radiation of Tycho's supernova remnant. Moreover, if the stripes are indeed produced by the mirror instability then the superdiffusive particle transport should be considered as a plausible mechanism in the supernova remnants.

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ANALYTICAL TIME-DEPENDENT INSTANTANEOUS FREQUENCY THEORY OF CHARGED PARTICLE MOTION IN ELECTROMAGNETIC FIELDS IN SPACE

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Abstract: A theoretical analysis has been carried out by means of novel differential equation for charged particles motion in electromagnetic fields in space plasma.

Keywords: Space, plasma, charged particle, electromagnetic field.

Introduction

The Solar System and the Universe in whole are filled with space plasma. Plasmas occupy the magnetospheres of the Earth and other planets, the solar corona and solar wind, tails of comets, the inter-stellar and inter-galactic media. Mainly, such plasmas contain both electron and ion components. A theoretical analysis of charged particles motion in electromagnetic fields remains high-demand in space plasma and ion physics, as well as in many other scientific fields like vacuum electronics, mass spectrometry, and so on. Computer simulation is a powerful tool for space plasma investigation, although analytical approach reveals more detailed understanding of actual physical processes in charged particles ensembles in free space. A possible approach is considered comprehensively for further development of analytical method produced via both the time-dependent non-linear phase (NP) and instantaneous frequency (IF) concepts describing the charged particle motion. Previously, Hilbert transform (HT) and Gabor's complex analytical signal (AS) approach were applied to Newtonian single particle motion equations in a framework of the frequency partition. [1]

Recently, using the HT and AS platform, differential equations (DE) were derived which describe time-dependent NP and IF of charged particles motion in a specific complex space without preliminary partitioning of frequency [2]. Firstly, the instantaneous frequency concept developed methods and technique of vacuum electron or ion devices, e.g., mass spectrometers. Now, several novel solutions appeared as appropriate tools for the space objects analysis.

Basic theory

For plasma in free space, one needs to contribute nonlinear and space charge fields, specific boundary conditions, relativistic effects, complex spectral compositions of ions, and so on. Here, we start with much simpler task. Its main and novel feature is time-dependent instantaneous non-linear phase and frequency. Since radial and azimuthal components of a moving particle time-dependent complex radius-vector in the plane transversal to uniform magnetic field $\tilde{r}(t) = r(t) \cdot \exp[-i(\varphi - \varphi_0)]$ are coupled by HT, then $\tilde{r}(t)$ is an AS by definition, and thus its properties allow determination IF in each point of the particle trajectory in a limited volume. Within our approach, the $\tilde{r}(t) = r_0 \exp[-i\Phi(t)]$ is associated with the constant amplitude wave with an unwrapped NP $\Phi(t)$ that complies with the relation $\Phi(t) = (\varphi - \varphi_0) + i \cdot \ln \frac{r(t)}{r_0}$.

The spectral function $S(\tilde{\omega})$ of a complex potential field in free space $\tilde{U}(t) = \frac{1}{4\pi\varepsilon_0\varepsilon} \cdot \frac{q}{\tilde{r}(t)}$ generated by a moving charge q could be revealed by Fourier Transform (FT) of $\tilde{U}(t)$:

$$S(\tilde{\omega}) = \frac{r_0 q}{4\pi\varepsilon_0\varepsilon} \int_0^{\infty} dt \cdot e^{i\Phi - i\omega t}.$$

The largest $S(\tilde{\omega})$ value is achieved with constant phase $[\Phi(t) - \omega t]$. Therefore, the set of frequency values $(cod\omega)$ is defined by $(cod\omega) = \tilde{\omega}(t) = \frac{d\Phi(t)}{dt}$. In the case of a linear time-dependent phase, one obtains $\frac{d\Phi(t)}{dt} = \frac{d(\omega t)}{dt} = \omega = const$. For NP, the IF $\tilde{\omega}(t)$ is a *codomain* generalization of a traditional notion of static frequency of stationary periodic signals. Contrary to Fourier frequency, the IF value reflects local NP deviations. Therefore, signals in the detection circuitry could be locally represented by static frequency and a time-dependent dispersion around this frequency. In case of charged particles motion, complex IF $\tilde{\omega}(t) = \dot{\varphi}(t) + i \frac{\dot{r}(t)}{r(t)}$ contains both angular and radial frequencies. In other words, IF is determined by radius amplitude (length) dynamics, and both radial and azimuthal charged particle velocities:

