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NORDSTROM'S THEORY OF GRAVITY AS OPPOSED TO EINSTEIN'S THEORY

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Nordstrom's theory (the scalar theory of gravity) has a lot of merits. It was rejected on the basis of pre-quantum understanding of propagation of light, i. e. erroneously.

In connection with intention to test Einstein's theory in cosmos we would like to regenerate interest in Nordstrom's theory, that was rejected evidently erroneously.

In 1914 A. Einstein reported in Zurich: "Two theories, that comply with the requirement of equality of inert and gravitational masses, was proposed, namely the theory of Nordstrom and the theory of Einstein and Grossman. The first is more simple and more evident in respect to the theory of special relativity, namely it is based on the assumption that frames of reference may be chosen so, that light travels in vacuum at the constant velocity c (the principle of the constancy of velocity of light).

The theory of Einstein and Grossman is more complicated because it does away with the principle of the constancy of velocity of light and arrives then at the necessity of extension of the theory of special relativity… The choice between two theories may be done by reference to experiments because gravitational field must lead to deflection of light rays in agreement with the theory of Einstein and Grossman and in conflict with Nordstrom's theory".

The last sentence reflects the opinion in 1914 that light moves along isotropic geodesics, which in conformal flat space of Nordstrom's theory are straight. But the quantum theory has other concepts.

The quadratic form of a centrally symmetrical field of mass m in general is

$$ds^2 = e^{2\nu}(dx^0)^2 - e^{2\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2),$$

where ν and λ are independent of one another and depend on α and r;

$\alpha = Gm/c^2$,

G and c are the gravitation and the electro-dynamic constants.

As a result the circular frequency ω and the local velocity of light

c_L

depend on α and r:

$$\omega = \omega_0 e^{(-\nu)}, \qquad (1)$$

where

$\omega_0 = \lim_{r\to\infty} \omega$

(it is the "red shift" if $\nu<0$);

$$c_L = ce^{\nu-\lambda} = c_L e^{\chi} \tag{2}$$

In the quantum mechanics the most probable path of photons is that, where eikonal

$$\psi = \int (k_i dx^i - \omega dt) \tag{3}$$

(i = 1, 2, 3) is stationary. The wave vector

$$k_i$$

and ω are related by the equations

$$\dot{x}^i = \partial\omega / \partial k_i \qquad \dot{k}_i = -\partial\omega / \partial x^i \tag{4}$$

Then we reason as follows.

1). Without loss of generality we can deal with the ray that lies in the plane $z = 0$, intersects the axis x orthogonal in the point R and is defined by the equation

$$x = R - \alpha g(y) \tag{5}$$

($g(y)$ is the required function) .

2). Since departure of the ray from straight line $\{x = R;\ z = 0\}$ is very small, we can assume that

$$k_y \approx -(\omega / c_L) = -(\omega_0 / c)e^{-\nu-\chi} \tag{6}$$

The sign « – » depends upon the fact that

$$k_i$$

is parallel to

$$\dot{x}_i$$

and anti-parallel to

$$\dot{x}^i$$

3). The magnitude

$$k_x$$

that vanishes at $y = 0$, depends on refraction that is defined by (2), and on gravitation of photons defined by (1) and (4).

Owing to refraction the ray bends toward mass through the angle

$$\gamma^{(\chi)}(y) = -x'_y \approx \int_{y=0}^{y} [(c_L),_x / c_L] dy = \int_{y=0}^{y} \chi,_r (x/r) dy \approx R \int_{y=0}^{y} (\chi,_r / r) dy.$$

Since

$$\gamma^{(\chi)} = -k_x^{(\chi)} / k_{y,}$$

then

$$k_x^{(\chi)} = (\omega_0 / c) R \int_{y=0}^{y} e^{-\nu-\chi} (\chi,_r / r) dy.$$

Owing to (1) and (4)

$$\dot{k}_x^{(\nu)} = -\partial\omega / \partial x = \omega_0 e^{-\nu} \nu,_r (x/r) \approx \omega_0 \,\mathrm{R}e^{-\nu}\, \nu,_r / r$$

and

$$k_x^{(\nu)} = \int \dot{k}_x^{(\nu)}\, dt = \int_{y=0}^{y} \dot{k}_x^{(\nu)}\, dy / c_L \approx (\omega_0 / c) R \int_{y=0}^{y} e^{-\nu-\chi} (\nu,_r / r) dy.$$

