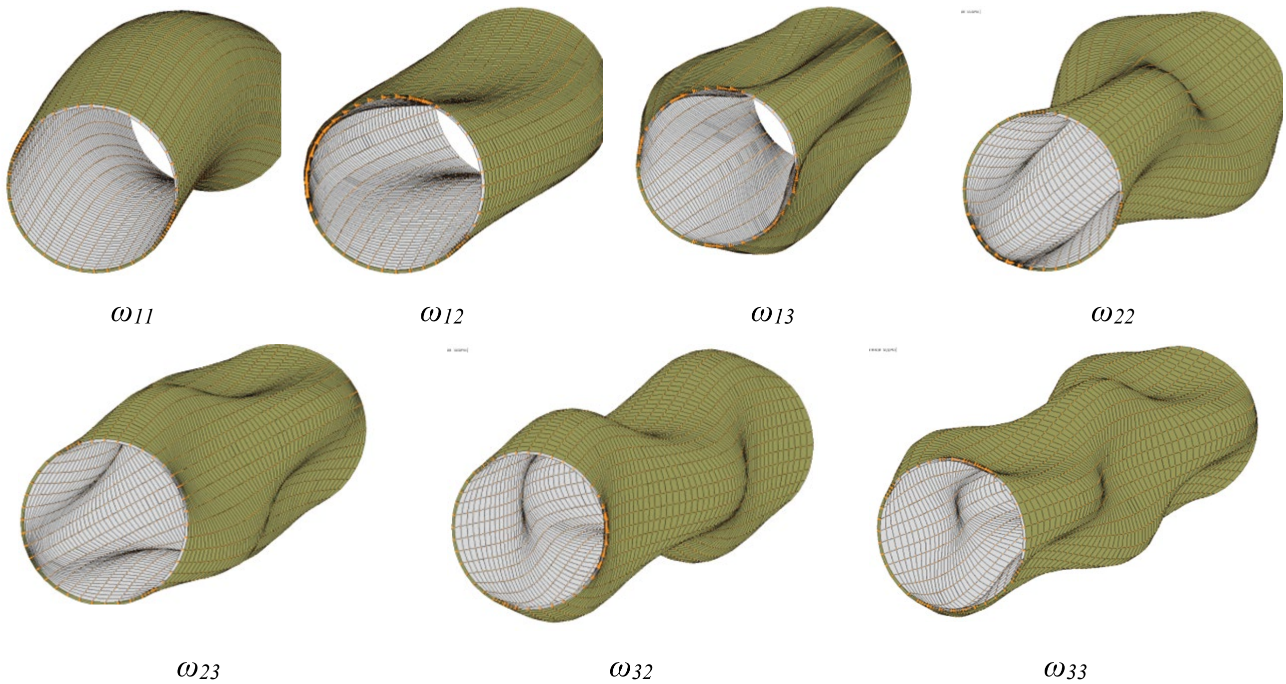


Figure 5. Waveforms for the pipeline section under consideration.

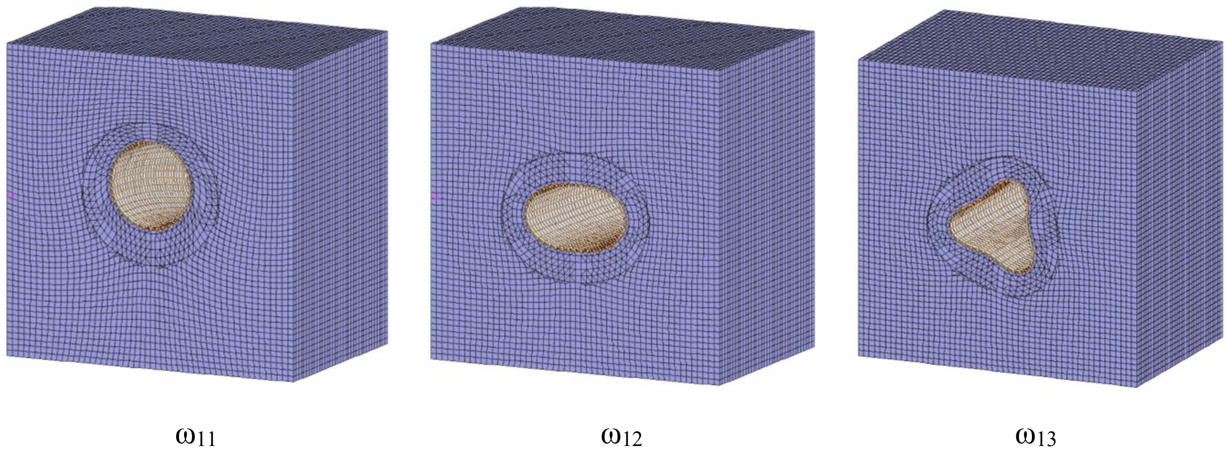


Figure 6. The waveforms for the pipeline section under consideration at n=1 in the soil mass (the size of the mass is 5.3×5.3 m in cross section).

4. Conclusion

1. The discrepancy between the frequencies of natural vibrations for the object of study determined by the analytical method and FEM does not exceed 10 %, and for the first 3 frequencies of the spectrum does not exceed 6 %, therefore, all the methods are applicable. The use of an analytical expression made it possible to obtain results an order of magnitude faster with the pastel coefficient than with the help of numerical soil modeling using volumetric elements.
2. When calculating the frequencies of natural vibrations by the FEM, the second method of setting ground conditions allows reducing the time of data input by 5–6 times with the same results.
3. Based on the analysis performed, it is recommended to use the analytical expression given in this paper when designing large-diameter pipeline transportation structures for the purpose of maximum productivity.

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