$$\tilde{\omega}(t) = |\tilde{\omega}| \cdot \exp \cdot i(\arg \tilde{\omega} + 2\pi k), \quad k = 0, \pm 1, \pm 2, ..., |\tilde{\omega}| = \sqrt{\dot{\varphi}^2 + \left(\frac{\dot{r}}{r}\right)^2},$$

$$\arg \tilde{\omega} = \arctan \frac{\dot{r}}{r\dot{\varphi}}.$$

Using the formalism described above, one can transform Newtonian motion equations into the first order DE that present IF spectra of the electrical field, generated by volume currents $q\dot{\tilde{r}}$ [2]:

$$i\dot{\tilde{\omega}} + \tilde{\omega}^2 - \tilde{\omega}\omega_c + \frac{q\tilde{E}(t)}{m\tilde{r}} = 0. \tag{1}$$

In Eqn. (1) m denotes a particle mass value, ω_c is cyclotron frequency, $\tilde{E}(t)$ is electric field value on the trajectory. The Eqn. (1) can be represented in the following exponential form:

$$\tilde{\omega}(t) = \omega_0 \cdot \exp\left[i\Phi - i\omega_c t + \int_0^t \frac{\eta\tilde{E}(t)}{\dot{\tilde{r}}} dt \right], \ \ \omega_0 = -\dot{\varphi}_0 + i\frac{\dot{r}_0}{r_0},$$

$$\tilde{\omega} \cdot \tilde{r} = r_0\omega_0 \cdot \exp\left[-i\omega_c t + i\int_0^t \frac{\eta\tilde{E}(t)}{\tilde{\omega}\cdot\tilde{r}} dt \right]. \tag{2}$$

In another form, solution of Eqn. (1) for any arbitrary electric fields can be found via Laplace transform following the standard approach of its application to DE solution [2]:

$$\tilde{\omega} - \tilde{\omega}_0 e^{-i\omega_c t} - i \cdot \left[\tilde{\omega}^2 + \frac{\eta\tilde{E}(t)\cdot u_0(t)}{\tilde{r}(t)} \right] * e^{-i\omega_c t} = 0,$$

$$\tilde{r}(t) = \tilde{r}(0) + \frac{\dot{\tilde{r}}(0)}{i\omega_c}\left(1 - e^{-i\omega_c t}\right) + \frac{\eta}{i\omega_c}[\tilde{E}(t)\cdot u_0(t)] * \left(1 - e^{-i\omega_c t}\right), \tag{3}$$

$$\dot{\tilde{r}}(t) = \dot{\tilde{r}}(0)\cdot e^{-i\omega_c t} + \eta[\tilde{E}(t)\cdot u_0(t)] * e^{-i\omega_c t}.$$

In Eqn. (3) the asterisk means convolution integral procedure, $\tilde{r}(0), \dot{\tilde{r}}(0)$ are initial complex radius-vector and velocity respectively, and $u_0(t)$ is the boxcar function of electric field, in the form of Heaviside unit step function, e.g., $u_0(t) = 0, t < 0, u_0(t) = 1, t \geq 0$. In the case of a zero magnetic field, one can put $\omega_c = 0$ in Eqns. (1–3).

Phase resonance and space charge field

Equations and solutions (1–3) contain any electric field $\tilde{E}(t)$, so the opportunity of nonlinearities and space charge fields is given.

The trajectory parameters and the motion dynamics scenario depend on an electric field $\tilde{E}(t)$ by combined Eqns. (1–3). In free space we can consider plasma potential in the form of cylindrical harmonics, so we receive azimuthally periodical solutions of charged particles motion. Further, among the analytical solutions of Eqns. (1–3), we have obtained two particular ones for the following cases: a linear field $\tilde{E} = const \cdot \tilde{r}$, and effective acceleration/deceleration field $\tilde{E} \times \tilde{r} = 0$. Then, using the approach mentioned above (which has led to Eqn. (1)), we received the DE of the same structure for NP and IF of charged particle oscillations in (r, z) plane. Therefore, NP and IF of the charged particle motion in electromagnetic field have been described as two coupled 2D non-linear oscillators, (r, φ) and (r, z).

Conclusion

The time-dependent non-linear phase and instantaneous frequency concepts could be applied for investigation of charged particle motion in space plasma. In order to improve such applications one needs further expansion of the developed theory for multicomponent instantaneous frequency approach, dissipative losses, non-uniform magnetic field, relativistic case, and so on.

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