We can assume that ν and χ, instead of ν and λ, are independent of one another. Related to them small effects must be added. Then

$$k_x = k_x^{(\chi)} + k_x^{(\nu)} \approx (\omega_0 / c) R \int_{y=0}^{y} e^{-\nu-\chi} [(\nu,_r + \chi,_r) / r] dy. \tag{7}$$

4). Taking into account (5), (6), (7) and

$$dt = \sqrt{1 + \alpha^2 (g'_y)^2}\, dy / c_L$$

we find the increment of eikonal (3)

$$d\psi = k_x dx + k_y dy - \omega dt = F dy$$

,form the equation of motion

$$F,_g - [F,_{g'_y}],_y = 0$$

and deduce the path that corresponds to stationarity of eikonal ψ.
The calculations are here.

a) In Nordstrom's theory $\nu = \lambda = -\alpha / r$, whence $\chi = 0$;

$$\omega = \omega_0 e^{\alpha / r}; \qquad c_L = c; \qquad k_y = -(\omega_0 / c) e^{\alpha / r}$$

Accurate to the second order of α,

$$k_x \approx (\omega_0 / c) \alpha \mathrm{Re}^{\alpha / r} \int_{y=0}^{y} dy / r^3 \approx (\omega_0 / c) \alpha y / Rr.$$

Accurate to the third order of α

$$F = -(\omega_0 / c)(\alpha^2 g'_y\, y / Rr + e^{\alpha / r} + e^{\alpha / r} \sqrt{1 + \alpha^2 (g'_y)^2})$$

$$F,_g = -(\omega_0 / c) 2 (e^{\alpha / r}),_r\, r'_x\, x'_g \approx -2 (\omega_0 / c) \alpha^2 R / r^3$$

$$[F,_{g'_y}],_y \approx [-(\omega_0 / c) \alpha^2 (y / Rr + g'_y)],_y = -(\omega_0 / c) \alpha^2 (R / r^3 + g''_{yy});$$

The equation of motion is

$$g''_{yy} = R / r^3$$

whence in view of $g(0) = 0$ $\qquad g = r / R - 1;$

$$\lim_{y \to \infty} (-x'_y) = \alpha / R$$

and the complete angle of curvature of light ray
(at $-\infty < y < \infty$) is $2\alpha / R$ ***(other then 0, that was considered in 1914).***

b) In Einstein's theory (Schwarzschild's metrics} $\nu = -\lambda = -\alpha / r$, then $\chi = -2\alpha / r$;

$$\omega = \omega_0 e^{\alpha / r}; \qquad c_L = c e^{-2\alpha / r}; \qquad k_y = -(\omega_0 / c) e^{3\alpha / r};$$

$$k_x = (\omega_0 / c) R \int_{y=0}^{y} e^{3\alpha / r} (3\alpha / r^3) dy \approx 3\alpha (\omega_0 / c) y / Rr;$$

$$F = -(\omega_0 / c)[3\alpha^2 g'_y \; y / Rr + e^{3\alpha / r} (1 + \sqrt{1 + \alpha^2 (g'_y)^2});$$

$$F,_g = -6(\omega_0 / c)\alpha^2 R / r^3 \qquad [F,_{g'_y}],_y = -(\omega_0 / c)\alpha^2 (3R / r^3 + g''_{yy});$$

The equation of motion is $g'' = 3R/r$ whence $g = 3(r/R - 1),$

$$\lim_{y\to\infty} (-x'_y) = 3\alpha / R$$

and the complete angle of light rays curvature is $6\alpha / R$ **(*other then 4α/R*)**

c). Einstein's calculations after 1913 corresponds to $\nu = 0,\ \chi = -2\alpha/r$ (they are free from attraction of photons to mass). The angle of light ray curvature is $4\alpha/R$. But it isn't Schwarzschild's metrics .

CONCLUSION. Results of Eddington's expedition in 1919 are well-known. Probably they must be analyzed. There were "photographic plates from Sobral", that gave the angle $\approx 2.13\ \alpha/R$, but "the small weight was assigned to this result".

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